Excited QCD, Zakopane
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Finite Temperature QCD with matter via Holography

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Introduction

- AdS−CFT or Holography

  3+1 dim. QFT (large Nc) with running coupling constants

  <−> 4+1 dim. Effective Gravity description

  Ex) 4 d QFT <−> 5d Gravity

(on "boundary" D−brane, open string)	(in "bulk", close string)
with $\beta(g^2)=0$ (conf. inv.)	in Anti−deSitter Space
with $\beta(g^2)\to 0$ (asym. freedom)	in asymptotic AdS
QCD	in ??

- AdS−CFT ： useful for strongly interacting QFT

  Ex) (Excited) QCD, Heavy Ion Phys., Condensed Matter Systems, etc.

  (*) AdS/CFT duality applied to QCD is called as AdS/QCD

- D branes

  Low Energy Dynamics of Nc D3 branes (+ others, etc.)

  −> 3+1 dim. SU(Nc) YM theory (DBI action) + (matter etc.)

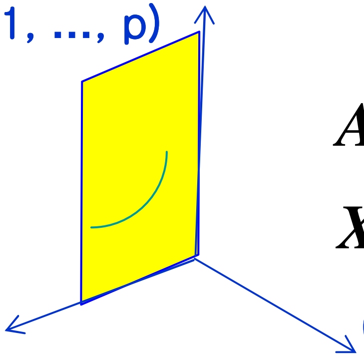
  Spacetime Geometry by large Nc D3 branes (+ others, etc.)

  −> AdS5 Space (etc. …)

D branes - Low Energy Dynamics -

* **Dp-branes :** p-dim. extended objects where open strings can end

Neumann boundary condition $\partial_\sigma X^\mu = 0$ for $\mu = 0, \ldots, p$,



$$A_\mu, \mu = 0, 1, \ldots, p$$

$$X_I, I = p+1, \ldots, q$$

P = 0, 2, 4, 6, 8 Type II A

−1, 1, 3, 5, 7, 9 Type II B

Dirichlet boundary conditior $X^\mu = 0$ for $\mu = p+1, \ldots, 9$

(Dp brane sitting at $x_{p+1} = \cdots = x_9 = 0$)

* Low Energy Dynamics of # Nc D3 branes

-> 3+1 dim. N=4 SUSY SU(Nc) YM Theories

$$L = -\frac{1}{4g_{YM}^2} Tr(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D_\mu \Phi^\Sigma D^\mu \Phi^I + \sum_{I,J} [\Phi^I, \Phi^J]^2) + Fermions$$

A_μ, Φ_I, Ψ :

1, ..., 6

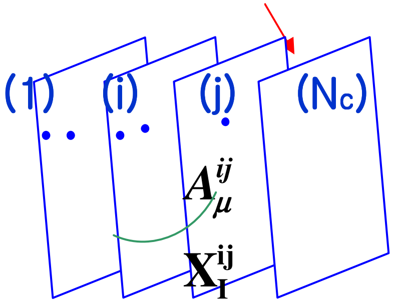
0, 1, ..., 3

: **Nc×Nc matirices, adj. repn. of U(Nc)**

Conformal symmetry : SO(4,2)

R-symmetry : SU(4) = SO(6)

#Dp-branes = Nc

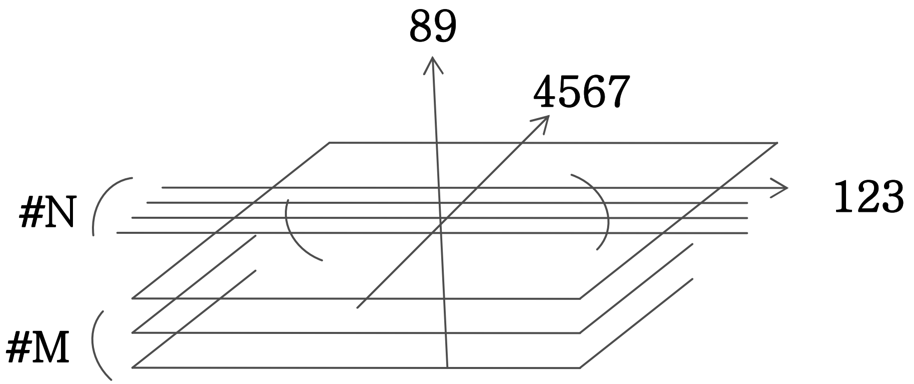


$$4\pi g_s = g_{YM}^2$$

Dp – Dp´System

Ex) D3-D7 : N=2 SUSY SU(Nc) Yang-MIlls

		0	1	2	3	4	5	6	7	8	9
#N	D3	—	—	—	—	X	X	X	X	X	X
#M	D7	—	—	—	—	—	—	—	—	X	X



↓ Low energy dynamics

N=2 SYM With #M

Still far from QCD !

Strings

CPX

3-3 : Aμ, Φ, λ, χ

0,1,2,3

(N=2 Vector multiplet)

Adjoint representation

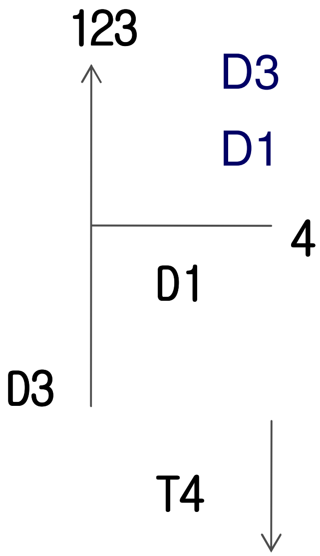
3-7 : Q, $\tilde{Q}$, ψ, $\tilde{4}$

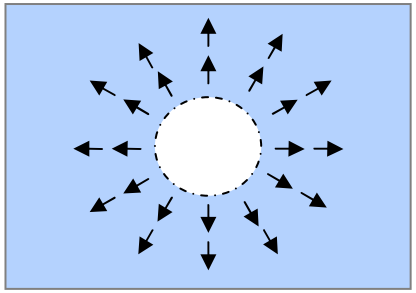
(N=2 hyper multiplet)

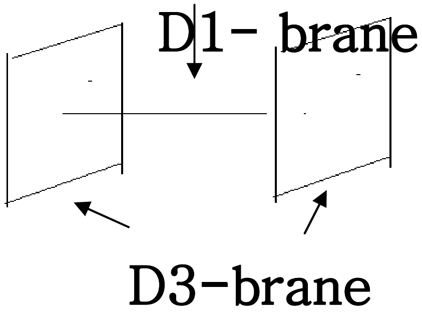
Matter in Fundamental

Ex
)magnetic monopole

	0	1	2	3	4	5	6	7	8	9
D3	—	—	—	—	X	X	X	X	X	X
D1	—	X	X	X	—	X	X	X	X	X

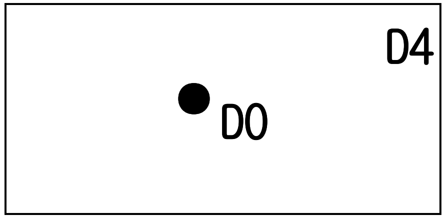



Field Theory



D-branes

instantons



D branes – Spacetime Geometry

Dp-brane solution in Supergravity

$$S = \int d^D x \sqrt{-g} \left[e^{-\phi} \{ R + \nabla_\mu \phi \nabla^\mu \phi \} - \frac{e^{m\phi}}{2n!} H_{\mu_1 \cdots \mu_n} H^{\mu_1 \cdots \mu_n} \right]$$

($m = 0$ for D-brane) (string frame)

$$ds^2 = f_p^{-1/2}(-dt^2 + dx_1^2 + \cdots + dx_p^2) + f_p^{1/2}(dx_{p+1}^2 + \cdots + dx_9^2) \ ,$$

$$e^{-2\phi} = f_p^{\frac{p-3}{2}} \ ,$$

$$A_{0 \cdots p} = -\frac{1}{2}(f_p^{-1} - 1) \ , \quad f_p = 1 + n c_p^{10}/r^{7-p}$$

(harmonic function)

Ex) p=3, near horizon limit : AdS x S5 geometry

- Bulk metric solution for D3-brane

$$ds^2 = f^{-1/2} dx_{||}^2 + f^{1/2}(dr^2 + r^2 d\Omega_5^2)$$

with

$$f = 1 + \frac{4\pi g N \alpha'^2}{r^4}$$

- in the Maldacena limit

$$\alpha' \to 0 \ , \qquad U \equiv \frac{r}{\alpha'} = \text{fixed}$$

reduces to the metric of AdS5 x S5

$$ds^2 = \alpha' \left[\underbrace{\frac{U^2}{\sqrt{4\pi g N}} dx_{||}^2 + \sqrt{4\pi g N} \frac{dU^2}{U^2}}_{\text{AdS5}} + \underbrace{\sqrt{4\pi g N} d\Omega_5^2}_{\text{S5}} \right]$$

the radius of S_5 = the radius of AdS_5 ($R_{sph}^2/\alpha' = \sqrt{4\pi g N}$)

- For $gN \gg 1$, we can trust this supergravity solution

Anti-de Sitter (AdS) Geometry : AdS_(p+2)

Gloabal Geometry as Hyperboloid

$$X_0^2 + X_{p+2}^2 - \sum^{p+1} X_i^2 = R^2$$

Isometry : SO(p+1,2)

$$\mathrm{d}s^2 = R^2(-\cosh^2\rho\,\mathrm{d}\tau^2 + \mathrm{d}\rho^2 + \sinh^2\rho\,\mathrm{d}\Omega^2)$$

Metric (Poincare Patch)

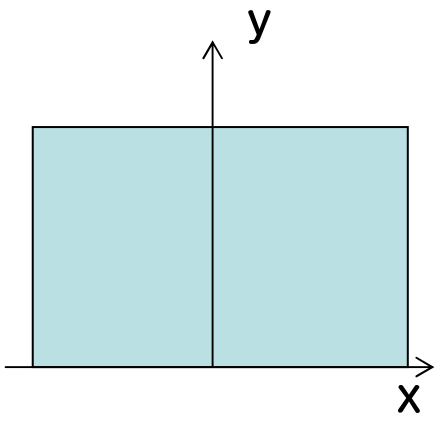
$$\mathrm{d}s^2 = R^2\left(\frac{\mathrm{d}u^2}{u^2} + u^2(-\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2)\right)$$

Euclidean AdS

$$\mathrm{d}s_{\mathrm{E}}^2 = R^2(\cosh^2\rho\,\mathrm{d}\tau_{\mathrm{E}}^2 + \mathrm{d}\rho^2 + \sinh^2\rho\,\mathrm{d}\Omega_p^2)$$

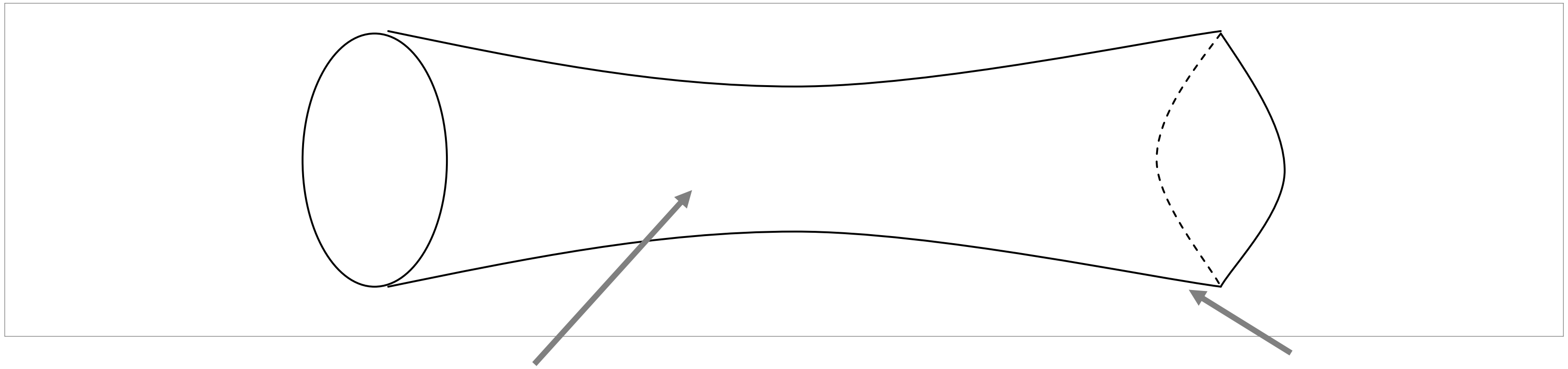
$$= R^2\left(\frac{\mathrm{d}u^2}{u^2} + u^2(\mathrm{d}t_{\mathrm{E}}^2 + \mathrm{d}\boldsymbol{x}^2)\right).$$

$$\mathrm{d}s^2 = R^2\left(\frac{\mathrm{d}y^2 + \mathrm{d}x_1^2 + \cdots + \mathrm{d}x_{p+1}^2}{y^2}\right) \quad u = 1/y$$



Contents

AdS/CFT – Holography Principle



AdS(p+ 2)

string Theory
(SUGRA, classical]

closed string

↔

Minkowski(p+ 1)

Gauge Theory [CFT]
(Quantum)

open string

Ex) AdS – QCD ?

AdS/CFT (large N)

– Gravity on the 5D bulk SUGRA on AdS5 x S5	– QFT on the 4D boundary N=4 SU(Nc) SYM
– Isometries of AdS x S5 SO(4,2) x SO(6)	– Conformal x R-symmetry SO(4,2) x SO(6)
– the classical gravity theory	– strongly coupled QFT

$$\boxed{4\pi g_s N = \frac{R^4}{\alpha'^2} = \lambda = g_{YM}^2 N}$$

Extension of the AdS/CFT

- the gravity theory on the asymptotically AdS space
  -> modified boundary quantum field theory
  (nonconformal, less SUSY, etc.) QCD ?

- the gravity theory on the black hole background
  -> corresponds to the finite temperature field theory

QFT with Asymptotic AdS SUGRA Duals

- N=1* Polchinski & Strassler,hep-th/0003136
- Cascading Gauge Theory Klebanov & Strassler, JHEP 2000
- D3-D7 system Kruczenski ,Mateos, Myers, Winters JHEP 2004
- D4-D8 system Sakai & Sugimoto 2005
  -> closely related to QCD

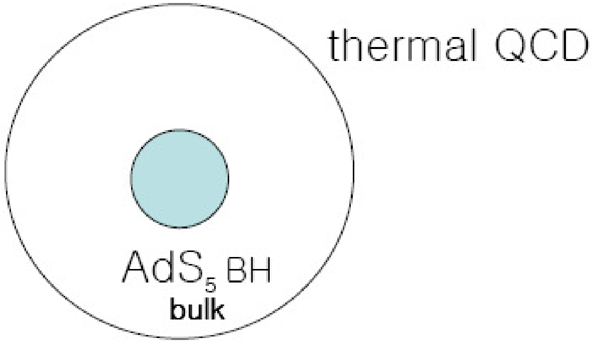
etc.

AdS-CFT at finite T

E. Witten (1998)

$$ds_5^2 = \frac{1}{z^2}\left(f^2(z)dt^2 - (dx^i)^2 - \frac{1}{f^2(z)}dz^2\right),$$

$$f^2(z) = 1 - (\frac{z}{z_T})^4 \qquad T = \frac{1}{\pi z_T}$$



- Can be applied to strongly coupled finite temperature YM
- Expt/Obser - RHIC, LHC, Nuetron (quark) stars
- Hawking-Page transition in the hard- & soft-wall models,

Herzog [hep-th/0608151

Hawking-Page transition

Witten ‘98

- Low temperature: no stable AdS black hole. thermal AdS background (Euclideanized zero temp. background)
- High temperature: AdS black hole

“confinement” phase ⟷ “de-confinement” phase

Thermal AdS ⟷ AdS-BH

Hawking-Page transition

Transition of bulk geometry at the same $\beta(=1/T)$.

AdS/CFT Dictionary

Witten 98; Gubser,Klebanov,Polyakov 98

Partition function
(semi-classial)

$$Z = \int_{\phi_0} \mathcal{D}\phi \exp\left(-S[\phi, g_{\mu\nu}]\right)$$

$$\phi_0(x) = \phi(x, z=0)$$

boundary value of
the bulk field ϕ

Generating functional for
the boundary operators $\mathcal{O}$

$$Z = \left\langle \exp \int d^d x \phi_0 \mathcal{O} \right\rangle$$
$$= \int \mathcal{D}[\Phi] \exp\{iS_4 + i \int \phi_0(x)\mathcal{O}\}$$

ϕ_0 : the source of the
boundary operator $\mathcal{O}$

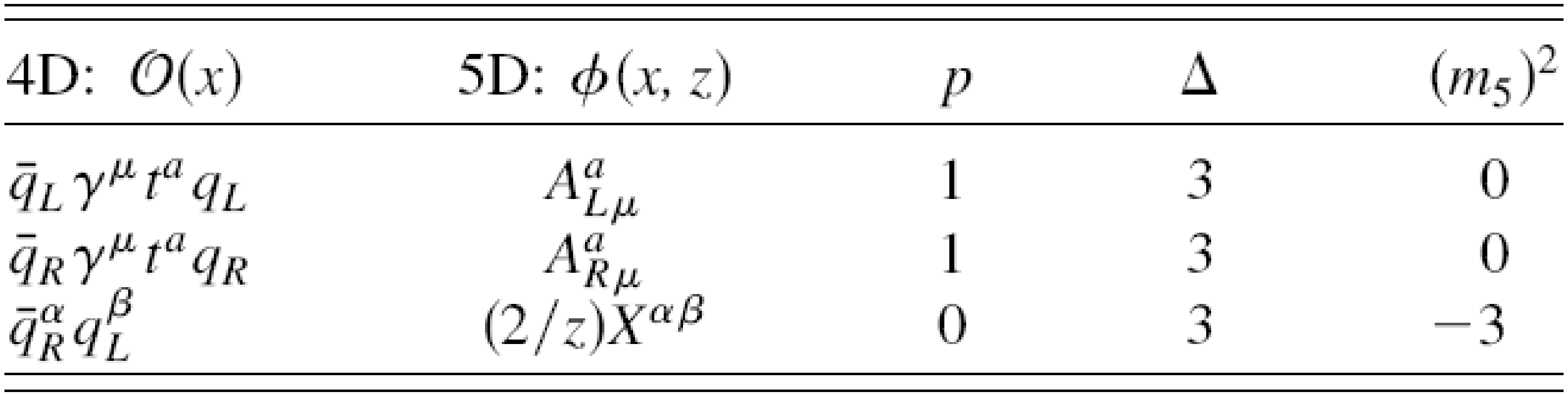
- 5D bulk field ϕ ←→ Operator $\mathcal{O}$
  5D mass ←→ [Operator]
- 5D gauge symmetry ←→ Current conservation
- small z ←→ Large Q
- Confinement ←→ (IR) cutoff z_m
- Kaluza-Klein states ←→ Excited, Resonant spectrum

$\mathcal{O}$(Operator in QFT) <-> ϕ(p-form Field in 5D)

Conformal dimension : Δ mass (squared): m_5^2

$$(\Delta - p)(\Delta + p - 4) = m_5^2$$

Ex)



4D: $\mathcal{O}(x)$	5D: $\phi(x, z)$	p	Δ	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3
$\langle \mathrm{Tr} G^2 \rangle$Gluon cond .	dilaton	0	4	0
$\bar{q}_R q_L$ Chiral cond.	scalar	0	3	−3
$\bar{q}_L \gamma^\mu q_L$ $\bar{q}_R \gamma^\mu q_R$ baryon density	vector w/ U(1)	1	3	0

the fluctuation field ϕ on the bulk space corresponds to a source for the QCD Operator $\mathcal{O}$.

AdS/QCD

Goal : model the spectrum and interactions of the light hadronic resonances in QCD based on the AdS/CFT correspondence

explaing Chiral Symmetry Breaking, Hidden local symmetry,

Large-N, Weinberg sum rules, etc.

Spectrum, Dynamics, etc.

Approaches :

Top-down Approach : rooted in string theory

Find the brane config. giving the gravity dual of QCD

Bottom-up Approach : phenomenological

Light-Front : radial direction of AdS <-> Parton momenta

(Brodsky, de Teramond, 2006)

Top-Down Approach

Observation :

- Nc of D3 branes
  AdS5 x S5 <-> N=4 SUSY YM
- Nc of D3 / orbifold, etc.
  AdS5 x X <-> N=2, 1 YM
- Nc of D3 + M of D7 (Meyers,…)
- Nc of D4 + M of D8 (Sakai-Sugimoto)

D3-D7 system

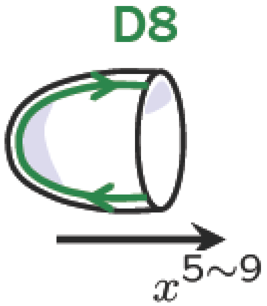
Kruczenski ,Mateos, Myers, Winters JHEP 2004

- YM theory: N=2 large-N_c SYM with quarks
- Flavor branes: N_f D7-branes
- Flavor symmetry: U(N_f)
- Quarks are massive (in general): m_q
- Probe approximation (N_c>>N_f)

  No back reaction to the bulk gometry from the flavor branes. (~ quenched approx.)
- Free energy ~ Flavor-brane action

D4–D8 system (Sakai–Sugimoto Model)

Sakai–Sugimoto, 2004



- Type IIA in Nc D4 Background (Witten)
  + Nf Probe D8 Branes along (x,z) x S4 (Nc >> Nf)
  as dual description to 4D QCD w/ Nf m=0 quarks
- Topology of the background : R(1,3) x R2 x S4
- Glueballs from closed strings
- Mesons from the open strings on D8

- The Effective Action : 5D U(Nf) YM CS theory

$$S_{\text{5dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

Bottom-Up Approach to AdS/QCD

- Introduce the contents (fields, etc.) as needed based on the AdS/CFT principle
- phenomenological

  Kaluza-Klein modes – radial excitations of hadrons identified by the symmetry properties of the modes

- Hard Wall Model

  Introduce IR brane for confinement

- Soft Wall Model – dilaton

Bottom-Up Hard Wall model

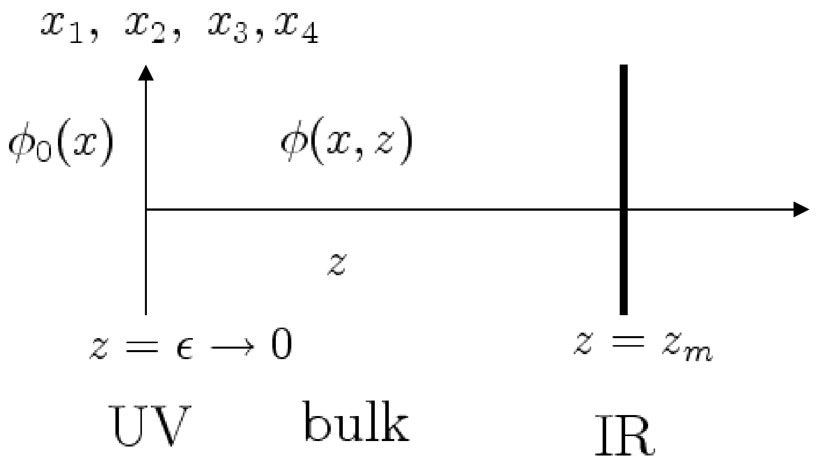
Erlich, Katz, Son, Stephanov, PRL (2005)
Da Rold, Pomarol, NPB (2005)

Ex) SU(2) x SU(2)

Infrared Brane at $z = z_m$

⟶ Confinement

(Polchinski & Strassler '00)



Metric – Slice of AdS metric

$$ds^2 = \frac{1}{z^2}(-dz^2 + dx^\mu dx_\mu), \qquad 0 < z \leq z_m$$

Operators/Fields Contents

$$(\Delta - p)(\Delta + p - 4) = m_5^2$$

4D: $\mathcal{O}(x)$	5D: $\phi(x, z)$	p	Δ	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

5d Action

$$S = \int d^5x \sqrt{g}\ \mathrm{Tr}\left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\}$$

$$D_\mu X = \partial_\mu X - iA_{L\mu} X + iXA_{R\mu} \qquad A_{L,R} = A^a_{L,R} t^a$$

$$X_0(z) = \frac{1}{2} M z + \frac{1}{2} \Sigma z^3 \qquad \Sigma = \sigma 1 \qquad M = m_q 1$$

Free Parameters $m_q,\ \sigma,\ z_m,\ g_5$

Matching the 5D gauge coupling

$$W_{4D}[\phi_0(x)] = S_{5D,\text{eff}}[\phi(x,\epsilon)] \quad \text{at} \quad \phi(x,\epsilon) = \phi_0(x)$$

Two point function

$$\left[\partial_z\left(\frac{1}{z}\partial_z V_\mu^a(q,z)\right) + \frac{q^2}{z} V_\mu^a(q,z)\right]_\perp = 0 \qquad V_z(x,z) = 0$$

$$S = -\frac{1}{2g_5^2}\int d^4x\left(\frac{1}{z}V_\mu^a \partial_z V^{\mu a}\right)_{z=\epsilon}$$

$$V^\mu(q,z) = V(q,z)V_0^\mu(q), \qquad V(q,\epsilon) = 1$$

$$\int_x e^{iqx}\langle J_\mu^a(x)J_\nu^b(0)\rangle = \delta^{ab}(q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_{\text{V}}(Q^2),$$

$$\Pi_{\text{V}}(-q^2) = -\frac{1}{g_5^2 Q^2}\frac{\partial_z V(q,z)}{z}\bigg|_{z=\epsilon} \qquad g_5^2 = \frac{12\pi^2}{N_c}$$

Observable	Measured (MeV)	Model A (MeV)	Model B (MeV)
m_π	139.6 ± 0.0004 [8]	139.6*	141
m_ρ	775.8 ± 0.5 [8]	775.8*	832
m_{a_1}	1230 ± 40 [8]	1363	1220
f_π	92.4 ± 0.35 [8]	92.4*	84.0
$F_\rho^{1/2}$	345 ± 8 [15]	329	353
$F_{a_1}^{1/2}$	433 ± 13 [6]	486	440
$g_{\rho\pi\pi}$	6.03 ± 0.07 [8]	4.48	5.29

Finite T QCD w/matter :Bottom-up Approach

- Hawking-Page(HP) transition in the bulk <-> confinement/deconfinement phase transition in QCD.
- Two asymp. AdS spaces : thermal AdS(tAdS) vs. AdS BH
- The dominant space is changed at the HP transition point.

<Set-up for bottom up>

Hawking-Page in a cut-off AdS$_5$

Witten, (1998), Herzog, PRL (2007)

the action density

$$I = -\frac{1}{2\kappa^2}\int d^5x\sqrt{g}\left(R+\frac{12}{L^2}\right) .$$

1) cut-off thermal AdS $ds^2 = L^2 \left(\frac{dt^2 + d\vec{x}^2 + dz^2}{z^2} \right)$

the action density I becomes

$$V_1(\epsilon) = \frac{4L^3}{\kappa^2} \int_0^{\beta'} dt \int_\epsilon^{z_0} dz\, z^{-5}$$

where $\beta' = \pi z_h \sqrt{f(\epsilon)}$ is fixed to have the same periodicity with the AdS black hole solution and $z = \epsilon$ represent the UV cut-off .

2) Cut-off AdS BH $ds^2 = \frac{L^2}{z^2} \left(f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)} \right)$ $f(z) = 1 - (z/z_h)^4$

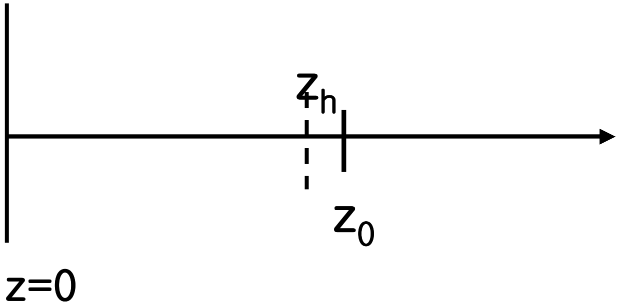
z_h horizon

the action density

$$V_2(\epsilon) = \frac{4L^3}{\kappa^2} \int_0^{\pi z_h} dt \int_\epsilon^{\min(z_0, z_h)} dz\, z^{-5}$$

The difference of the actions

$$\Delta V = \lim_{\epsilon \longrightarrow 0} (V_2(\epsilon) - V_1(\epsilon))$$
$$= \begin{cases} \frac{L^3 \pi z_h}{\kappa^2} \frac{1}{2z_h^4} & z_0 < z_h \\ \frac{L^3 \pi z_h}{\kappa^2} \left(\frac{1}{z_0^4} - \frac{1}{2z_h^4} \right) & z_0 > z_h \end{cases}$$



If ΔV is positive(negative), tAdS (AdS BH) is stable.

1) For $z_0 < z_h$

there is no HP transition, tAdS always stable

2) For $z_0 > z_h$

the HP transition occurs at $T_c = 2^{1/4}/(\pi z_0)$

at low T ($T < T_c$), tAdS (confining phase) is dominant

at high T ($T > T_c$), AdS BH is dominant (deconf. phase)

Quark flavor number dependence of Tc

Include the bulk matter

$$S_{\text{matter}} = M_5 \int d^5x \sqrt{g}\, \mathrm{Tr}\Big[\frac{1}{2}|D_\mu \Phi|^2 + \frac{1}{2} M_\Phi^2 |\Phi^2 + \frac{1}{4}(F_L^2 + F_R^2)\Big]$$

Scalar field solution

$$v(z) = \langle \Phi \rangle = az + bz^3$$

Mass of the boundary quark

Chiral condensate $\langle \bar{q} q \rangle$

Scalar Contribution to the Action

$$V_{1m} = \frac{L^3 \pi z_h}{2} \cdot 3 M_5 N_f b^2 z_{\mathrm{IR}}^2$$ for thermal AdS

$$V_{2m} = 0$$ for AdS BH

Action difference

$$\Delta V_m = -\frac{L^3 \pi z_h}{\kappa^2} \cdot \frac{b_t^2 L^2 N_f z_{\mathrm{IR}}^2}{32 N_c}$$

$z_{\mathrm{IR}} < z_h$ $\qquad$ $z_{\mathrm{IR}} > z_h$

Total difference of Action

$$\Delta V = \begin{cases} \frac{L^3 \pi z_h}{2\kappa^2}\left[\frac{1}{z_h^4} - \left(\frac{N_f}{N_c}\right)\frac{\xi^2}{16 z_{\mathrm{IR}}^4}\right] & z_{\mathrm{IR}} < z_h, \\ \frac{L^3 \pi z_h}{2\kappa^2}\left[\frac{2}{z_{\mathrm{IR}}^4} - \frac{1}{z_h^4} - \left(\frac{N_f}{N_c}\right)\frac{\xi^2}{16 z_{\mathrm{IR}}^4}\right] & z_{\mathrm{IR}} > z_h. \end{cases}$$

Critical Temperature modified by mesons

$$T = \begin{cases} T_0\left(\frac{\xi^2}{32}\frac{N_f}{N_c}\right)^{1/4} & z_{\mathrm{IR}} < z_h, \\ T_0\left(1 - \frac{\xi^2}{32}\frac{N_f}{N_c}\right)^{1/4} & z_{\mathrm{IR}} > z_h. \end{cases} \qquad T_0 = \frac{1}{\pi z_h} = \frac{2^{1/4}}{\pi z_{\mathrm{IR}}}$$

Smaller than that of the pure AdS gravity

Hawking-Page type Transition at Finite Density

Equation of Motion for time component of U(1) vector

$$\partial_z\left[\frac{1}{z}\partial_z V_\tau(z)\right] = 0$$

solution

$$V_\tau = c_1 + c_2 z^2$$

the Action

$$V_{v1} = \pi z_h M_5 N_f L^5 c_2^2 z_{\rm IR}^2$$ for thermal AdS

$$V_{v2} = \begin{cases} \pi z_h M_5 N_f L^5 c_2^2 z_h^2 & z_h < z_{\rm IR} \\ \pi z_h M_5 N_f L^5 c_2^2 z_{\rm IR}^2 & z_h > z_{\rm IR} \end{cases}$$ for AdS BH

Action difference

$$\Delta V_v = \begin{cases} -\pi z_h M_5 N_f L^5 c_2^2 (z_{\rm IR}^2 - z_h^2) & z_h < z_{\rm IR}, \\ 0 & z_h > z_{\rm IR}. \end{cases}$$

Total difference of Action

$$\Delta V = \frac{L^3 \pi z_h}{\kappa^2}\left[\frac{1}{z_{\mathrm{IR}}^4} - \frac{1}{2z_h^4} - \frac{L^4 N_f c_2^2}{48N_c}(z_{\mathrm{IR}}^2 - z_h^2)\right] \qquad \text{for } z_h < z_{\mathrm{IR}}$$

Critical Temp.

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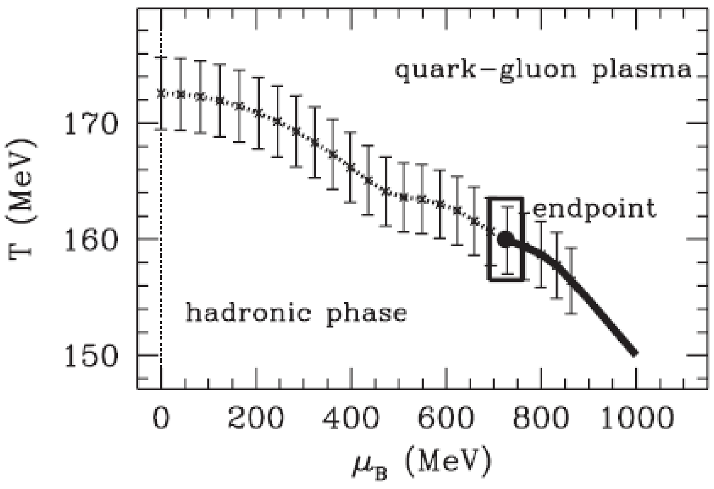
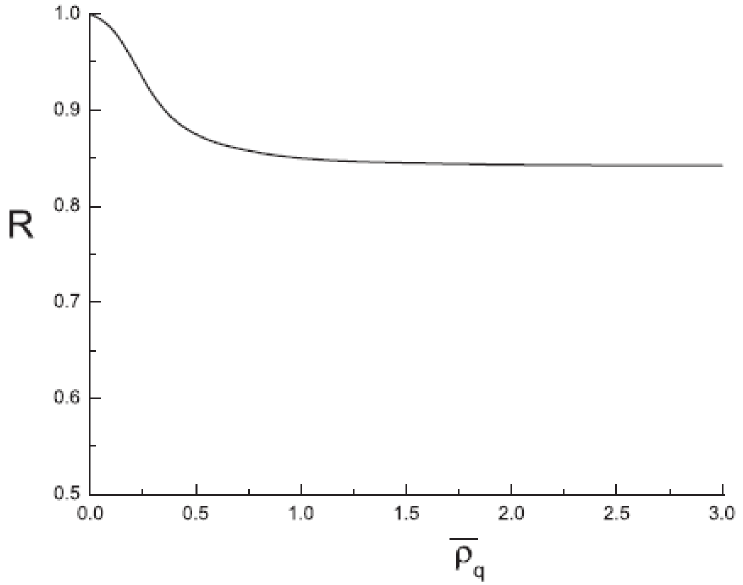
$$0 = T^4 - \frac{2}{\pi^4}\left(\frac{1}{z_{\mathrm{IR}}^4} - \frac{L^4 N_f}{48N_c} c_2^2\left(z_{\mathrm{IR}}^2 - \frac{1}{\pi^2 T^2}\right)\right)\Bigg|_{T=T_c} \qquad T_0 = \frac{1}{\pi z_h} = \frac{2^{1/4}}{\pi z_{\mathrm{IR}}}$$

$$T_c(\rho_q) = \frac{2^{1/4}}{\pi}\left(\frac{1}{z_{\mathrm{IR}}^4} - \frac{L^4 N_f z_{\mathrm{IR}}^2}{48N_c} c_2^2\right)^{1/4}$$



Tc from lattice simulations



Density dependence of Tc

Gluon Condensation at finite temperature

5D Gravity Action with a dilaton field

$$S = \gamma \frac{1}{2\kappa^2} \int d^5x \sqrt{g} \left[\mathcal{R} + \frac{12}{R^2} - \frac{1}{2} \partial_M \phi \partial^M \phi \right]$$

Metric Back reaction

$$ds^2 = \frac{R^2}{z^2} \left[dz^2 + \left(1 - f^2 z^8\right)^{1/2} \left(\frac{1 + f z^4}{1 - f z^4} \right)^{a/2f} \left(d\vec{x}^2 - \left(\frac{1 - f z^4}{1 + f z^4} \right)^{2a/f} dt^2 \right) \right]$$

$$\phi(z) \quad = \quad \phi_0 + \frac{c}{f} \sqrt{\frac{3}{2}} \log \left(\frac{1 + f z^4}{1 - f z^4} \right) \qquad\qquad c = \sqrt{f^2 - a^2}$$

The solution well defined in the range $0 < z < f^{-1/4} := z_f$

IR cutoff

This solution reduces to

- pure AdS space-time for $a = c = 0$
- AdS black hole solution for $c = 0, \; f = a$
- dilatonic AdS solution for $a = 0$

According to the holographic renormalization, the boundary energy-momentum tensor is related to the $g^{(4)}$, the coefficient of the z^4 in the metric

$$T_{\mu\nu} = \frac{N_c^2}{2\pi^2} g^{(4)}{}_{\mu\nu} = \mathrm{diag}(\rho, p, p, p)$$

- In our case, $g^{(4)}{}_{\mu\nu} = a \cdot \mathrm{diag}(3, 1, 1, 1)$

  => Using the Stephan-Boltzmann's law,

$$a = \frac{1}{4}\pi^4 T^4$$

- Gluon condensation at finite temperature

Near the boundary z~0, the dilaton profile is

$$\phi(x,z) \to z^{d-\Delta}\phi_0(x) + z^{\Delta}A(x)$$

which implies that A is related to the vev of the corresponding operator $\langle \mathcal{O} \rangle = (2\Delta - d)A(x)$

-> In our case ($d = 4 \text{ and } \Delta = 4$), we obtain

$$A = \frac{1}{(2\Delta - 4)} \cdot \sqrt{2\kappa^2/R^3}\,\left\langle \mathrm{Tr}\, G^2 \right\rangle$$

Comparing with expansion of the exact solution, we find

$$A = \sqrt{6}c$$

Then, the gluon condensation is given by

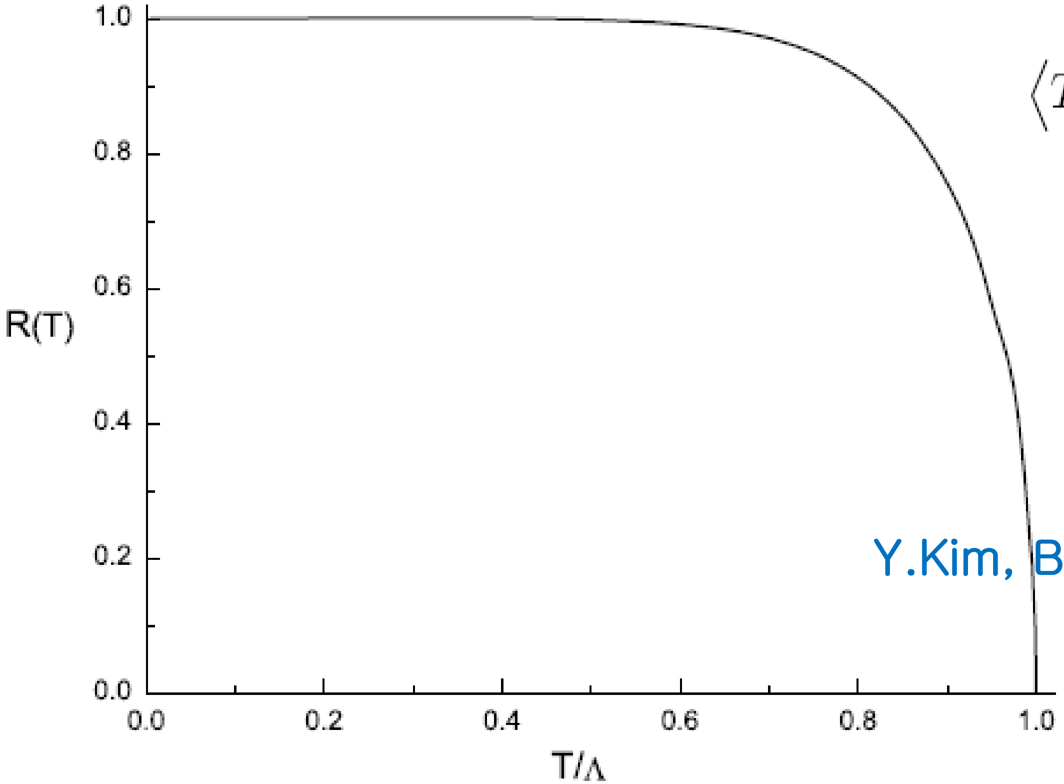
$$\left\langle \mathrm{Tr}\, G^2 \right\rangle = 4\sqrt{\frac{3R^3}{k^2}} \cdot c$$

$$= \frac{\sqrt{3}}{2}\pi^3 \cdot N_c \Lambda^4 \sqrt{1 - T^8/\Lambda^8}$$

where $f \equiv (\pi\Lambda)^4/4$ and $\frac{R^3}{\kappa^2} = \frac{4N_c^2}{\pi^2}$

The ratio of the gluon condensation at the finite temperature to the one at zero temperature

$$R(T) = \langle \mathrm{Tr} G^2 \rangle_T / \langle \mathrm{Tr} G^2 \rangle_{T=0}$$



$$\langle TrG^2 \rangle = 4\sqrt{\frac{3R^3}{\kappa^2}\left(\Lambda^8 \quad - \frac{\pi^8 T_H^8}{16}\right)}$$

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The temperature dependence of the condensate agrees very with Lattice QCD results, and we are working on Hawking-Page transition.

Summary

- Holographic Principles ：

  (d+1 dim.) (classical) Sugra ↔ (d dim.) (quantum) YM theories

- SUGRA w/ BH ↔ Finite Temperature

  Witten; Hawking-Page transition

- AdS/QCD : can be a powerful tool for QCD

  - Top-down Approach & Bottom-up Approach
  - Finite T QCD by cut-off AdS black hole set-up
    - 4D vector meson mass
    - T dependent masses & coupling constants in AdS/QCD
    - the critical temperature.
    - Dense matter : U(1) chemical potential→ baryon density
  - etc.

Thank You !