Comparative analysis of Large N_c QCD and Quark model approaches to baryons

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Introduction

- Description of baryons
  - Large N_c QCD: $N_c \to \infty$, model-independent

    Group theory

    G. 't Hooft, Nucl. Phys. **72**, 461 (1974); E. Witten, Nucl. Phys. B **160**, 57 (1979)

    R. Dashen and A.V. Manohar, Phys. Lett. B **315**, 425 (1993); **315**, 438 (1993)

    E. E. Jenkins, Phys. Rev. D **54**, 4515 (1996)
  - Quark Model: model-dependent

    Hamiltonian dynamics
  - Compatibility of both approaches
- Light and heavy baryons

Light baryons

C. Semay, F. Buisseret, N. Matagne and Fl. Stancu,
Phys. Rev. D **75**, 096001 (2007) [hep-ph/0702075].

C. Semay, F. Buisseret and Fl. Stancu,
Phys. Rev. D **76**, 116005 (2007) [arxiv:0708.3291].

Large N_c expansion (I)

- When $N_c \to \infty$, exact $SU(2N_f)$ symmetry
  - Baryons: N_c quarks
- Large but finite N_c
  - $SU(2N_f)$ broken, $1/N_c$ expansion
- Mass formula $M = \sum_i c_i \hat{O}_i$
  - Some operators

    $\hat{O}_1 = N_c \mathbf{1}$ $\qquad \hat{O}_2 = \frac{1}{N_c} \ell^i S^i$ $\qquad \hat{O}_4 = \frac{1}{N_c} S^i S^i$
  - $1/N_c^2$ neglected

    Quark model ?
  - c_i to be fitted. Contain the QCD dynamics.

Large N_c expansion (II)

- Excited baryons
  - Labelled by an integer K, quantum of excitation

    Harmonic oscillator picture

    $K = 0$ for ground state baryons

    $P = (-1)^K$

    $$c_i = c_i(K)$$

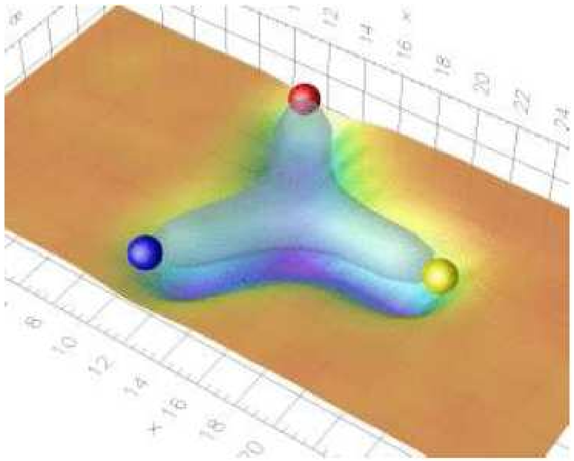
- Ground state baryons (N and Δ)

$$M = c_1 N_c 1 + c_4 \frac{S^2}{N_c} + O(N_c^{-3})$$

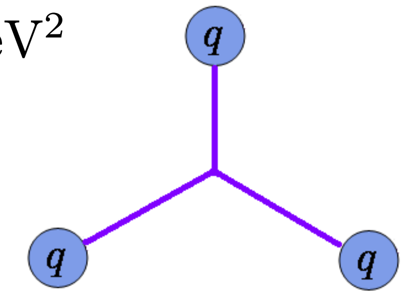
Quark model for baryons (I)

- Dominant order: $H = \sum_i \sqrt{\vec{p}_i^{\,2} + m_i^2} + a|\vec{x}_i - \vec{x}_Y|$
  - Spinless Salpeter Hamiltonian
  - Y-junction as long-range potential
- Lattice QCD

F. Bissey *et al.*, Phys. Rev. D **76**, 114512 (2007) [hep-lat/0606016]



$a \approx 0.16 - 0.2\ \mathrm{GeV}^2$



Quark model for baryons (II)

- Light quarks $H = \sum_i \sqrt{\vec{p}_i^{\,2}} + a|\vec{x}_i - \vec{R}|$
  - Toricelli point $\approx$ Center of mass

B. Silvestre-Brac *et al*., Eur. Phys. J. C **32**, 385 (2003) [hep-ph/0309247]

- How to get analytical relations ?
  - Auxiliary field technique
    $H \to H(\mu_j, \nu_j) = \sum_j \frac{\vec{p}_j^{\,2}}{2\mu_j} + \frac{a^2(\vec{x}_j - \vec{R})^2}{2\nu_j} + \frac{\mu_j}{2} + \frac{\nu_j}{2}$
  - Elimination

$$\delta_{\mu_k} H(\mu_j, \nu_j) = 0, \quad \mu_k = \sqrt{\vec{p}_k^{\,2}}$$

Kinetic energy

$$\delta_{\nu_k} H(\mu_j, \nu_j) = 0, \quad \nu_k = a|\vec{x}_k - \vec{R}|$$

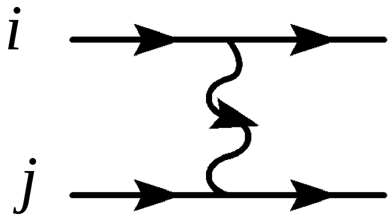
String energy

  - If seen as numbers… Just a harmonic oscillator

Mass formula (I)

- Y-junction $\begin{cases} M_0 = 6\mu_0 = \sqrt{2\pi a(K+3)} \\ K = 2(n_1+n_2)+(\ell_1+\ell_2) \end{cases}$

- Short distances: One gluon exchange



$$\Longrightarrow \quad V_{ij}(r_{ij}) = -\frac{2}{3}\frac{\alpha_s}{r_{ij}} + \text{ corrections}$$

  - $\alpha_s \approx 0.2 - 0.4$ remains small once confinement is separated
  - In perturbation, $\Delta M_{oge} = -\frac{2\alpha_s}{3}\sum_{i<j}\left\langle \frac{1}{|\vec{x}_i - \vec{x}_j|}\right\rangle$

$$\approx -\frac{\pi\alpha_s a}{3\sqrt{3}\mu_0}$$

Mass formula (II)

- Self-energy Yu. A. Simonov, Phys. Lett. B **515**, 137 (2001)

$$\Delta M_{qse} = -\frac{f\,a}{\pi}\sum_i \frac{\eta(m_i/\delta)}{2\mu_i}$$

$$f \in [3,4], \quad \delta \approx 1\ \mathrm{GeV}$$

$$\mu_i = \langle\sqrt{\vec{p}_i^{\,2} + m_i^2}\rangle$$

  - Light quarks

$$\Delta M_{qse} = -\frac{f\,a}{4\mu_0}$$

- Squared mass

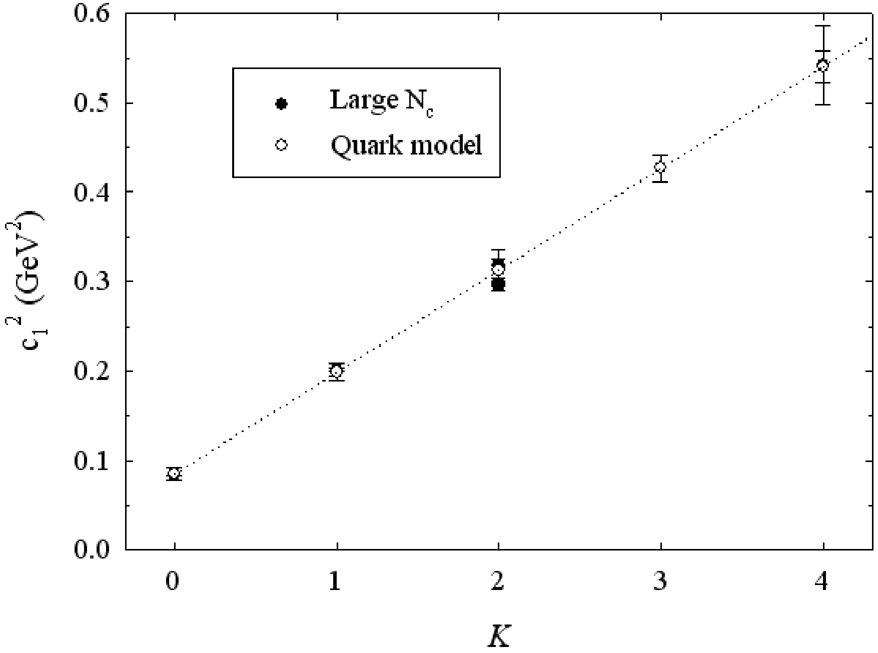
$$M^2 \approx 2\pi a(K+3) - \frac{4}{\sqrt{3}}\alpha_s - \frac{12}{(2+\sqrt{3})} f a$$

K → Excitation number

First comparison

- Spin-independent part
  - Large N_c: $M^2 = c_1^2 \, N_c^2$
  - Quark model: $M^2 = \beta \, K + \gamma$

Do we have $c_1^2 = (\beta K + \gamma)/N_c^2$?



OK for

a = 0.163 GeV²

α_s = 0.4

f = 4.0

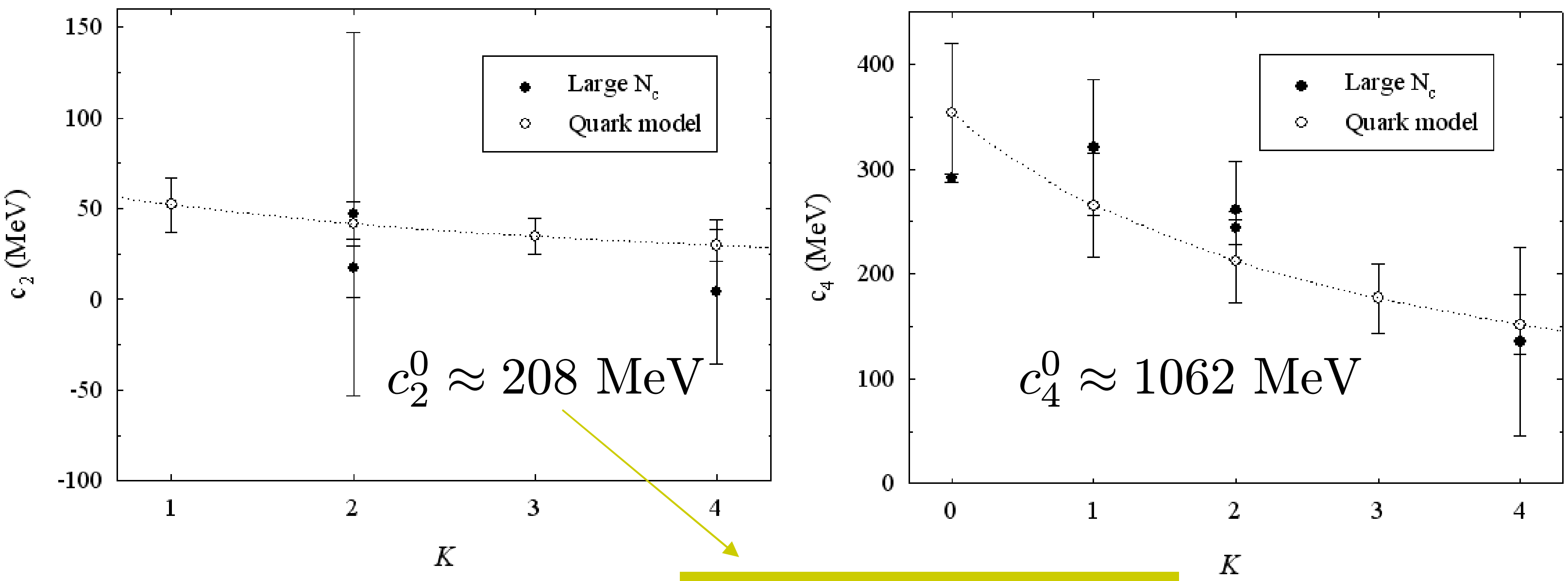
Standard values

Spin-dependent terms

- Corrections in $1/\mu_0{}^2$ Yu. A. Simonov, hep-ph/9911237

$$c_2 = \frac{c_2^0}{K+3}, \quad c_4 = \frac{c_4^0}{K+3}$$

- Expected:



Small spin-orbit term

Large N$_c$ and strangeness

- $SU(2N_f)$ symmetry with three flavors (u, d, s)
  - Mass formula $M = \sum_i c_i \hat{O}_i + \sum_j d_j \hat{B}_j$

    ↓ $SU(3)$ breaking
  - Strange quarks contribution $n_s \Delta M_s = \sum_j d_j \hat{B}_j$
- Classification number K assumed

Quark model with strangeness

- Analytic results at order m_s^2
- Mass formula $M = M_q + n_s \Theta \frac{m_s^2}{\mu_0}$

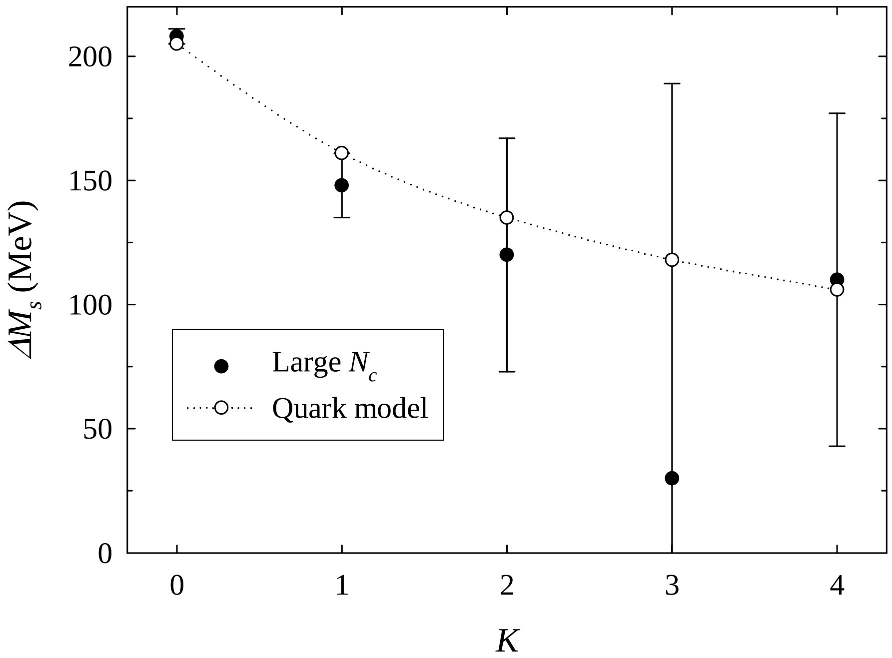
Strange quarks

Nonstrange baryon

$\Theta = \Theta(K, \ldots)$

- K is still a good quantum number

OK with m_s = 243 MeV



Charm and bottom baryons

C. Semay, F. Buisseret, and Fl. Stancu,
Phys. Rev. D **78**, 076003 (2008) [arXiv:0808.3349].

Experimental data

- In 2007-2008: New heavy baryons

One c quark	One b quark	
$\Lambda_c = 2286.46 \pm 0.14$ MeV,	$\Lambda_b = 5620.2 \pm 1.6$ MeV,	Nonstrange
$\Sigma_c = 2453.56 \pm 0.16$ MeV,	$\Sigma_b = 5811.5 \pm 1.7$ MeV,	
$\Sigma_c^* = 2518.0 \pm 0.8$ MeV,	$\Sigma_b^* = 5832.7 \pm 1.8$ MeV,	
$\Xi_c = 2469.5 \pm 0.3$ MeV,	$\Xi_b = 5792.9 \pm 3.0$ MeV.	$n_s = 1$
$\Xi_c' = 2576.9 \pm 2.1$ MeV,		
$\Xi_c^* = 2646.4 \pm 0.9$ MeV,		
$\Omega_c = 2697.5 \pm 2.6$ MeV,	$\Omega_b = 6165 \pm 23$ MeV	$n_s = 2$
$\Omega_c^* = 2768.3 \pm 3.0$ MeV.		

Large N_c and heavy quarks

- Heavy baryon
  - $N_c - 1$ light quarks, $1/N_c$ expansion
  - One heavy quark: $1/m_Q$ expansion
- Mass formula $M = m_Q + \Lambda_{qq} + \lambda_q + \lambda_Q$

$$\begin{cases} \Lambda_{qq} = c_0 N_c + \frac{c_2}{N_c} J_{qq}^2 \\ \lambda_q = \frac{c_0'}{2m_Q} + \frac{c_2'}{2N_c^2 m_Q} J_{qq}^2 \\ m_Q \text{ and } \lambda_Q = 2\frac{c_2''}{N_c m_Q} \vec{J}_{qq} \cdot \vec{J}_Q \end{cases}$$

Light quarks (first two lines)

Heavy quark (third line)

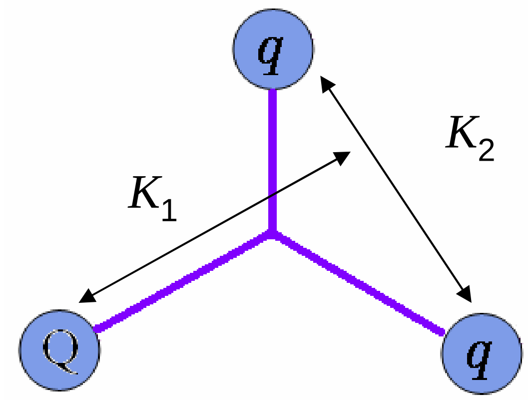
Quark model

- Mass formula with Y-junction
  - Auxiliary fields + 1/ m_Q expansion

$$M_{qqQ} = m_Q + 4\mu_1 + \frac{\pi\, a}{12 m_Q} G(K_1, K_2),$$

$$\mu_1 = \sqrt{\frac{\pi a(K_1+K_2+3)}{12}},$$

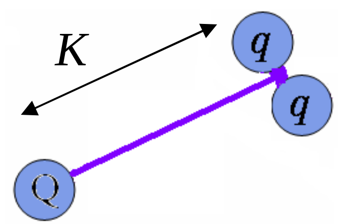
$$G(K_1, K_2) = \sqrt{2K_2+3}\left(\sqrt{2(K_1+K_2+3)} - \sqrt{2K_2+3}\right)$$



Minimal mass for $K_2 = 0$, $K_1 = K$

Heavy quark – diquark picture for excited states

Explanation of *K* introduced in Large N_c QCD



Back to Regge trajectories

- Heavy baryons

$$(M - m_Q)^2 \approx \frac{4\pi a}{3} K \approx 1.3\pi a\, K$$

  - Smaller slope than light baryons

$$M^2 \approx 2\pi a\, K$$

- Mesons
  - Light $q\bar{q}$ $\quad M^2 \approx 2\pi a\, K$
  - Heavy $Q\bar{q}$ $\quad (M - m_Q)^2 \approx \pi a\, K$

Additional terms

- OGE
  - $\alpha_s(qq) \neq \alpha_s(Qq)$
  - Simple choice $\alpha_s(Qq) = 0.7\,\alpha_s(qq)$

    C. Semay and B. Silvestre-Brac, Phys. Rev. D **52**, 6553 (1995)
- QSE for heavy quark $\Delta M_{qse} \propto m_Q^{-3} \approx 0$
- Strangeness
  - Power expansion in m_s^2

$$\Delta M_s = n_s \Theta(K)\, \frac{m_s^2}{\mu_1}$$

Comparison (I)

$$M = m_Q + c_0\, N_c + \frac{c_2}{N_c} J^2_{qq} + \frac{c_0'}{2m_Q} + \frac{c_2'}{2N_c^2 m_Q} J^2_{qq} + \frac{2c_2''}{N_c m_Q} \vec{J}_{qq} \cdot \vec{J}_Q$$

$$M_{qqQ} = m_Q + 4\mu_1 + \cdots + \frac{a}{2m_Q} G(K, K_2 = 0) + \ldots$$

- Matching between the coefficients
  - Spin effects neglected
- Quark model parameters fixed from light baryons
- Heavy quark masses fitted on $\Lambda_c,\ \Lambda_b$ $(J^2_{qq} = 0)$

Comparison (II)

- $K = 0$

	Large N_c (MeV)	Quark Model (MeV)	δ (%)
m_c	1315	1252	4.7
m_b	4642	4612	0.6
c_0	324	333	2.7
c'_0	96	91	5.2
ΔM_s	206	170	17.5

- Satisfactory agreement

Conclusion

Summary

- Compatibility between Large N_c mass formula and quark model for light and heavy baryons
  - Support for the quark model assumptions
  - Physical interpretation of the coefficients in Large N_c mass formula
- Dynamical origin of the classification number K understood from quark model
  - Light baryons: total excitation number
  - Heavy baryons: heavy quark- light diquark picture

Outlook

- Future predictions in the heavy baryon sector
  - $K = 1$ ⟶ 5 coefficients in the Large N_c formula
    - Can be fitted on experiment BUT…
  - Quark model parameters fitted on ground state heavy baryons
    - Prediction of mass formula coefficients for excited baryons ($K = 1$)

- Masses of excited baryons from a combined Large N_c - Quark model approach, without fit.