Hydrodynamics with a Critical End Point

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Excited QCD 2009, Zakopane

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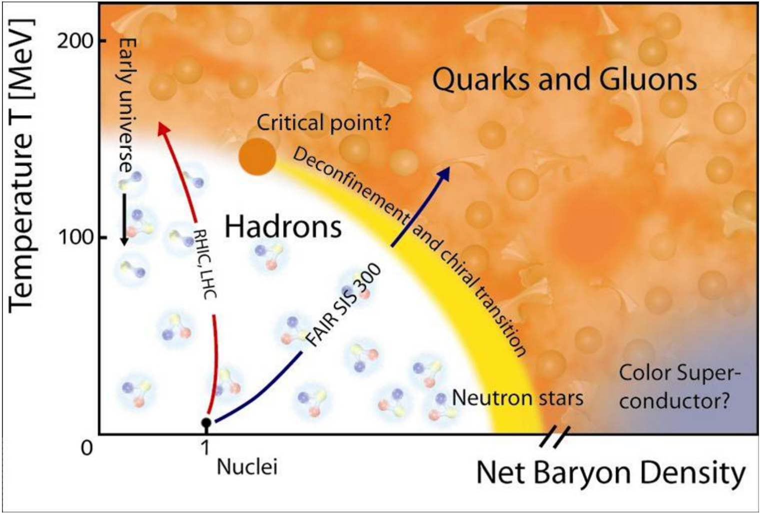
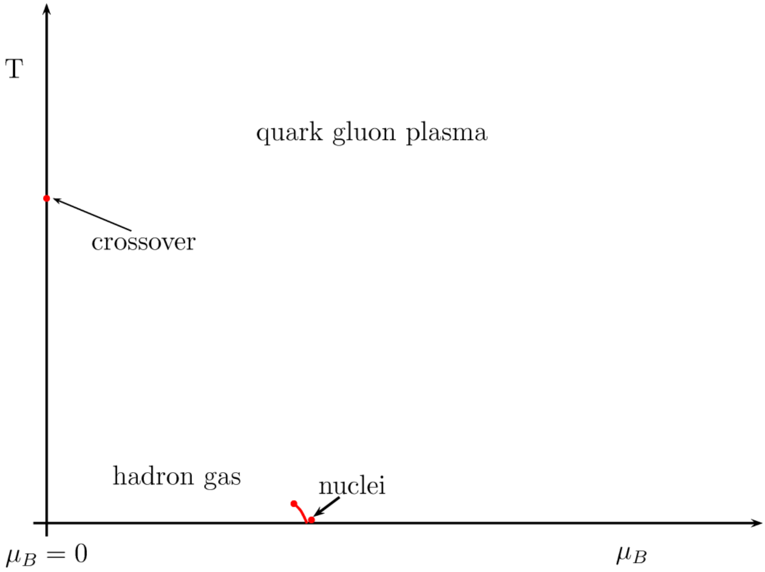
The QCD Phase Diagram

-suggested-



The QCD Phase Diagram

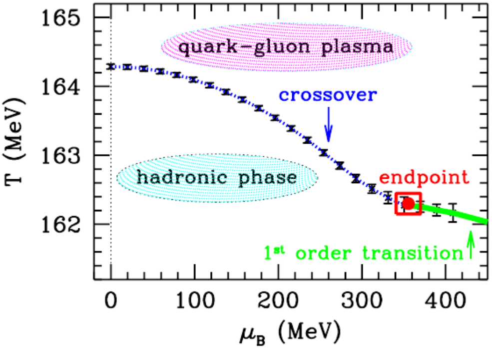
-what we really know-



The Critical Point

location

lattice QCD at finite μ_B

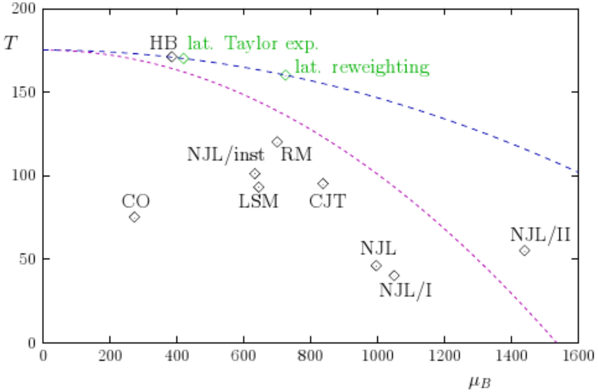


(Z. Fodor and S.D. Katz, JHEP 0404:050,2004)

The Critical Point

location

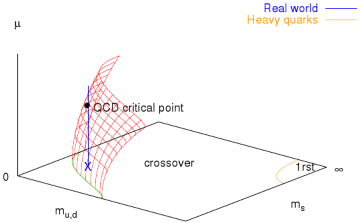
lQCD calculations generally agree on $\frac{\mu_b^c}{T^c(\mu_b=0)} \gtrsim 2$



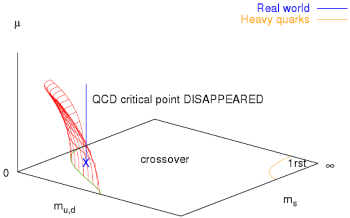
(M. Stephanov, hep-lat/0701002)

The Critical Point

lattice QCD



conventionally, first-order region expands with μ.



exotic scenario: first-order region shrinks!

(de Forcrand, Philipsen, hep-lat/0607017) singularity of thermodynamic functions

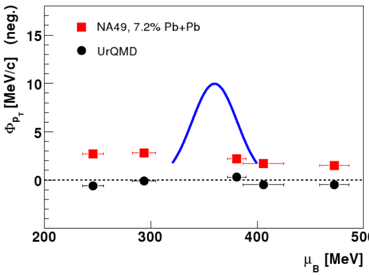
$\Rightarrow$ diverging correlation length ξ

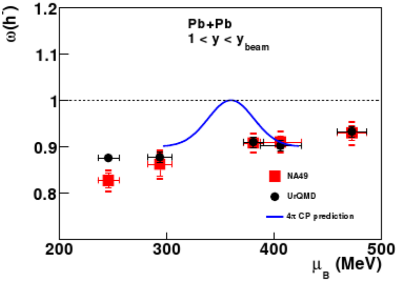
$\Rightarrow$ fluctuations increase/decrease non-monotonically vs beam energy

including finite size and critical slowing down $\Rightarrow \xi \approx 2 - 3$fm

(M.Stephanov, K.Rajagopal, E.Shuryak, Phys. Rev D60,114028,1999), (B.Berdnikov, K.Rajagopal, Phys. Rev. D61,105017,2000)

experimental signatures:



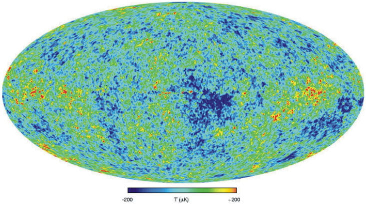


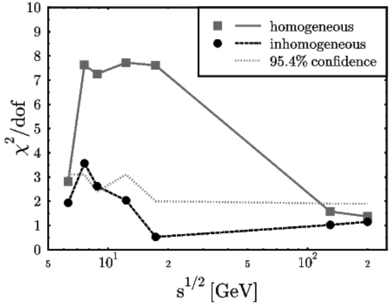
(NA49 collaboration J.Phys.G35:104091,2008)

The Phase Transition

density inhomogeneities

- inhomogeneous freeze-out surface of hadrons
- superposition of grand-canonical ensembles with different T and μ_b
- fitting ratios of particle multiplicities



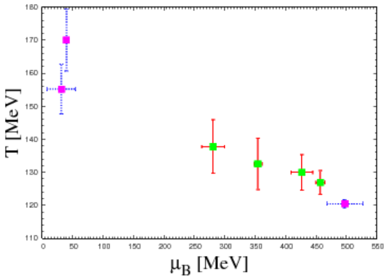


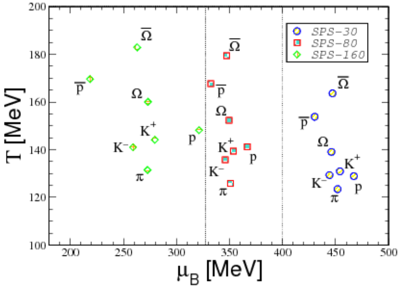
(A. Dumitru, L. Portugal, D.Zschiesche, Phys. Rev. C73,024902,2006)

The Phase Transition

chemical freeze-out

at chemical freeze-out there are significant fluctuations around the mean T and μ_B

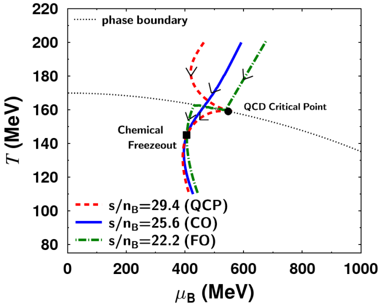


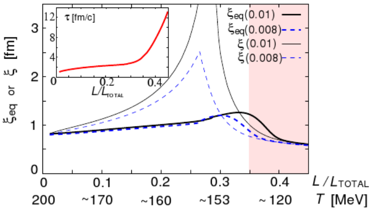


(A. Dumitru, L. Portugal, D.Zschiesche, Phys. Rev. C73,024902,2006)

Hydrodynamics - Nonaka

trajectories





- construct an equation of state with a critical point form the universality class (3d Ising)
- isentropic expansion trajectories $s/n_B = \mathrm{const.}$
- focussing effect near the critical point
- correlation length ξ stays finite
- no dynamic fluctuations

(M.Asakawa, C.Nonaka, Nucl. Phys. A774,753-756,2006)

Motivation

usually considered:

- thermodynamics in a grand-canonical scenario OR
- dynamics

experimental situation:

- finite system
- finite life time
- no global equilibrium
- dynamics
- observables in a finite phase space

$\Rightarrow$ use hydrodynamics with fluctuations dynamically driven through a critical point:
couple a hydrodynamic quark fluid to field equations!

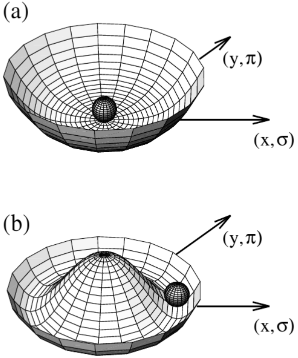
The Linear Sigma-Model

chiral symmetry

$$\mathcal{L} = \overline{q}\left[\mathrm{i}\gamma^{\mu}\partial_{\mu} - \mathrm{g}\left(\sigma + \gamma_5 \tau \vec{\pi}\right)\right] q + 1/2\left(\partial_{\mu}\sigma\right)^2 + 1/2\left(\partial_{\mu}\vec{\pi}\right)^2 - U\left(\sigma, \vec{\pi}\right)$$

$$U\left(\sigma, \vec{\pi}\right) = \frac{\lambda^2}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 - h_q \sigma - U_0$$

(M.Gell-Mann, M.Levy, Nuovo Cim. 16, 705,1960)



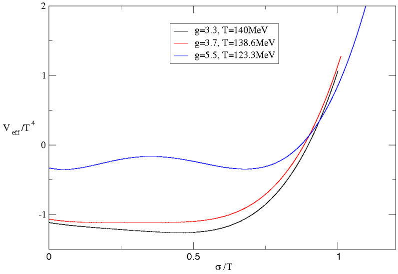
$SU_L(2) \otimes SU_R(2)$ chiral symmetry spontaneously broken in vacuum
$\langle \sigma \rangle = f_\pi = 93\mathrm{MeV}$
$\langle \vec{\pi} \rangle = 0$
explicit symmetry breaking by
$h_q = f_\pi m_\pi^2$

The Linear Sigma-Model

thermodynamics

grand canonical partition function at $\mu_b = 0$

$$\mathcal{Z} = \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}\sigma\mathcal{D}\vec{\pi} \exp\left[\int_0^{1/T} \mathrm{d(it)}\right] \int_V \mathrm{d}^3 x \mathcal{L}$$



grand canonical potential

$$V_{\mathrm{eff}} = \Omega = -{}^{T}/{}_{V} \log \mathcal{Z}$$

$$= - d_q T \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \log(1 + \mathrm{e}^{-E/T})$$

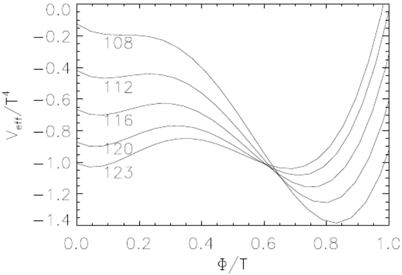
$$+ U(\sigma, \vec{\pi})$$

with $E = \sqrt{p^2 + g^2\phi^2}$

The Linear Sigma-Model

a first order phase transition

- for $T_{sp} < T < T_c$ the σ-field can be in a metastable minimum of restored symmetry
- transition via nucleation of thermally activated bubbles of the broken symmetry phase



(O. Scavenius, A. Dumitru, E.S. Fraga, J.T. Lenaghan, A.D. Jackson, Phys.Rev. D63 (2001) 116003)

The Linear Sigma-Model

equations of motion

classical equation of motion for the fields: $\phi = (\sigma, \vec{\pi})$

$$\partial_\mu \partial^\mu \phi + \frac{\delta U}{\delta \phi} = -g^2 \phi d_q \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{E} f_{FD}(p) = -g \rho_\phi$$

with the (pseudo-)scalar density

$$\rho_\phi = g \phi d_q \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{E} f_{FD}(p)$$

solved by a staggered leap-frog algorithm

Chiral Hydrodynamics

coupled equation

equations of relativistic hydrodynamics:

$$\partial_\mu (T^{\mu\nu}_{\text{fluid}} + T^{\mu\nu}_{\text{field}}) = 0 \;\Rightarrow\; \partial_\mu T^{\mu\nu}_{\text{fluid}} = g\rho_\phi \partial^\nu \phi$$

with the stress-energy tensor for an ideal fluid

$$T^{\mu\nu}_{\text{fluid}} = (e + p) u^\mu u^\nu - p g^{\mu\nu}$$

equation of state from self-consistency conditions

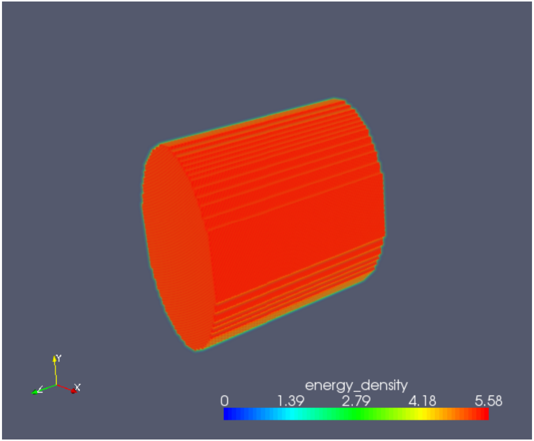
$$\begin{aligned} e(\phi, T) &= T \frac{\partial p(\phi, T)}{\partial T} - p(\phi, T)\,, \\ p(\phi, T) &= -V_{\text{eff}}(\phi, T) + U(\phi) \end{aligned}$$

(K.Paech, A.Dumitru,H.Stöcker, Phys.Rev.C68:044907,2003)

Initial Conditions

energy density

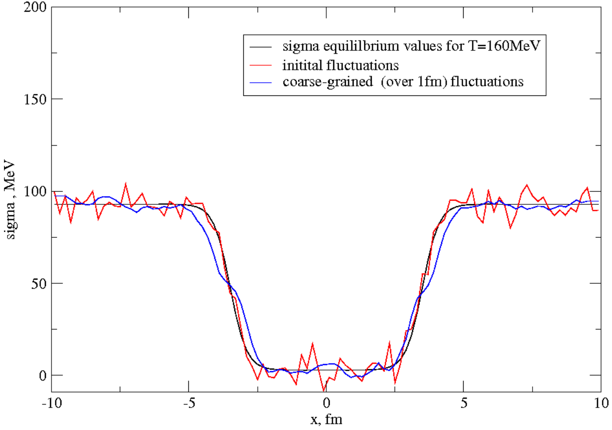
ellipsoidal initial conditions for the quark fluid



Initial Conditions

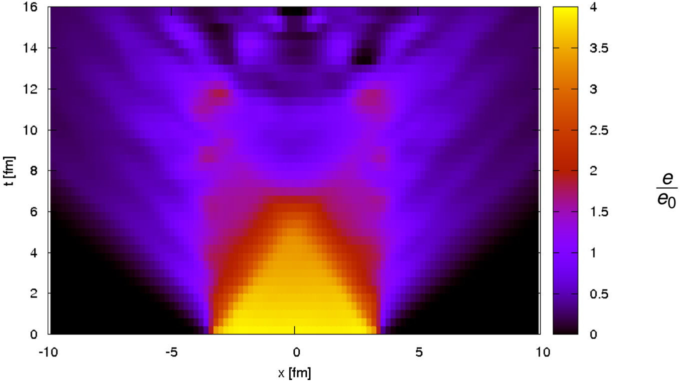
chiral fields

Wood-Saxon like initial distribution for the sigma field



Energy Density

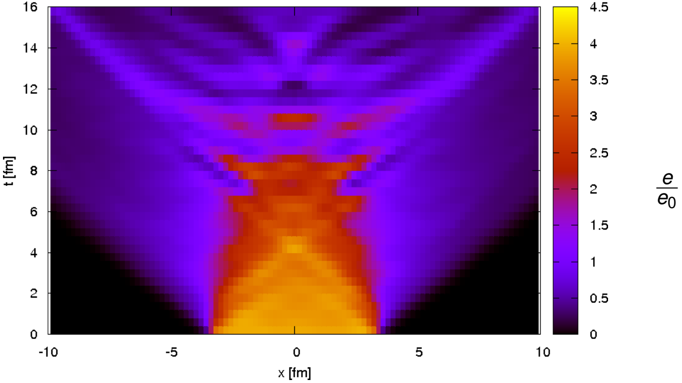
first order phase transition



along a trajectory through a first order phase transition (g=5.5)

Energy Density

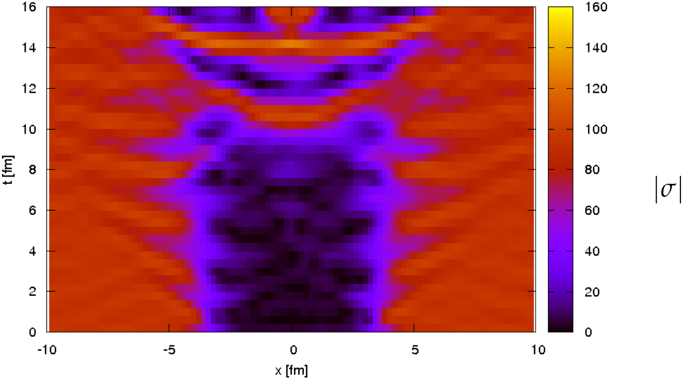
critical point



along a trajectory near a critical point (g=3.7)

Chiral Field

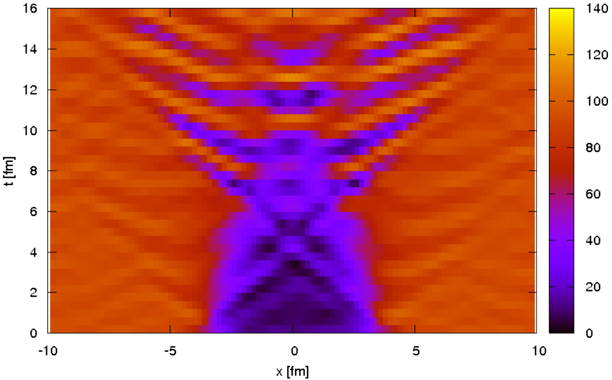
first order phase transition



for a first order phase transition (g=5.5), ($m_q = g|\sigma|$)

Chiral Field

Critical Point

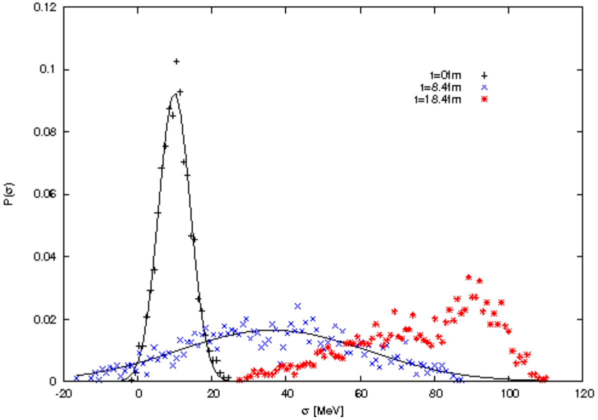


$|\sigma|$

near a critical point (g=3.7), ($m_q = g|\sigma|$)

Field Distributions

critical point

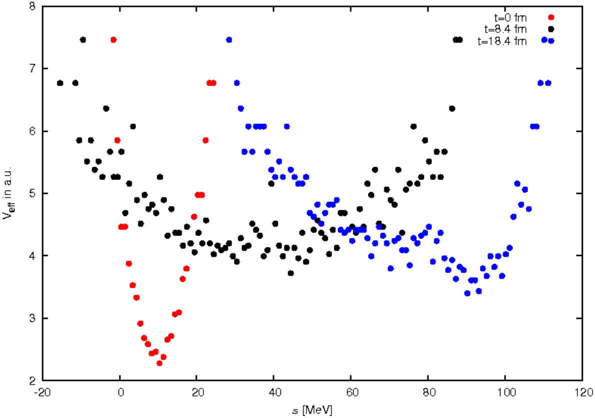


at $t = 0\mathrm{fm}$: Gaussian distribution with $v = 4.2\mathrm{MeV}$
at $t = 8.4\mathrm{fm}$: Gaussian distribution with $v = 25.5\mathrm{MeV}$
at $t = 18.4\mathrm{fm}$: not Gaussian anymore

Dynamic Effective Potential

critical point

$$V_{eff}^{dyn} = a_0 + a_1\tilde{\sigma} + a_2\tilde{\sigma}^2 + ... + a_n\tilde{\sigma}^n \quad \text{with} \quad \tilde{\sigma} = \sigma - \sigma_{eq}$$

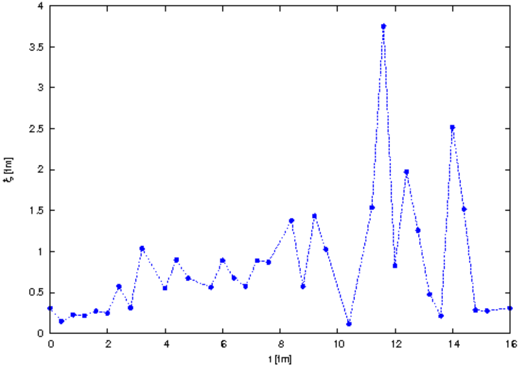


Correlation Length

critical point

$$\frac{1}{\xi^2} = \frac{\partial^2 V_{eff}}{\partial \tilde{\sigma}^2}\Big|_{\tilde{\sigma}=0}$$

- ξ stays finite
- at $t = 0\mathrm{fm}$:
  $\xi = 0.31\mathrm{fm} \Rightarrow$
  $m_\sigma = 636.5\mathrm{MeV}$
- at $t = 8.4\mathrm{fm}$:
  $\xi = 1.4\mathrm{fm} \Rightarrow$
  $m_\sigma = 142.9\mathrm{MeV}$



Conclusions

The critical point of QCD is an interesting target to shoot at, both theoretically and experimentally!

- formation of high energy density droplets due to a first order phase transition
- long-range fluctuations of the sigma field for trajectories near a critical end point
- the dynamical and non-equilibrium effects of a chiral phase transition can be investigated within a hydrodynamic model

upcoming experiments:

- low energy run @RHIC (2011)
- CBM@FAIR (2012)