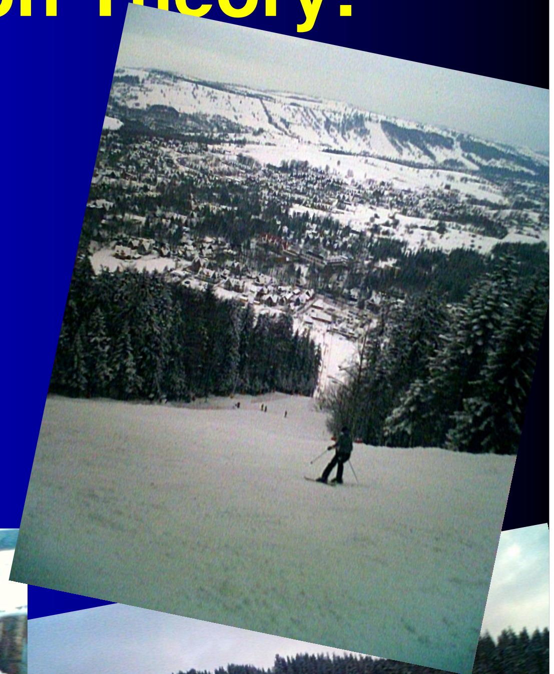
Departamento de Física Teórica II. Universidad Complutense de Madrid

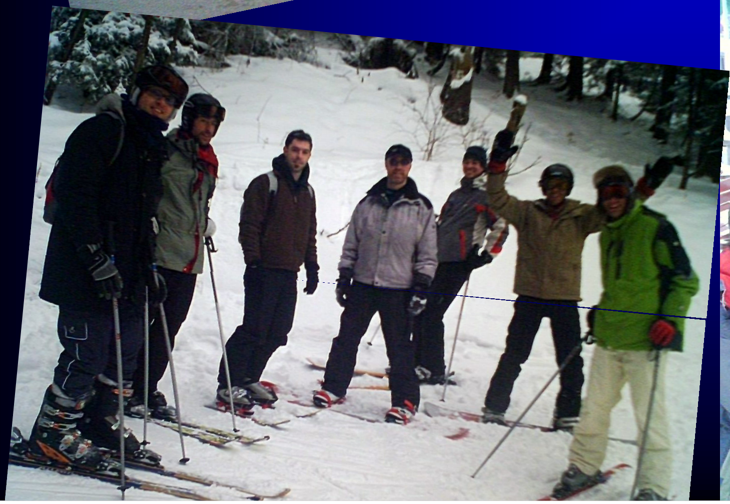
LIght scalars dependence on quark masses and N_c from Unitarized Chiral Perturbation Theory:



José R. Peláez

In colaboration with
C. Hanhart & G. Ríos

EXCITED QCD 09
Zakopane



Intro

Unitarized Chiral Perturbation Theory

The Inverse Amplitude Method (IAM)

1st Part

N_c behavior of scalars

One loop SU(3) ChPT: scalar nonet JRP, Phys.Rev.Lett. 92:102001,2004

Two loop SU(2) ChPT: the σ G. Ríos and JRP, Phys.Rev.Lett.97:242002,2006.

2nd Part

Chiral Extrapolation: the quark mass dependence

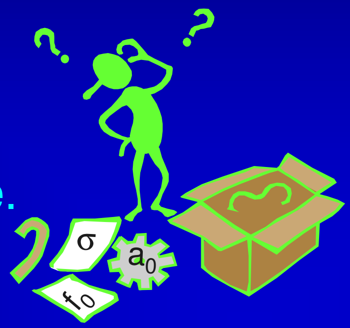
C. Hanhart, G. Ríos and JRP, Phys.Rev.Lett.100:152001,2008.

NEW!

The clasification of light scalar mesons is a long standing issue...

- There are many kind of possible states:
  - quark-antiquark
  - Four quarks
  - Meson molecules
  - Glueballs

- All these states do mix and their masses change.



- Some octets may seem to have too many particles...

- And for others we are not even sure if there is a particle or not

Is there a σ ? Is there a kappa?
What ere they made of?

Some light on these issues has recently
from meson-meson scattering in a chiral context

Chiral Perturbation Theory in the meson sector

Weinberg, Gasser & Leutwyler

π, K , η Goldstone Bosons
of the spontaneous
chiral symmetry breaking
$SU(N_f)_V \times SU(N_f)_A \rightarrow SU(N_f)_V$

QCD degrees of freedom
at **low energies << 4πf~1.2 GeV**

ChPT is the most general expansion in energies
of a lagrangian made only of pions, kaons and etas
compatible with the QCD symmetry breaking

Leading order parameters: breaking scale f_0 and masses

At 1-loop, QCD dynamics encoded in
chiral parameters: $L_1 \ldots L_8$
Determined from EXPERIMENT
leading $1/N_c$ behavior known from QCD

ChPT is the QCD Effective Theory
but is limited to low energies

Unitarity and the Inverse Amplitude Method: One channel –one loop

NAIVE DERIVATION

Partial wave unitarity

(On the real axis above threshold)

$$\mathrm{Im}\, t = \sigma |t|^2$$

$$\mathrm{Im}\frac{1}{t} = -\sigma$$

$$t \approx \frac{1}{\mathrm{Re}\, t^{-1} - i\sigma}$$

exactly unitary !!

$$t \approx \frac{t_2^2}{t_2 - \mathrm{Re}\, t_4 - i\sigma t_2^2}$$

ChPT = series in p^2

$$t = t_2 + t_4 \ldots$$

perturbative unitarity

$$\mathrm{Im}\, t_4 = \sigma |t_2|^2$$

provides ....

$$\mathrm{Re}\, t^{-1} \approx t_2^{-2}(t_2 - \mathrm{Re}\, t_4 + \ldots)$$

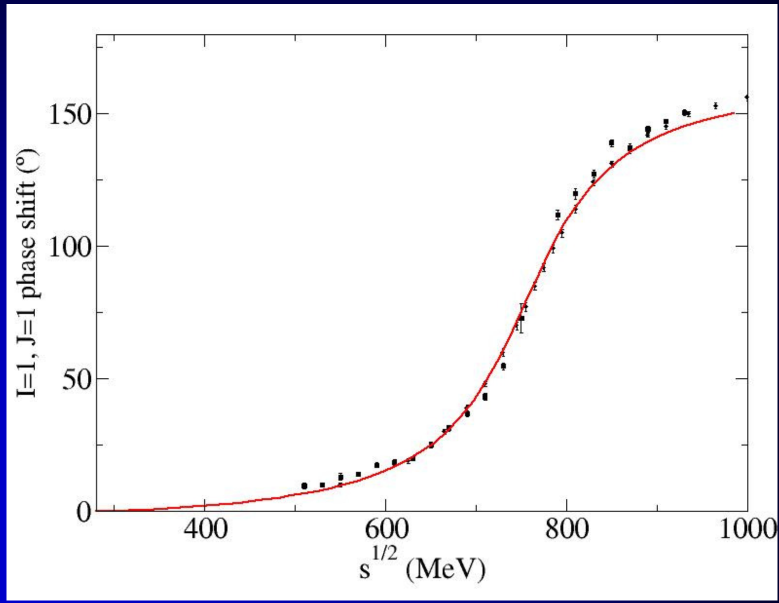
IAM

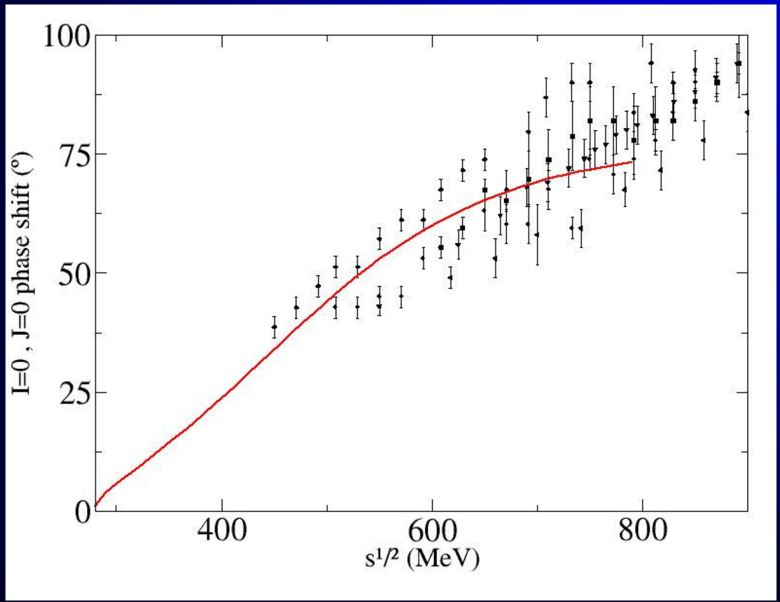
$$t \approx \frac{t_2^2}{t_2 - t_4}$$

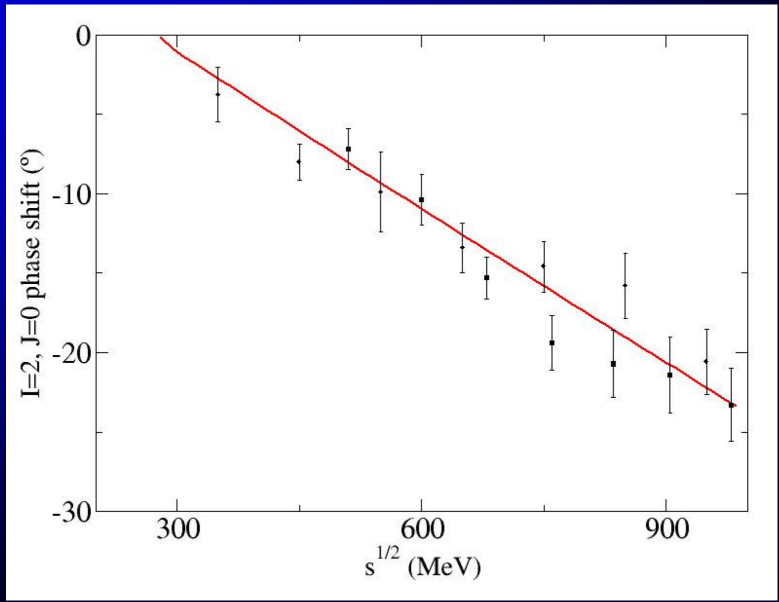
The Inverse Amplitude Method: Results for one channel

Truong '89, Truong,Dobado,Herrero,'90, Dobado, JRP,'93,'96

Fit pion scattering data



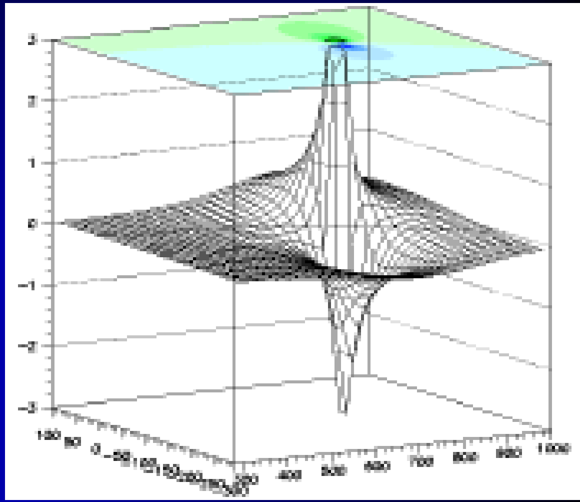




The Inverse Amplitude Method: Results for one channel

Truong '89, Truong,Dobado,Herrero,'90, Dobado JRP,'93,'96

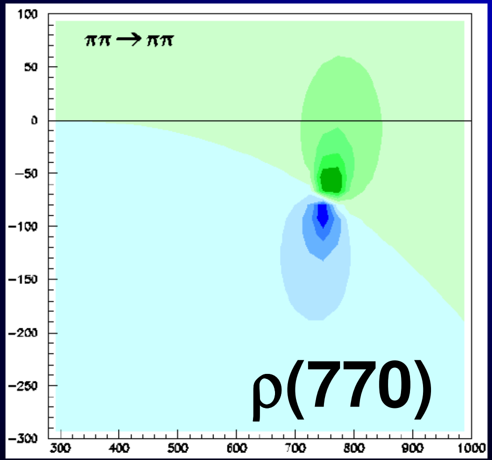
- EXTREMELY SIMPLE
- Unitarity + Chiral Low energy expansion
- Systematic extension to higher orders

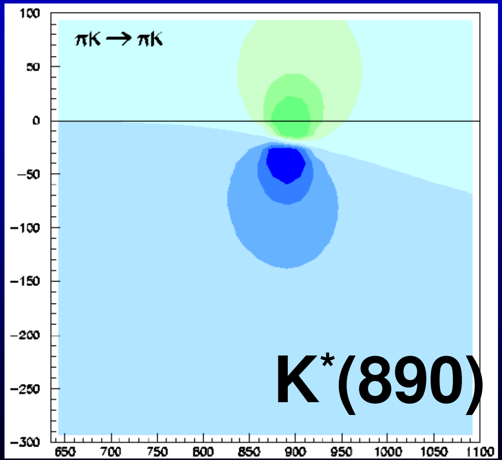


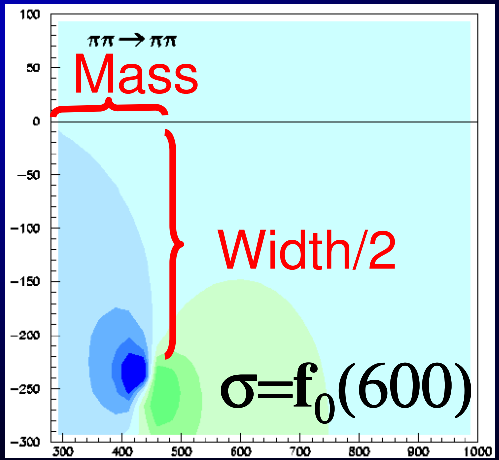
- Originally obtained from dispersion relation
  This allows us to go to the complex plane.
- Dynamically Generates Poles:
  Resonances: $f_0(600)$ or "sigma", K*, ρ

Dobado, Pelaez '96

$f_0(600)$ pole: 440-i245 MeV







The Inverse Amplitude Method: Dispersive Derivation: THE REAL THING

- We have just seen that, for physical s

$$\mathrm{Im}\frac{1}{t} = -\sigma \quad \text{and} \quad \mathrm{Im}\, t_4 = \sigma\, t_2^{\,2}$$

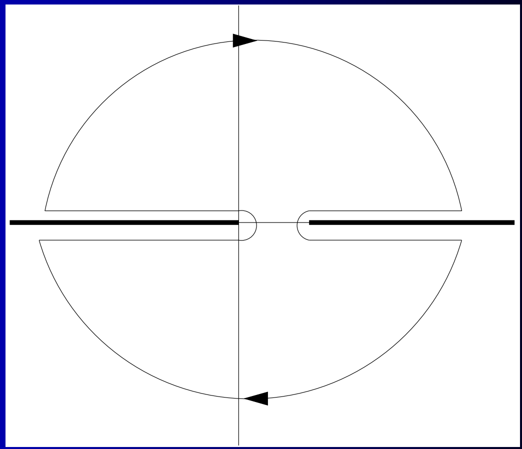
- Define $G \equiv \frac{t_2^{\,2}}{t}$,

$$\mathrm{Im}\, t_4 = \sigma\, t_2^{\,2} = -\mathrm{Im}\, G$$

- Write dispersion relations for G and t_4

$$t_{IJ}^{(4)} = b_0 + b_1 s + b_2 s^2 + \frac{s^3}{\pi}\int_{s_{th}}^{\infty}\frac{\mathrm{Im}\, t_{IJ}^{(4)}(s')ds'}{s'^3(s'-s-i\epsilon)} + LC(t_{IJ}^{(4)}).$$

$$G(s) = G_0 + G_1 s + G_2 s^2 + \frac{s^3}{\pi}\int_{s_{th}}^{\infty}\frac{\mathrm{Im}\, G(s')ds'}{s'^3(s'-s-i\epsilon)} + LC(G) + PC,$$



Subtraction Constants from ChPT expansion OK since s=0 $G(0)=t_2(0)-t_4(0)$

PHYSICAL cut EXACTLY Opposite to each other

Up to NLO ChPT Opposite to each other

PC is $O(p^6)$ and we neglect it or use ChPT

All together…we find AGAIN

IAM

$$t \approx \frac{t_2^2}{t_2 - t_4}$$

Unitarity and the Inverse Amplitude Method: Multiple channels–one loop

Oller, Oset, JRP, PRL80(1998)3452, PRD59,(1999)074001

Partial wave unitarity

(On the real axis above **all thresholds**)

$$\mathrm{Im}\, T = T\, \Sigma\, T^*$$

$$\mathrm{Im}\, T^{-1} = -\Sigma$$

$$T \approx (\mathrm{Re}\, T^{-1} - i\,\Sigma)^{-1}$$

exactly unitary !!

$$T = T_2 (T_2 - \mathrm{Re}\, T_4 - i\, T_2 \Sigma T_2)^{-1} T_2$$

ChPT = series in p²

$$T = T_2 + T_4 \ldots$$

perturbative unitarity

$$\mathrm{Im}\, T_4 = T_2\, \Sigma\, T_2$$

provides ....

$$\mathrm{Re}\, T^{-1} \approx T_2^{-1} (T_2 - \mathrm{Re}\, T_4 + \ldots) T_2^{-1}$$

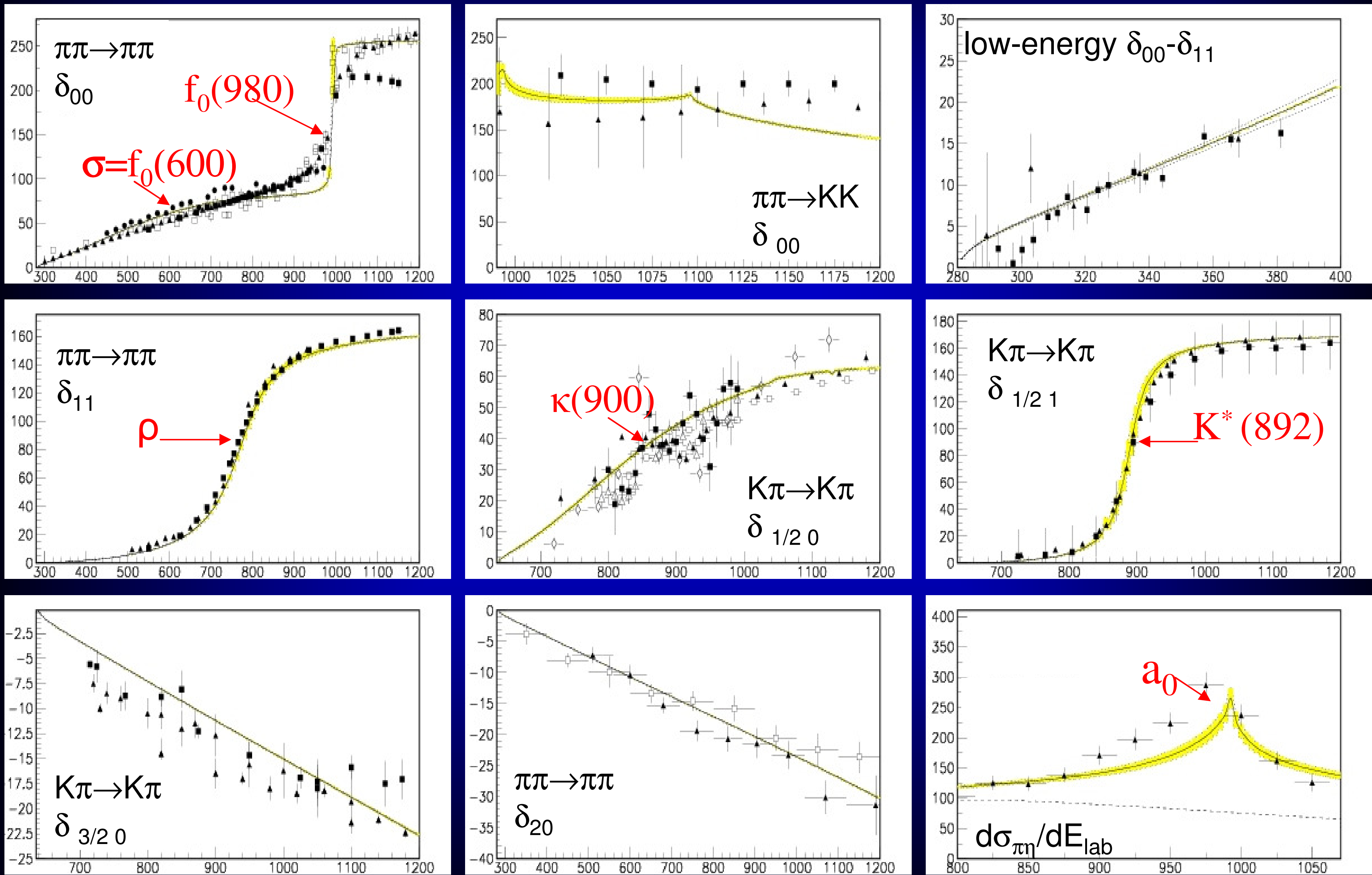
Coupled channel IAM

$$T \approx T_2 (T_2 - T_4)^{-1} T_2$$

To the DATA !!

One-loop ChPT IAM fit to meson-meson scattering

(+3% syst.) Gómez-Nicola, JRP, Phys. Rev. D65:054009, (2002)



Gómez-Nicola, JRP, Phys. Rev. D65:054009, (2002)

- MINUIT fit :
  - Incompatible sets of Data.
    Customarily add systematic error: 1%, 3%, 5% } Identical curves but variation in parameters
  - Final error: MINUIT error + Systematic error

	ChPT(μ=M_ρ)	IAM fit (+3%)	IAM fits
L_1	0.4± 0.3	0.561± 0.008	0.6± 0.1
L_2	1.35± 0.3	1.211± 0.001	1.2± 0.1
L_3	-3.5±1.1	-2.79± 0.02	-2.79± 0.14
L_4	-0.3± 0.5	-0.36± 0.02	-0.36± 0.17
L_5	1.4± 0.5	1.39± 0.02	1.4± 0.5
L_6	-0.2± 0.3	0.07± 0.03	0.07± 0.08
L_7	-0.4± 0.2	-0.444± 0.03	-0.44± 0.15
L_8	0.9± 0.3	0.78± 0.02	0.8± 0.2

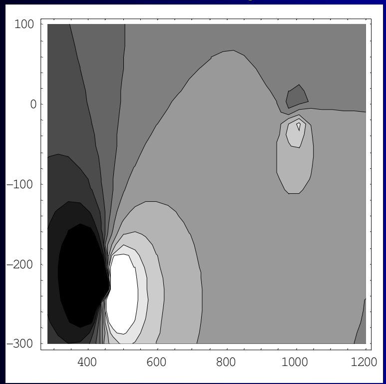
Simultaneous description of low energy and resonances

Fully renormalized and with parameters compatible with ChPT.

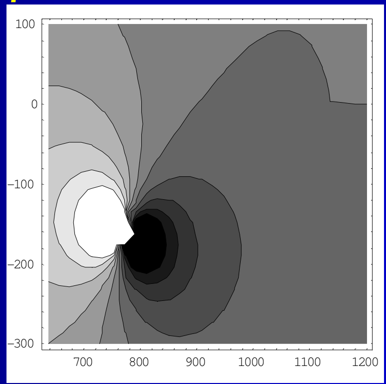
Oller, Oset, JRP, PRL80(1998)3452, PRD59,(1999)074001
A. Gomez Nicola and J.R.P.. PRD65:054009, (2002)

- With the full one-loop SU(3) unitarized ChPT, we GENERATE, the following resonances, not present in the ChPT Lagrangian, as poles in the second Riemann sheet

f0(600) and $f_0(980)$



$\rho(770)$

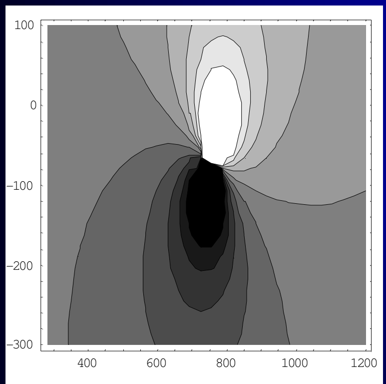


without a priori assumptions on on their existence or nature

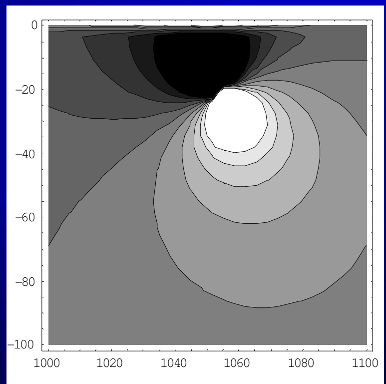
J.R.P, hep-ph/0301049. AIP Conf.Proc.660:102-115,2003

Brief review: Mod.Phys.Lett.A19:2879-2894,2004

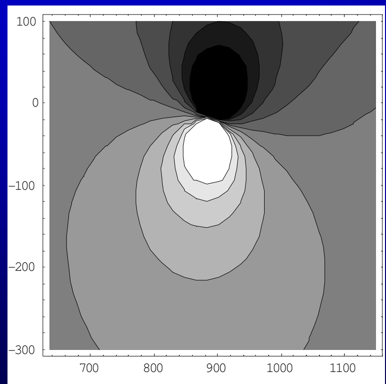
κ



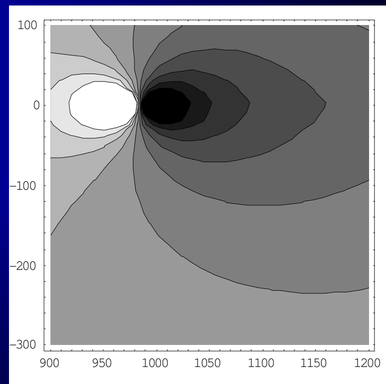
a_0



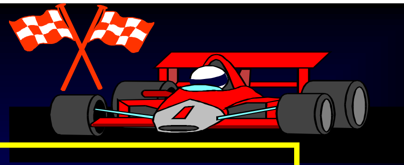
K*



ϕ (octet)



Resonance POLE POSITIONS...



	$\mathrm{Re}\sqrt{s_{pole}} \approx M$ (MeV) Complete IAM	$-\mathrm{Im}\sqrt{s_{pole}} \approx \Gamma/2$ (MeV) Complete IAM
ρ	754 ± 18	74 ± 10
K^*	889 ± 13	24 ± 4
$\sigma = f_0(600)$	440 ± 8	212 ± 15
κ	753 ± 52	235 ± 33
$f_0\,(980)$	973^{+39}_{-127}	11^{+189}_{-11}
$a_0\,(980)$	1117^{+24}_{-320} cusp?	12^{+43}_{-12}

Light scalar nonet

(Only statistical errors)

Outline

Intro

Unitarized Chiral Perturbation Theory

The Inverse Amplitude Method (IAM)

Description of meson-meson scattering compatible with ChPT

No model dependence in one-channel (not in coupled channels)

Both LIGHT VECTORS and SCALARS as poles

NLO (1 loop) and NNLO(2 loops) IAM available

No supurious parameter dependence in cutoffs or other parameters

1st Part

N_c behavior of scalars

One loop SU(3) ChPT: scalar nonet — JRP, Phys.Rev.Lett. 92:102001,2004

Two loop SU(2) ChPT: the σ — G. Ríos and JRP, Phys.Rev.Lett.97:242002,2006.

Large N_c expansion

We cannot obtain the L_i from QCD, BUT their 1/Nc expansion, is known and Model Independent

$(\times 10^{-3})$	ChPT ($\mu=M_\rho$)	IAM fits	Large N_c SCALING
$2L_1 - L_2$	-0.6 ± 0.6	0.0 ± 0.2	O(1)
L_2	1.4 ± 0.3	1.2 ± 0.1	$O(N_c)$
L_3	-3.5 ± 1.1	-2.79 ± 0.14	$O(N_c)$
L_4	-0.3 ± 0.5	-0.36 ± 0.17	O(1)
L_5	1.4 ± 0.5	1.4 ± 0.5	$O(N_c)$
L_6	-0.2 ± 0.3	0.07 ± 0.08	O(1)
L_7	-0.4 ± 0.2	-0.44 ± 0.15	O(1)
L_8	0.9 ± 0.3	0.8 ± 0.2	$O(N_c)$

The qqbar meson masses M=O(1) and their decay constants $f=O(\sqrt{N_c})$

Pions, kaons and etas states: $M \approx O(1), \Gamma \approx O(1/N_c)$

Our IAM ChPT amplitudes **do not** have any other parameter hiding Nc dependence like cutoffs, subtractions, etc...

We can thus study the Nc scaling of the resonances

LIGHT VECTOR MESONS

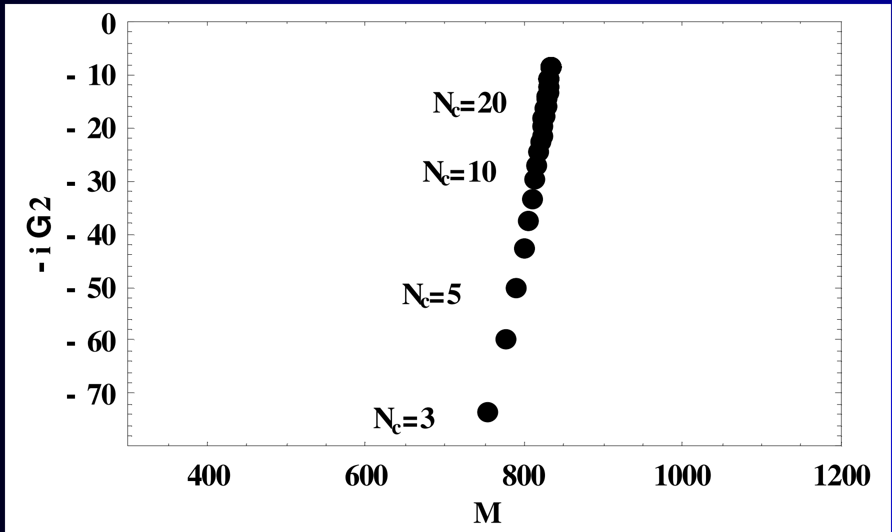
qqbar states:

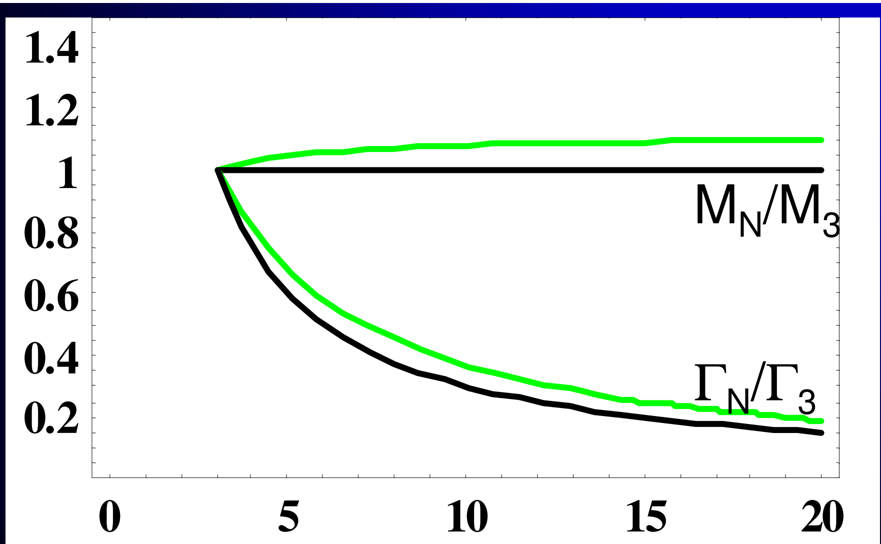
$$M \approx O(1), \Gamma \approx O(1/N_c)$$

The IAM generates the expected N_c scaling of established qq states

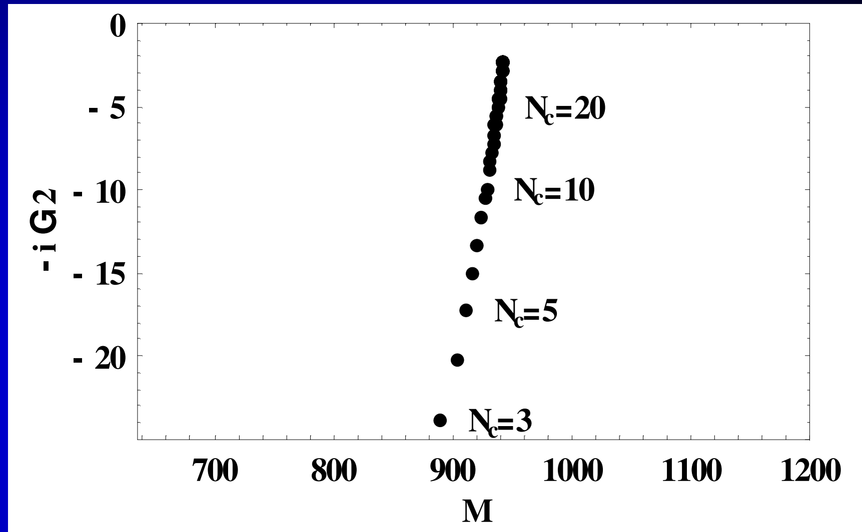
JRP, Phys.Rev.Lett. 92:102001,2004

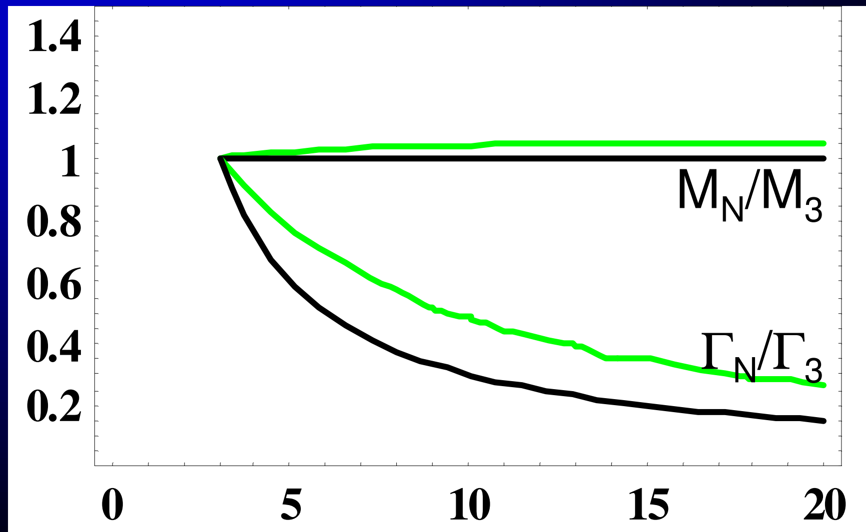
The ρ(770)





The K*(892)

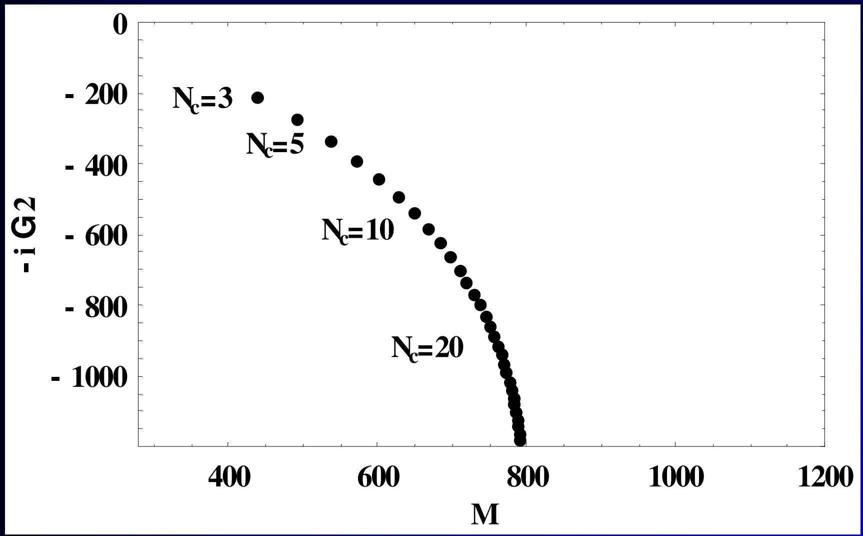




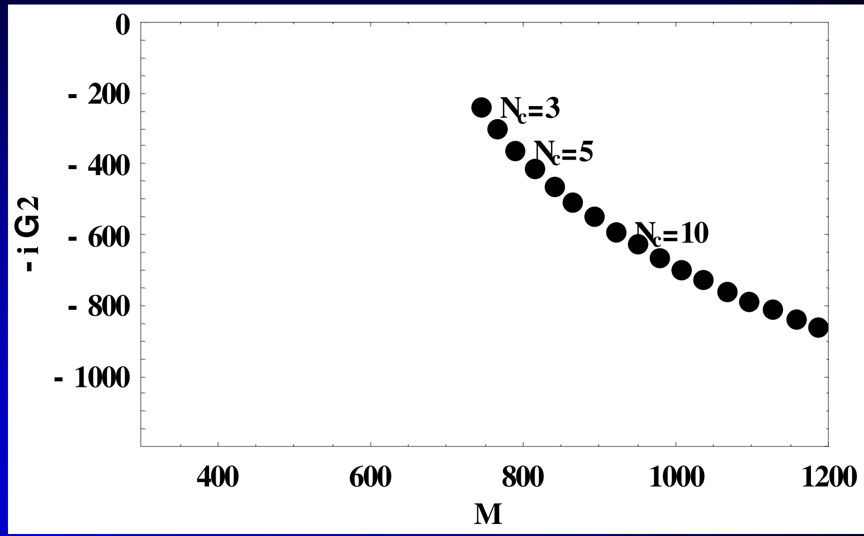
What about scalars ?

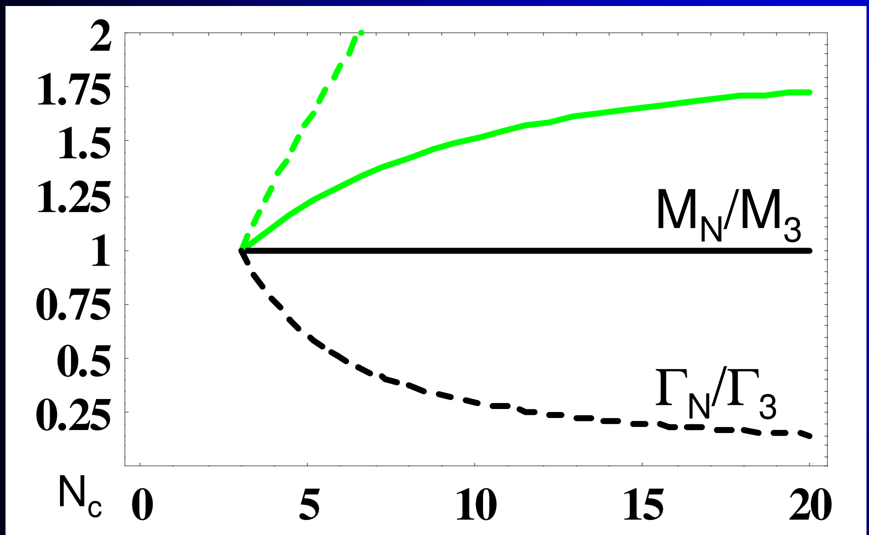
JRP, Phys.Rev.Lett. 92:102001,2004

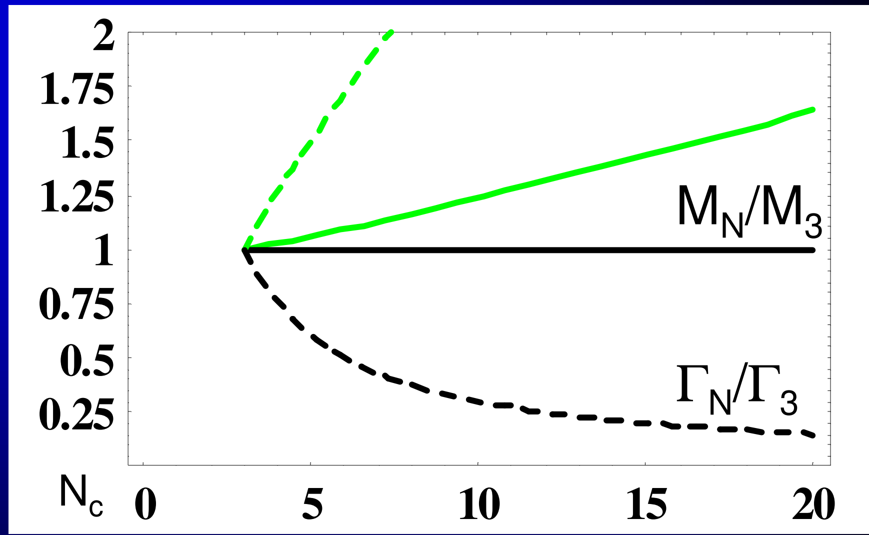
The σ (μ=770MeV)



The κ (μ=500MeV)







Similar results follow for the $f_0(980)$ and $a_0(980)$

Complicated by the presence of THRESHOLDS and except in a corner of parameter space for the $a_0(980)$

G. Ríos and JRP, Phys.Rev.Lett.97:242002,2006.

Since for quark antiquark states $M \approx O(1), \Gamma \approx O(1/N_c)$

$$
\begin{aligned}
M_{N_c}^{\bar{q}q} &\simeq M_{N_c-1}\left[1+\epsilon_M\left(\frac{1}{N_c}-\frac{1}{N_c-1}\right)\right] \\
&\equiv M_{N_c-1}+\Delta M_{N_c}^{\bar{q}q}, \\
\Gamma_{N_c}^{\bar{q}q} &\simeq \frac{\Gamma_{N_c-1}\left(N_c-1\right)}{N_c}\left[1+\epsilon_\Gamma\left(\frac{1}{N_c}-\frac{1}{N_c-1}\right)\right] \\
&\equiv \frac{\Gamma_{N_c-1}\left(N_c-1\right)}{N_c}+\Delta\Gamma_{N_c}^{\bar{q}q}.
\end{aligned}
$$

χ^2-like function to measure how close a resonance is to a quark antiquark

$$
\bar{\chi}_{\bar{q}q}^2=\frac{1}{2n}\sum_{N_c=4}^{n}\left[\left(\frac{M_{N_c}^{\bar{q}q}-M_{N_c}}{\Delta M_{N_c}^{\bar{q}q}}\right)^2+\left(\frac{\Gamma_{N_c}^{\bar{q}q}-\Gamma_{N_c}}{\Delta\Gamma_{N_c}^{\bar{q}q}}\right)^2\right]
$$

$$
\bar{\chi}^2 \leq 1 \quad is\ q\bar{q}, \qquad \bar{\chi}^2 >> 1 \quad is\ NOT\ q\bar{q}
$$

This χ^2-like can be used to:

1) check if a resonance behaves as quark-antiquark

2) Try to force a resonance to behaves as quark-antiquark by tuning parameters

	IAM Fit	$10^3 l_1^r$	$10^3 l_2^r$	$10^3 l_3^r$	$10^3 l_4^r$	χ^2_{data}	χ^2_{LECS}	$\chi^2_{\rho,\bar{q}q}$	$\chi^2_{f_0(600),\bar{q}q}$
One loop	$O(p^4)$ Only Data	-3.8	4.9	0.43	7.2	1.1	0.08	0.26	140
	$O(p^4)$ ρ as $q\bar{q}$	-3.8	5.0	0.42	6.4	1.2	0.03	0.22	143
	$O(p^4)$ $f_0(600)$ as $q\bar{q}$	-3.9	4.6	2.6	15	1.4	5.6	0.32	125
Two loops	$O(p^6)$ ρ as $q\bar{q}$	-5.4	1.8	1.5	9.0	1.1	1.9	0.93	15
	$O(p^6)$ $f_0(600)$ as $q\bar{q}$	-5.7	2.6	-1.7	1.7	1.4	2.1	2.0	3.5
	$O(p^6)$ ρ, $f_0(600)$ as $q\bar{q}$	-5.7	2.5	0.39	3.5	1.5	1.4	1.3	4.0

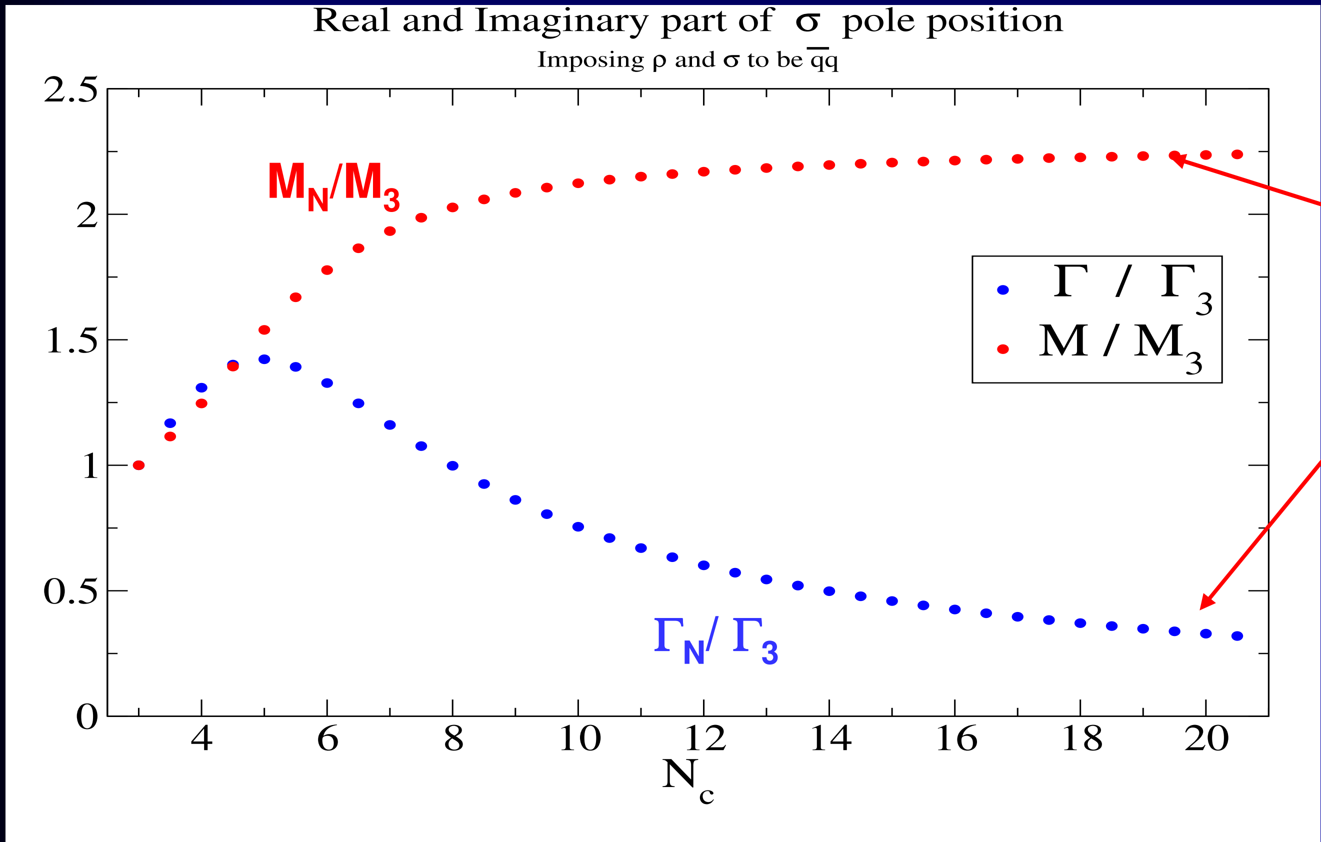
The rho always comes out naturally as a quark-antiquark

The sigma cannot be made to behave as a quark-antiquark even by forcing it

The sigma:

G. Ríos and JRP, Phys.Rev.Lett.97:242002,2006.

Large Nc behavior of UNITARIZED $\pi\pi\rightarrow\pi\pi$ TWO LOOP ChPT



quark-antiquark mixing may emerge at larger Nc at M≈1 GeV

The $f_0(600)$ still does NOT behave DOMINANTLY as quark-antiquark

BUT, from Nc>8 or 10, the $f_0(600)$ we might be seeing a quark-antiquark **subdominant** component whose large Nc mass is ≥ 1 GeV

These results suggest what LIGHT SCALARS ARE NOT predominantly made of...

The light scalar nonet
DOMINANT component
is NOT a quark-antiquark state

Results consistent at higher orders.
The σ cannot be forced to behave as a quark-antiquark.

At two loops, a subdominant quark-antiquark component emerges above 1 GeV!!
(consistent with two nonet picture, the lightest non-quark antiquark)

In agreement with similar conclusions of quark models by Rupp-Van Beveren et al.
or unitary chiral approach by Oset-Oller. Or report of Tornqvist & Close J.PhysG28:R249(2002)

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2nd Part

Chiral Extrapolation: the quark mass dependence

C. Hanhart, G. Ríos and JRP, Phys.Rev.Lett.100:152001,2008.

NEW!

Motivation for Chiral extrapolation

- The LATTICE provides rigorous and systematic QCD results in terms of quarks and gluons with growing interest in scattering and the scalar sector.

  Caveat: small, realistic, quark masses are hard to implement.

- Anthropic considerations...

ChPT provides the correct QCD dependence of quark masses
as an expansion...

We can study the scalars in Unitarized ChPT for
larger quark masses
(chiral extrapolation)
and provide a reference for lattice studies

Changing the pion mass (Applicability)

How high can we make pion mass?

We do not want to spoil the chiral expansion

SU(3) ChPT works well with kaon masses ~ 494 MeV → with m_π < 500 MeV we are OK (And we are using **Unitarized** ChPT!)

But we are working with SU(2) and there are not kaons. We do not want to reach the kaon thershold

$$\left(\frac{m_\pi}{m_\pi^{phys}}\right)^2 \simeq \frac{m_q}{m_q^{phys}}$$

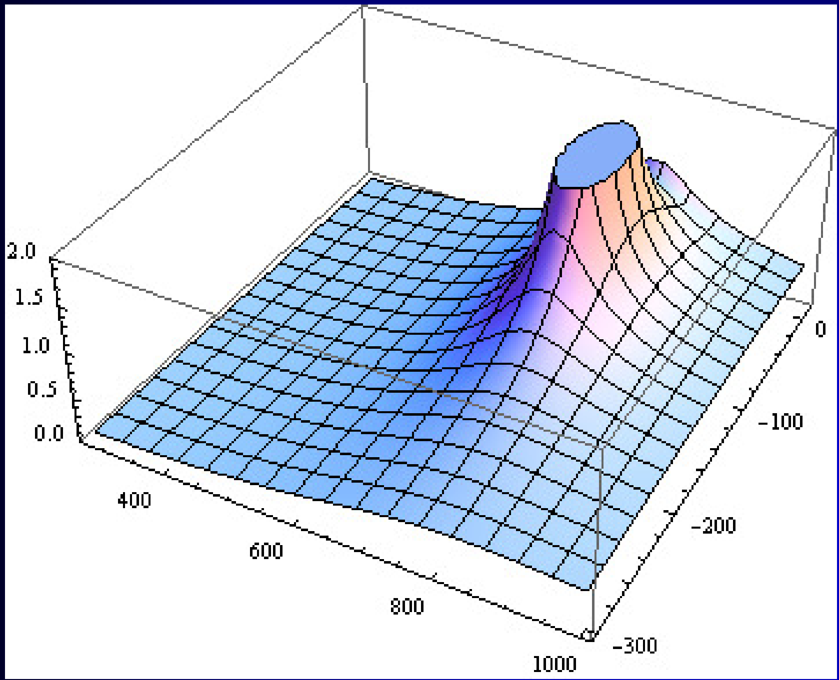
$$\left(\frac{m_K}{m_K^{phys}}\right)^2 \simeq \frac{m_q + m_s}{m_q^{phys} + m_s} \simeq \frac{\left(\frac{m_\pi}{m_\pi^{phys}}\right)^2 + \frac{m_s}{m_q^{phys}}}{1 + \frac{m_s}{m_q^{phys}}}$$

For m_π = 500 MeV we have m_K ~ 600 MeV

m_π = 500 MeV is our limit of applicability

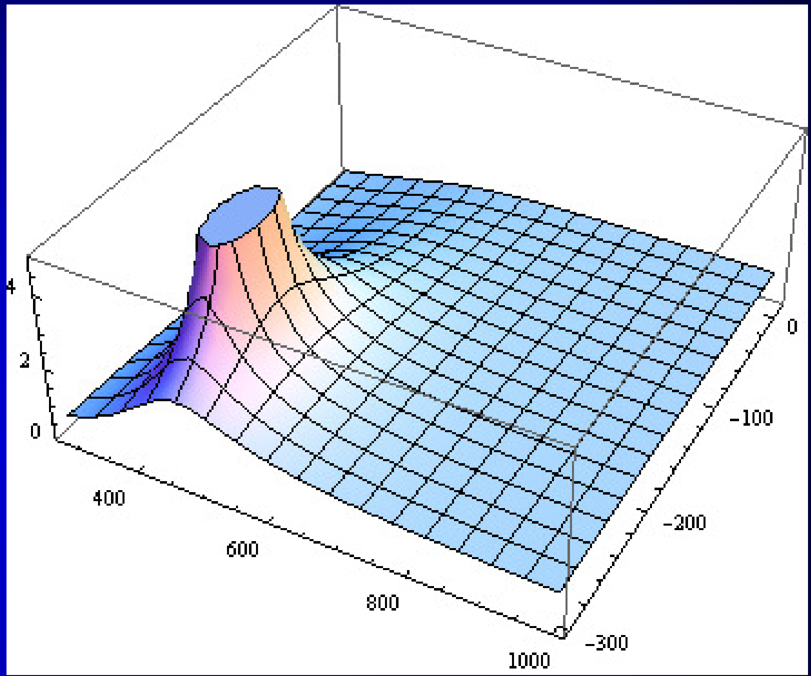
Pole movements with increasing m_π

The $\rho(770)$



Becomes narrower,
and gets closer to the
new threshold
At some points enters the first
sheet (bound state)

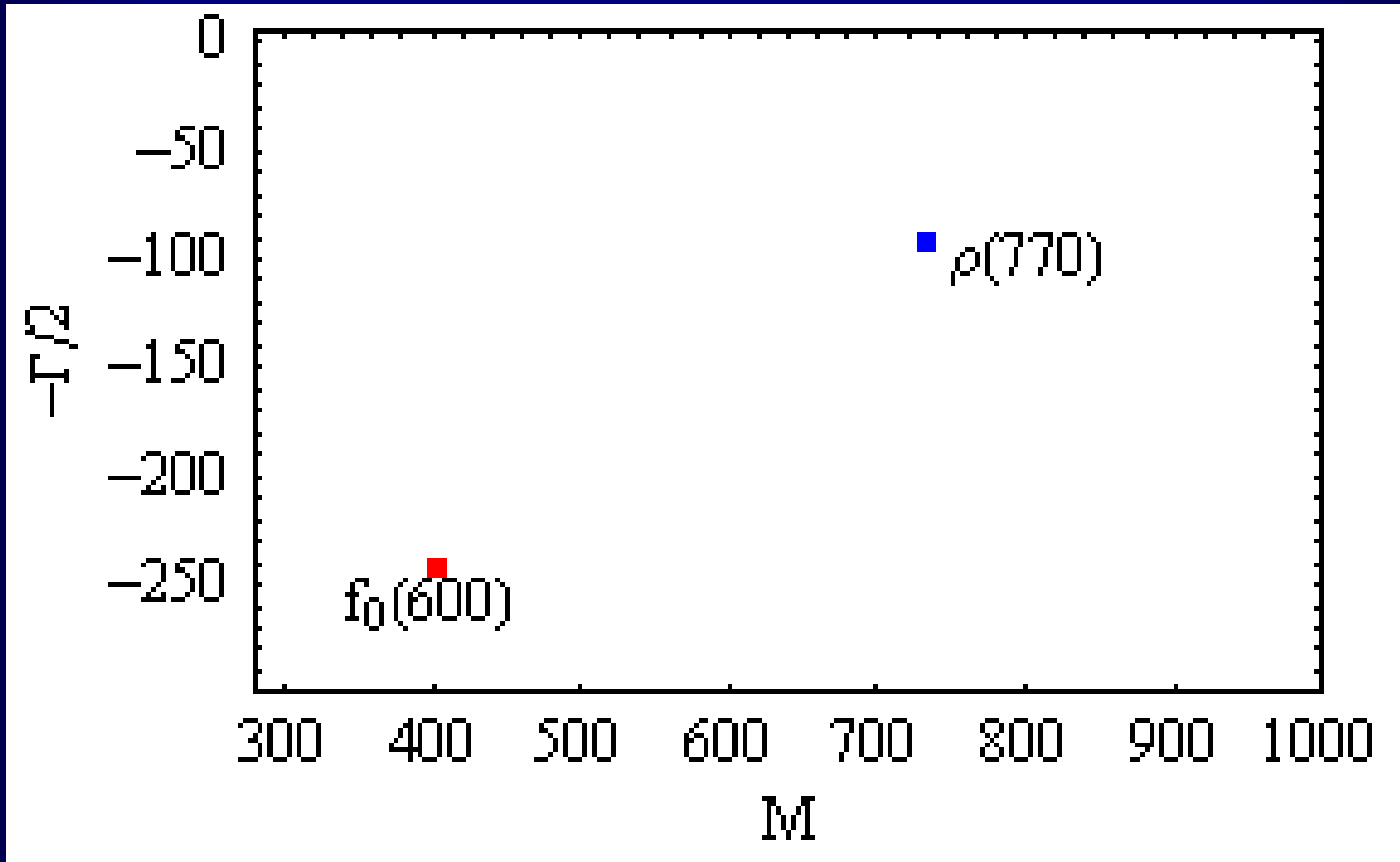
The $f_0(600)$ or σ



Becomes narrower.
When in real axis
“splits” in two real poles

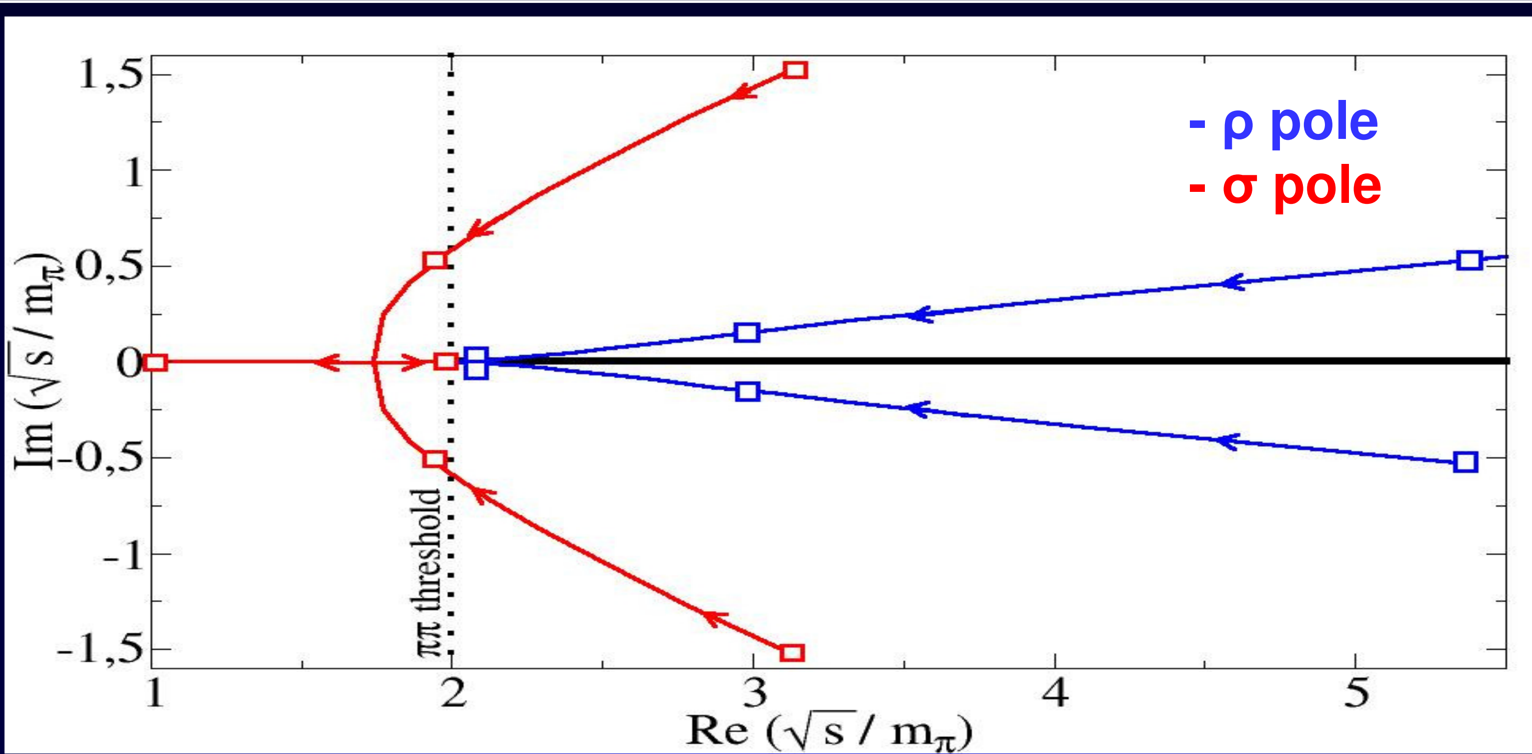
Pole movements with increasing m_π

ρ(770) versus f0(600)



UChPT SU(2) single channel calculation

Pole movements with increasing m_π



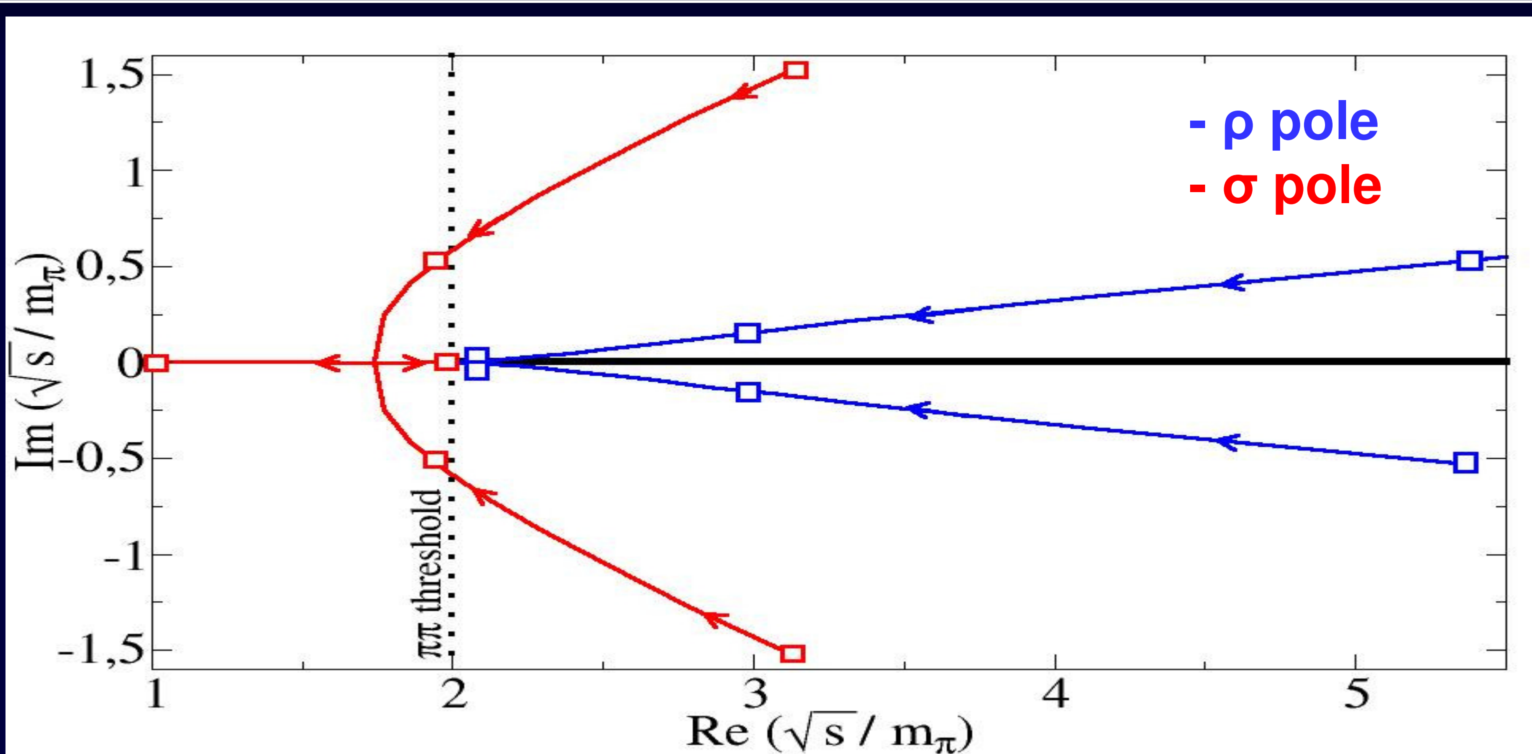
To follow the position relative to threshold: normalize to m_π units

<u>The rho:</u> Conjugate poles reach the real axis AT THRESHOLD:

- one pole in the 1st sheet (bound state).

- another in the 2nd sheet in almost the same position

Pole movements with increasing m_π



The sigma: 1) Conjugate poles reach the real axis BELOW threshold:

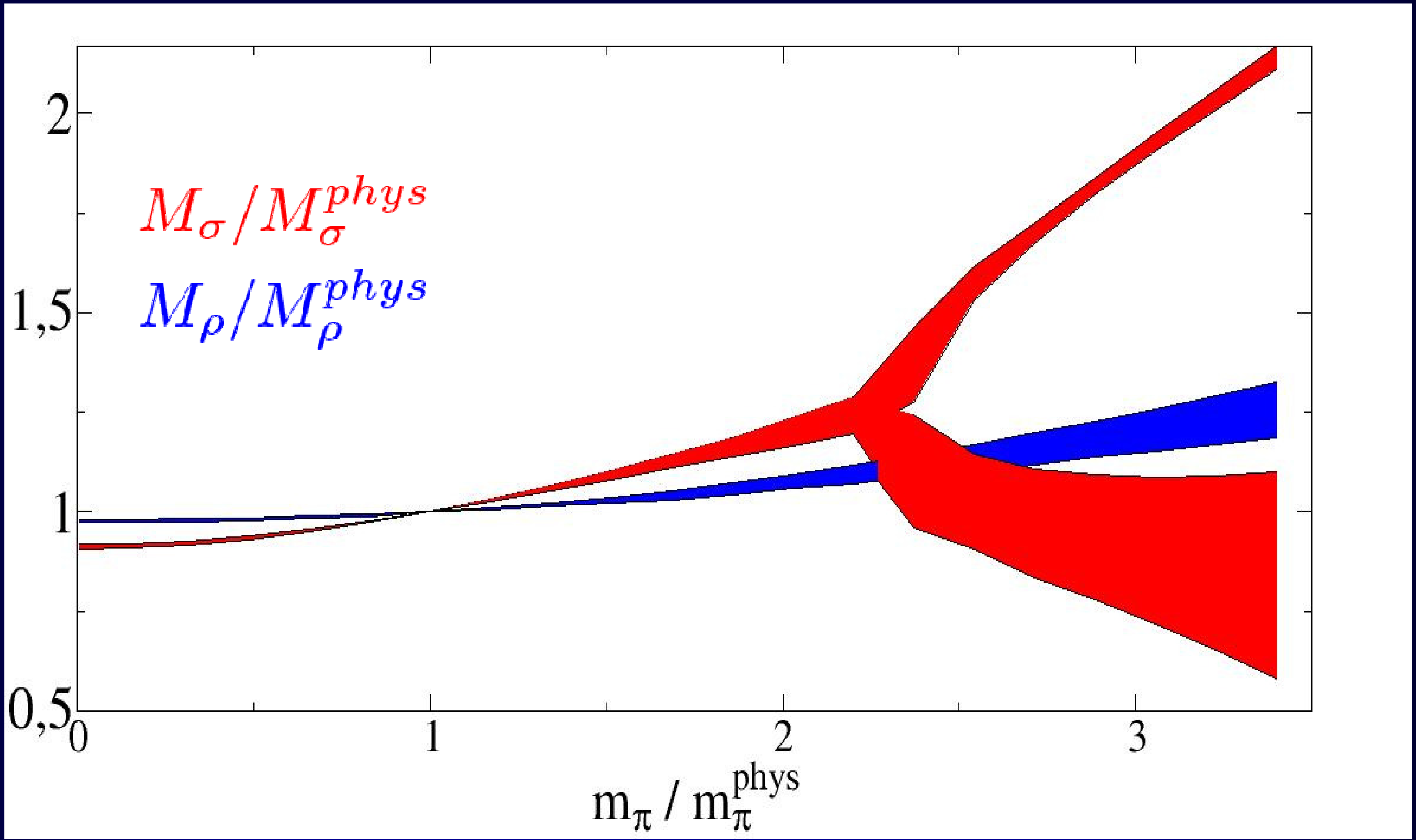
2) TWO real POLES on the 2nd sheet: "Splitting" typical of scalars.

3) One moves towards threshold until it jumps to the 1st sheet.
The other remains on the 2nd sheet in ASYMMETRIC position

If very asymmetric: sizable "molecular" component

Morgan, Pennington. PRD**48** (1993) 1185

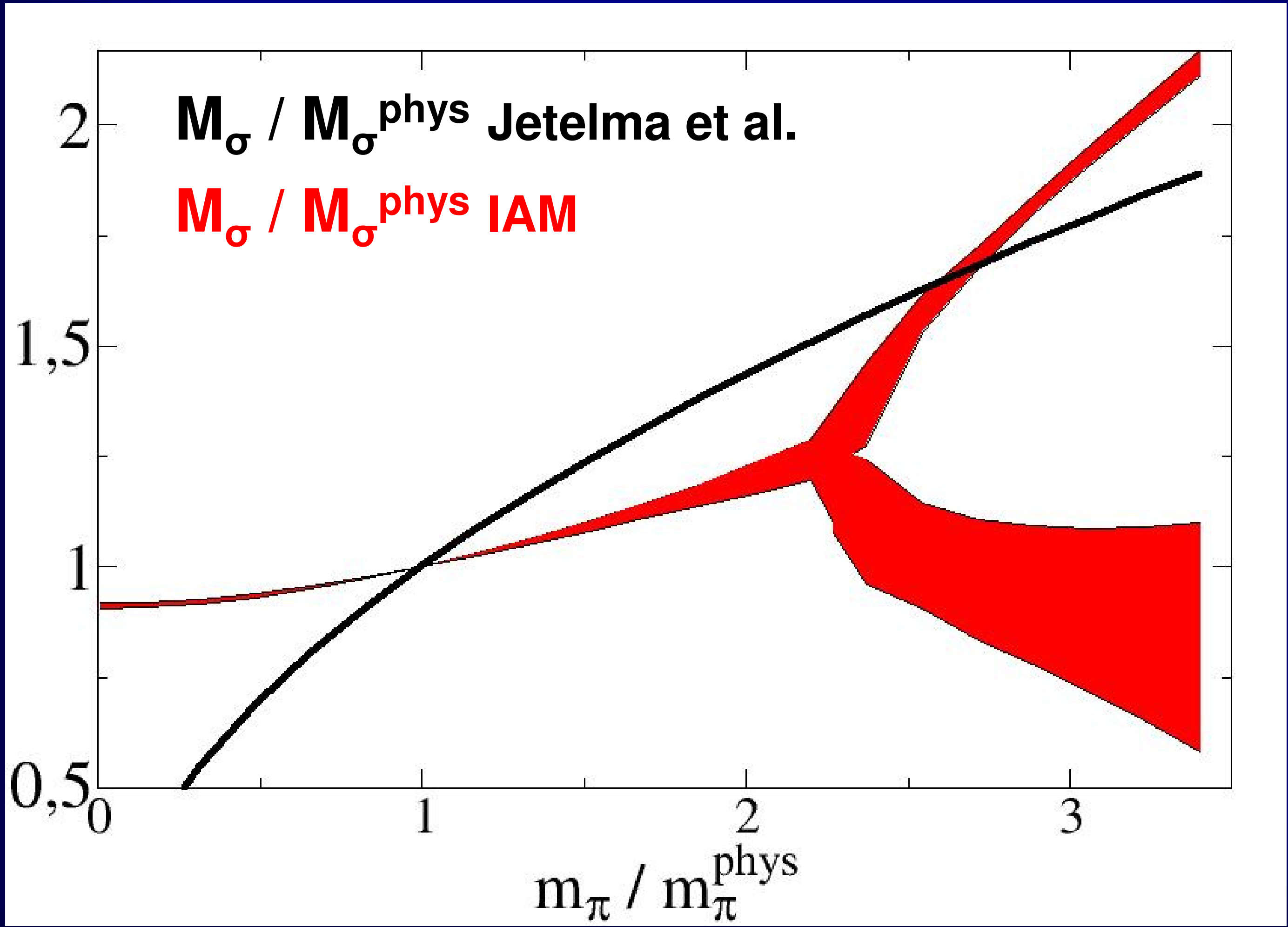
Resonane mass m_π dependence



There is a “non-analyticity” in the sigma m_π dependence.

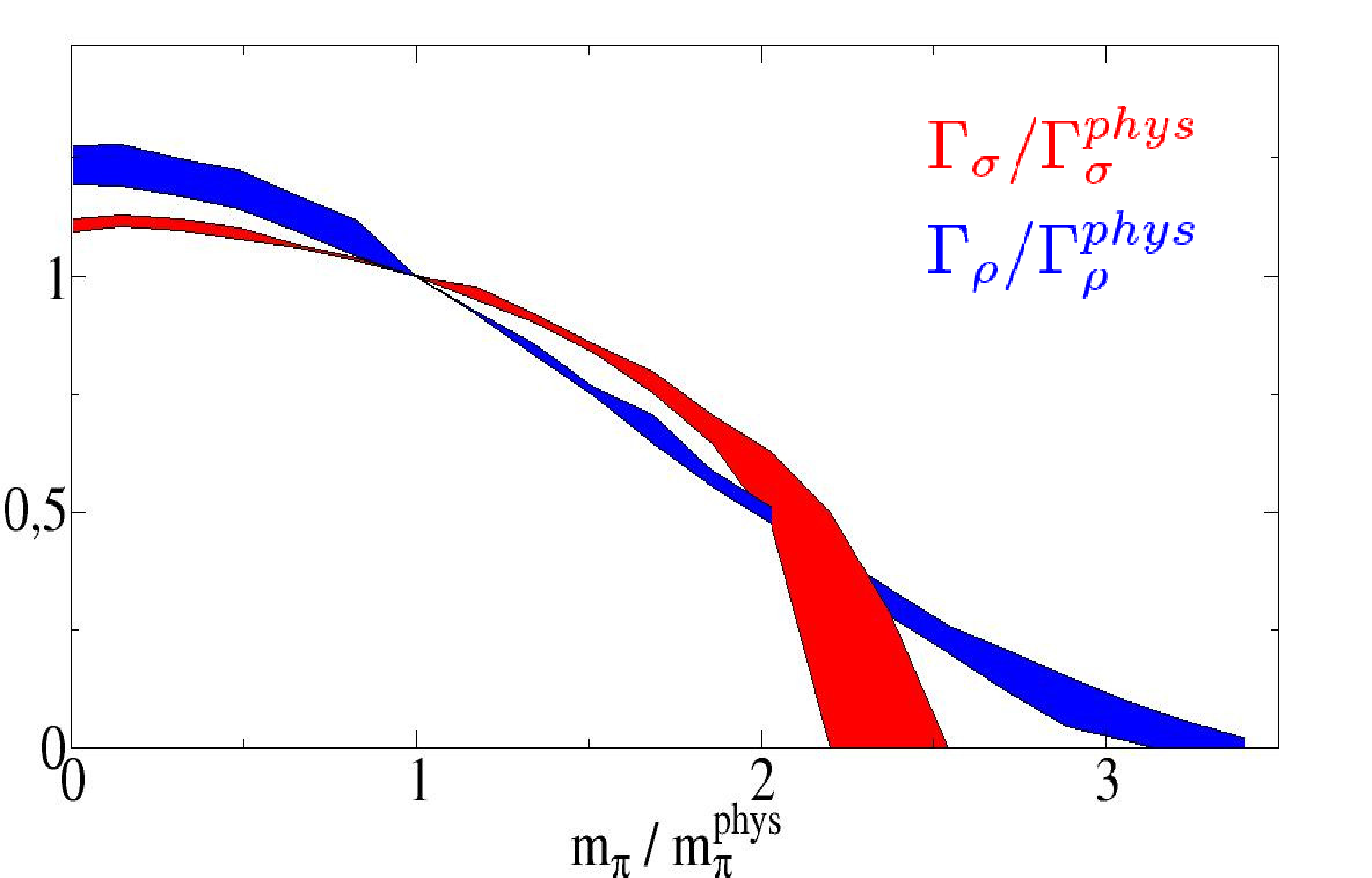
The rho mass grows slower than sigma

The sigma mass behavior is much softer than in estimates inspired in the LσM (E. Jetelma and M. Sher, PRD61,017301 (1999)) and shows a “non analyticity”



This may have some anthropic principle consequences…

Resonance width m_π dependence

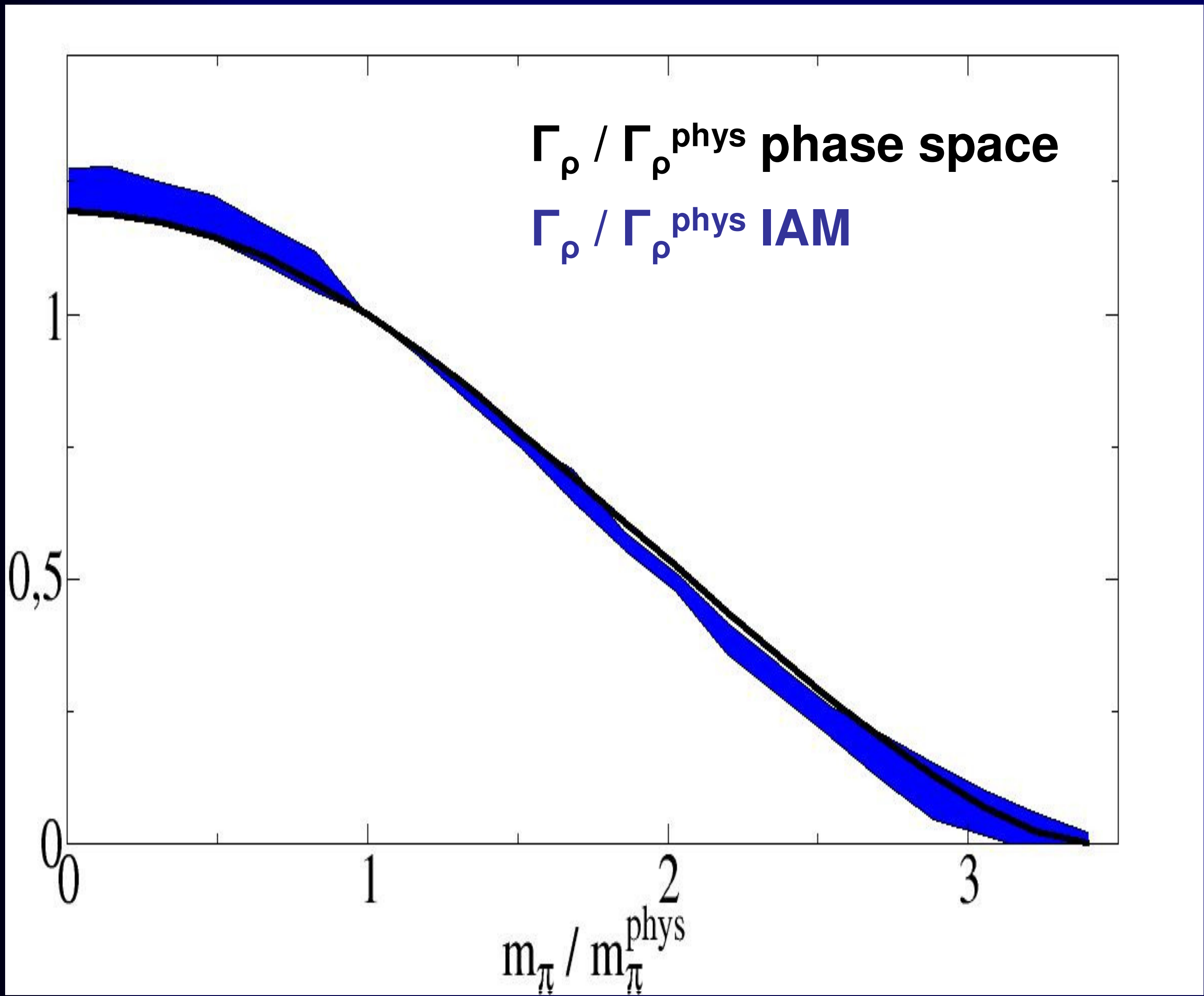


For a narrow vector particle (like the rho) the decay width is given by

$$\Gamma_\rho = \frac{g^2\,|\vec{p}|^3}{6\pi M_\rho^2} \quad \begin{cases} \dfrac{|\vec{p}|^3}{M_\rho^2} & \text{Phase space} \\ g^2 & \text{Coupling to pions} \end{cases}$$

We can calculate the width variation due to phase space reduction and compare with our results. The difference gives the dependence of the coupling constant on the pion mass

Rho width m_π dependence vs. phase space

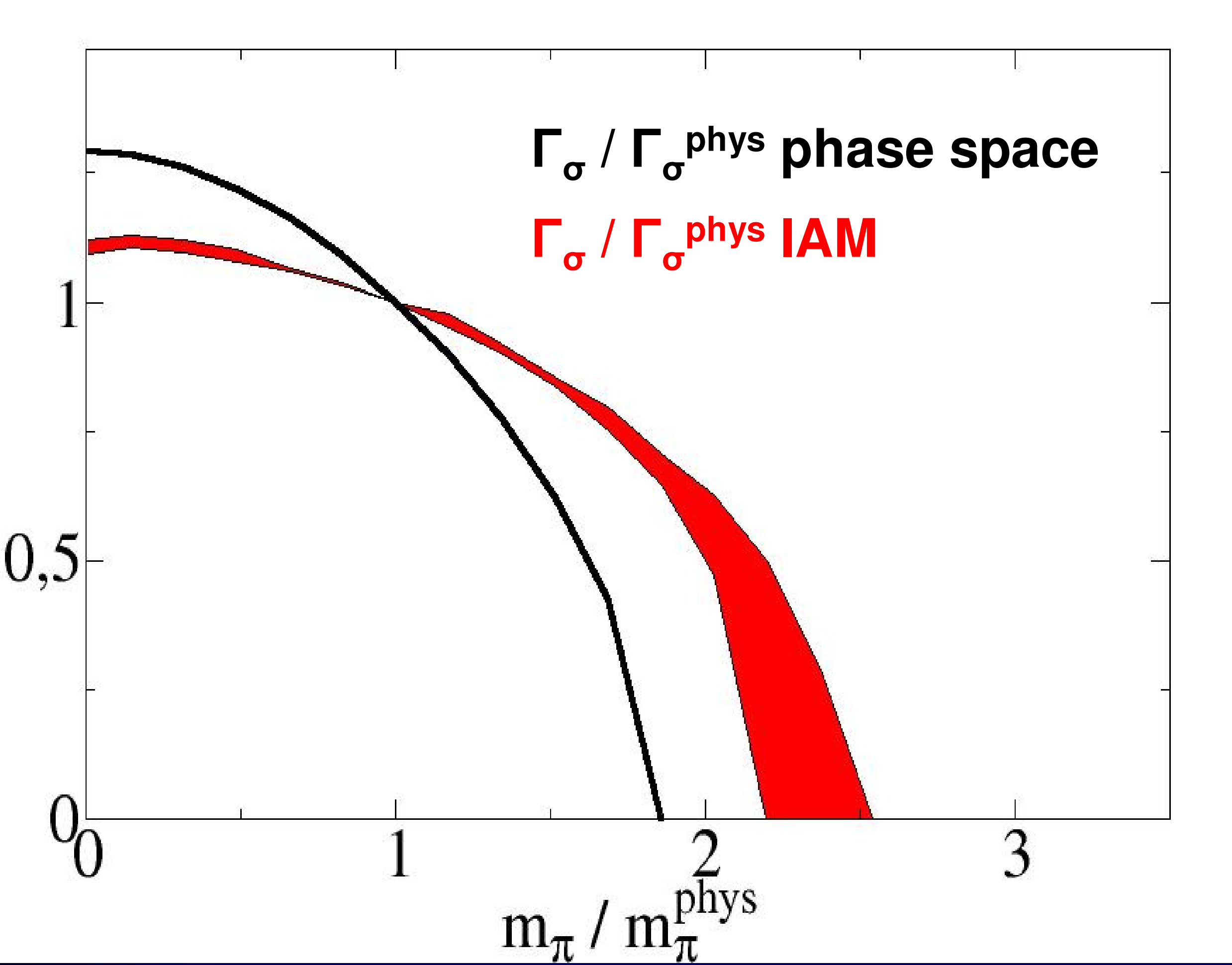


Width behavior explained by phase space

$\rho \rightarrow \pi\pi$ coupling almost independent of m_π

(assumption in some lattice calculations)

It **does not follow** the phase space decrease of a Breit-Wigner:



$$\Gamma_\sigma = \frac{g^2 |\vec{p}|}{8\pi M_\sigma}$$

Very bad approximation for a wide resonance as the sigma

g dependence on m_π

The **dynamics** of the sigma decay depends strongly on the pion(quark) mass
(Recall that some pion-pion vertices in ChPT depend on the pion mass).

Rho mass dependence on pion mass Bruns, Meissner EPJ C**40** (2005) 97

$$M_\rho = M_\rho^0 + c_1 m_\pi^2 + c_2 m_\pi^3 + c_3 m_\pi^4 \log\left(\frac{m_\pi^2}{M_\rho^2}\right) + O(m_\pi^4)$$

Natural O(1) values expected for the c_i parameters

We can fit our extrapolation and make a prediction for the paremeters.

We only fit the first two terms

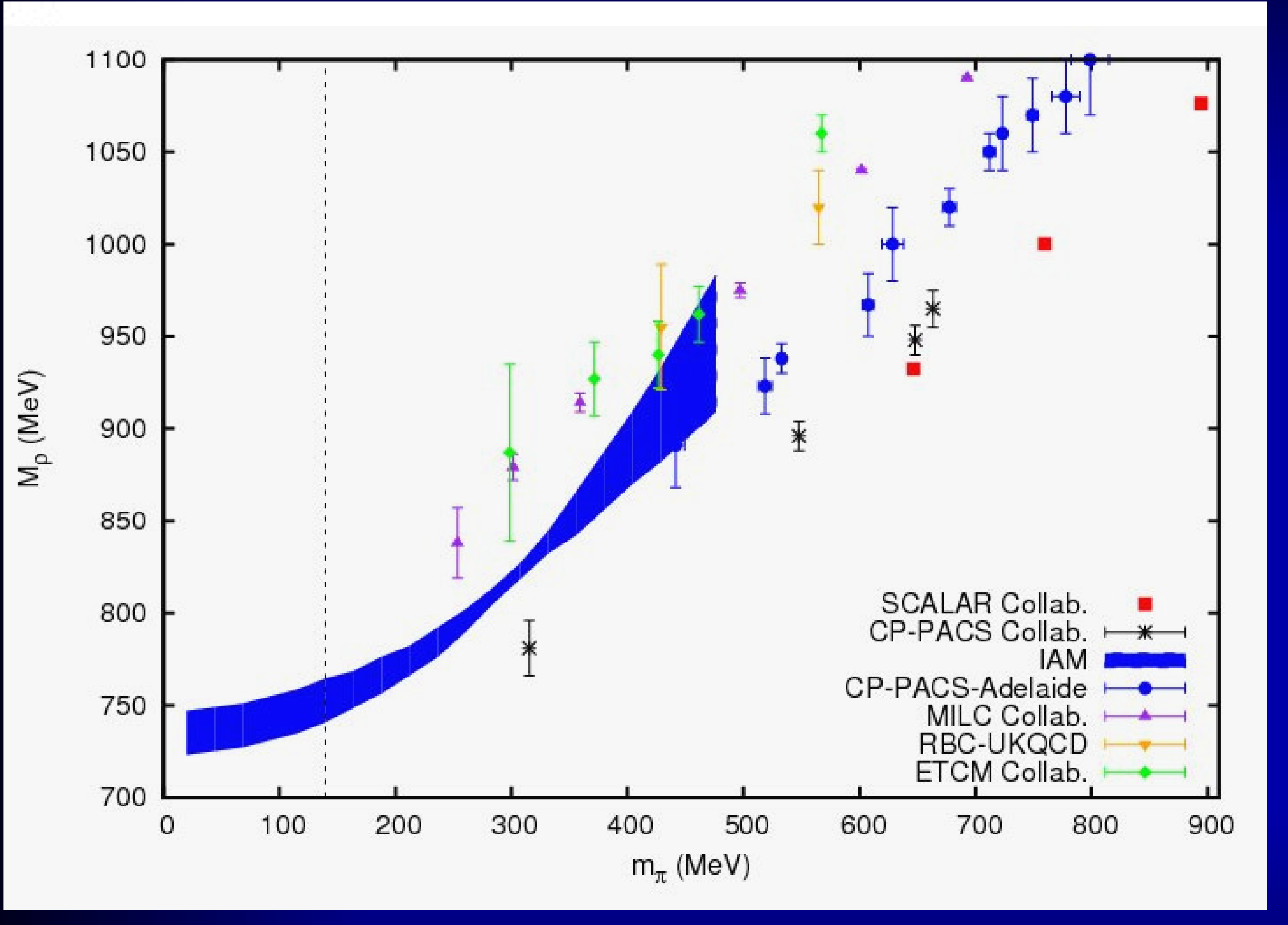
$$M_\rho^0 = 0.735 \pm 0.0017 \text{ GeV}$$
$$c_1 = 0.90 \pm 0.17 \text{ GeV}^{-1}$$

Compares well
with fits to lattice

Bruns, Meissner EPJ C**40** (2005) 97

$$M_\rho^0 = 0.65 \text{ to } 0.80 \text{ GeV}$$
$$c_1 = -1.2 \text{ to } 2.2 \text{ GeV}^{-1}$$

Comparison with lattice results for the rho



CAUTION!!!

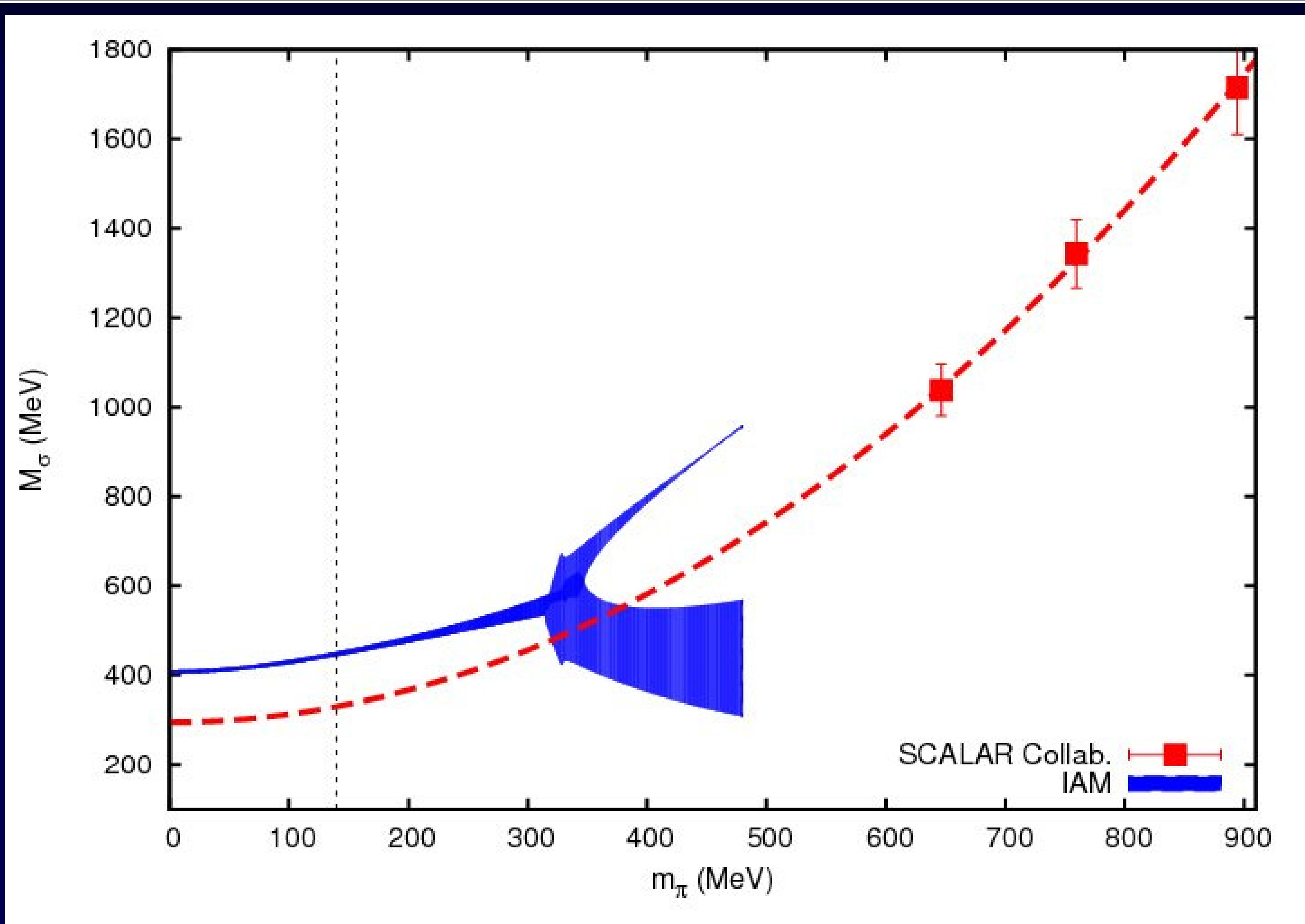
We give POLE MASS in complex plane

Lattice caveats:

Improved actions,
Lattice spacing...
Finite volume...
WIDTHLESS rho

The best would be to use ChPT on the lattice....future work

Comparison with lattice results for the sigma



AGAIN CAUTION!!!

We give POLE MASS in complex plane + usual lattice caveats

IMPORTANT REMARK
Extrapolations should take care of known scalar mass “splitting” non-analyticity

Summary:Scalars in Unitarized Chiral Perturbation Theory

Intro

- Simultaneously resonances and low energy meson-meson scattering with parameters compatible with ChPT
- Generates Light Scalar Nonet and vector octet

1st Part

N_c behavior of light resonances

- quark-antiquark remarkably good for vectors
- SCALARS predominantly NOT quark-antiquark states

  SUBDOMINANT quark-antiquark component around 1.1 GeV.
  (Suggests mixing with heavier ordinary scalar nonet)

2nd Part

Pion mass dependence of ρ(770) and f0(600) mass and width

- $M\rho$ behavior qualitatively similar to lattice,
  Caution: lattice non-zero width and that we use pole masses
- We predict the parameters giving the $M\rho$ dependence on $M\pi$.
  Consistent with chiral fits.
- $\Gamma\rho$ just obeys the BW phase space reduction whereas $\Gamma\sigma$ does not.
  Dynamical $M\pi$ effects through 2 pion couplings!
- The sigma mass dependence is stronger than for the rho

In progress...

SU(3), Coupled channels, finite volume effects, strange resonances....

Spare transparencies...

My results suggest that...

One type of Jaffe's **tetraquarks** large Nc behavior is **qualitatively** consistent with mine

but I DO NOT CLAIM that...

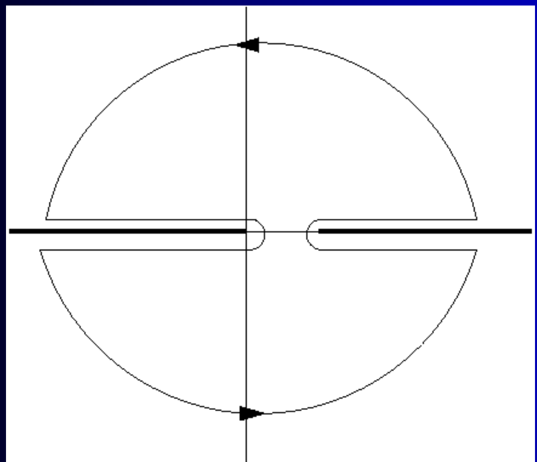
The whole sigma vanishes in the large Nc ONLY the DOMINAT COMPONENT

scalars are tetraquarks

Jaffe's model is right or wrong

The analytic structure of 1/t (right cut, left cut and possible poles) allows us to write a dispersion relation for 1/ t substracted at the Adler zero, s_A

$$\frac{1}{t(s)} = \frac{s-s_2}{\pi}\int_{RC}\frac{-\text{Im}\, t_4(s')/t_2^2(s')\, ds'}{(s'-s_2)(s'-s-i\epsilon)} + LC(1/t) \quad + PC(1/t)$$



On the right cut we use elastic unitarity $\quad \text{Im}\,\frac{1}{t} = -\sigma = -\text{Im}\,\frac{t_4}{t_2^2}$

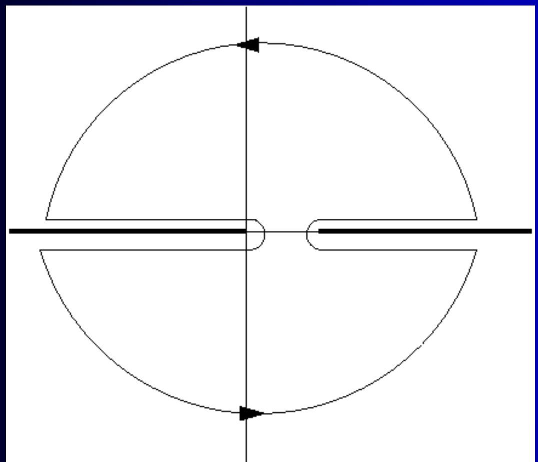
and approximate the Adler zero by its LO approximation $\quad s_A \simeq s_2$

Right cut imaginary part **known exactly**

$$\frac{s-s_A}{s'-s_A} \simeq \frac{s-s_2}{s'-s_2}$$ LO very good approximation for s' far from s_A

The analytic structure of 1/t (right cut, left cut and possible poles) allows us to write a dispersion relation for 1/ t substracted at the Adler zero, s_A

$$\frac{1}{t(s)} = \frac{s-s_2}{\pi}\int_{RC}\frac{-\mathrm{Im}\,t_4(s')/t_2^2(s')\,ds'}{(s'-s_2)(s'-s-i\epsilon)} + LC(t_4/t_2^2) + PC(1/t)$$



On the left cut we use ChPT

$$LC(1/t) \simeq -LC(t_4/t_2^2)$$

Left cut weighted at low energies where **ChPT valid**

even more suppresed when s is in the physical and resonace region (near right cut)

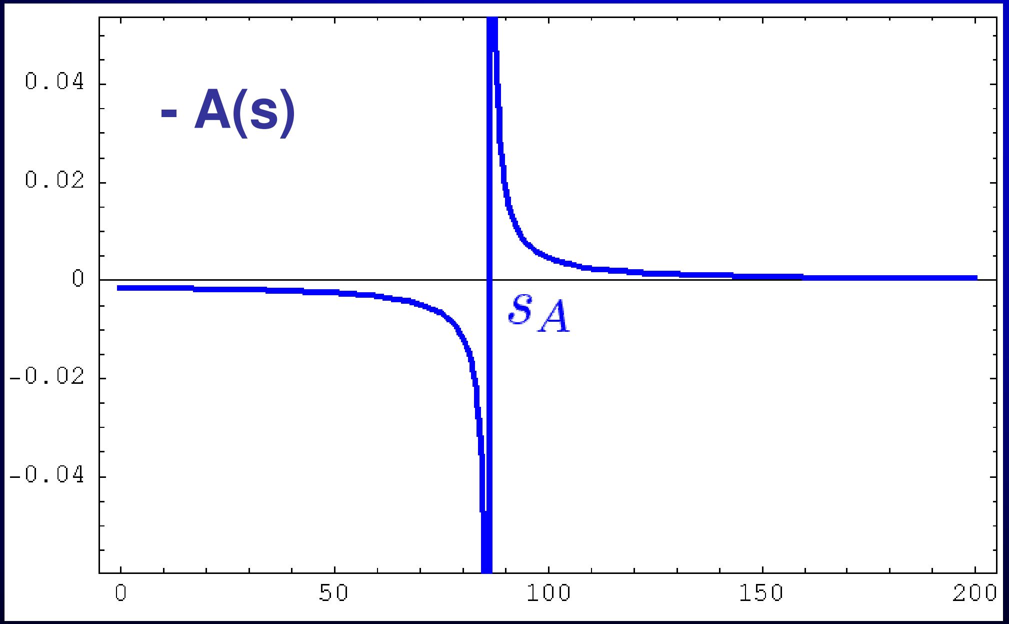
FOR MIKE PENNINGTON's eyes only:

If we do NOT neglect the pole contribution: $t \simeq \frac{t_2^2}{t_4 - t_2 + A}$

$$A(s) = t_4(s_2) - \frac{(s_2 - s_A)(s - s_2)}{s - s_A}[t_2'(s_2) - t_4'(s_2)]$$

If we set $A(s)=0$ we get the standard IAM. This is the case of the p wave

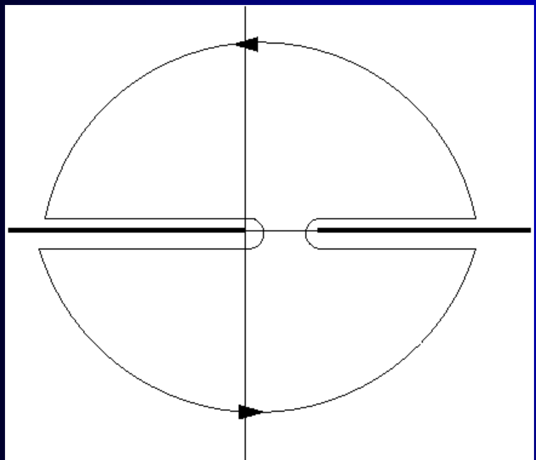
A(s) counts O(p^6) and is **numerically very small** except near Adler zero



The differences with the standard IAM are less than 1% in the physical and resonance region

The analytic structure of 1/t (right cut, left cut and possible poles) allows us to write a dispersion relation for 1/ t substracted at the Adler zero, s_A

$$\frac{1}{t(s)} = \frac{s-s_2}{\pi}\int_{RC} \frac{-\text{Im}\, t_4(s')/t_2^2(s')\, ds'}{(s'-s_2)(s'-s-i\epsilon)} - LC(t_4/t_2^2) + PC(1/t)$$



This is exactly the dispersion relation for $-t_4$ / t_2^2 except for the pole contribution

$$= -\, t_4(s) + PC(t_4/t_2^2)$$

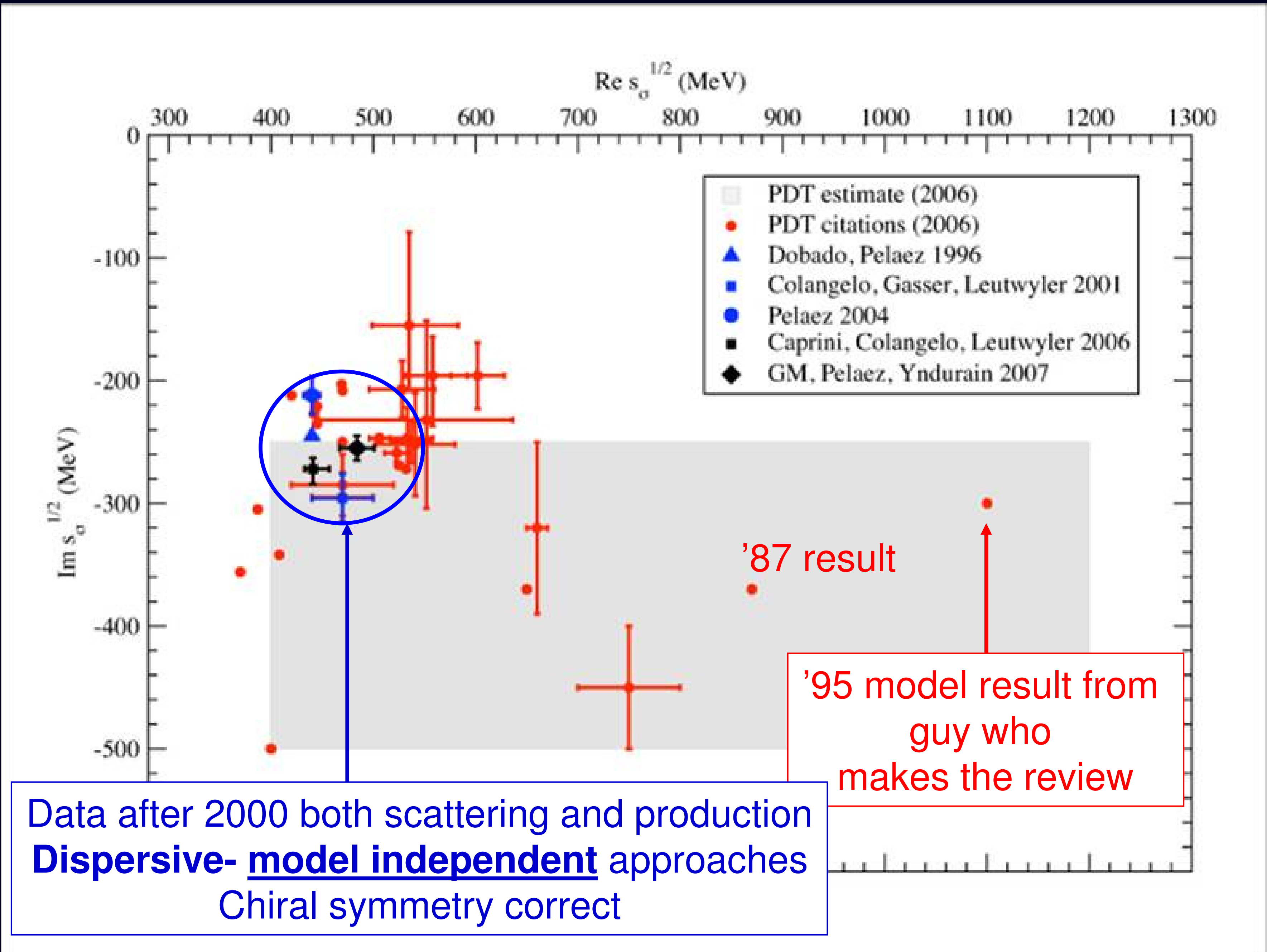
$$\frac{1}{t(s)} = -t_4(s) + PC(t_4/t_2^2) + PC(1/t)$$

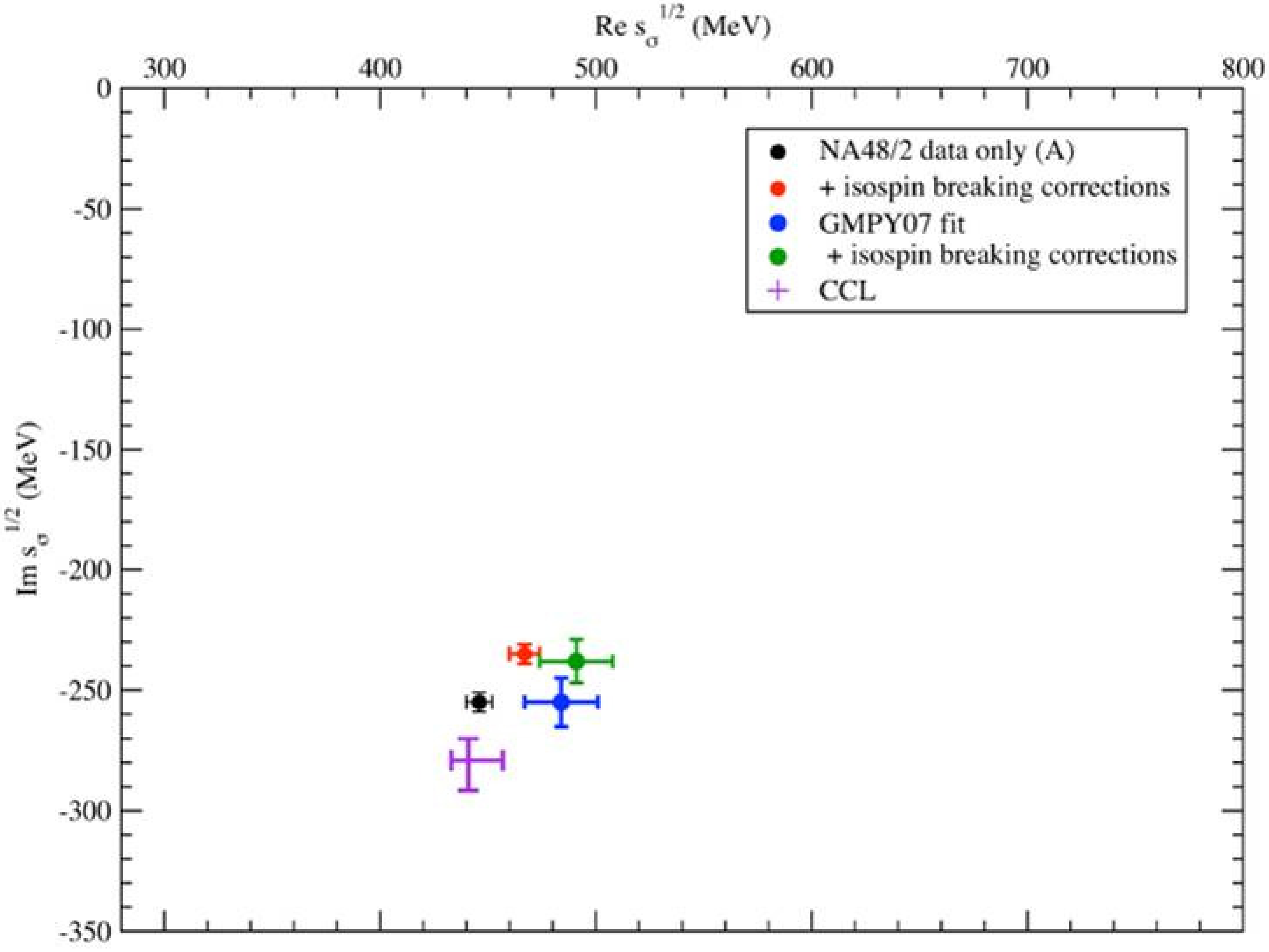
The pole contributions read

$$PC(t_4/t_2^2) = \frac{t_4(s_2)}{t_2'(s_2)^2(s-s_2)^2} + \frac{t_4'(s_2)}{t_2'(s_2)^2(s-s_2)} + \frac{t_4''(s_2)}{2t_2'(s_2)^2}$$

$$PC(1/t) = \frac{1}{t'(s_A)(s-s_A)} - \frac{t''(s_A)}{2t'(s_A)^2}$$

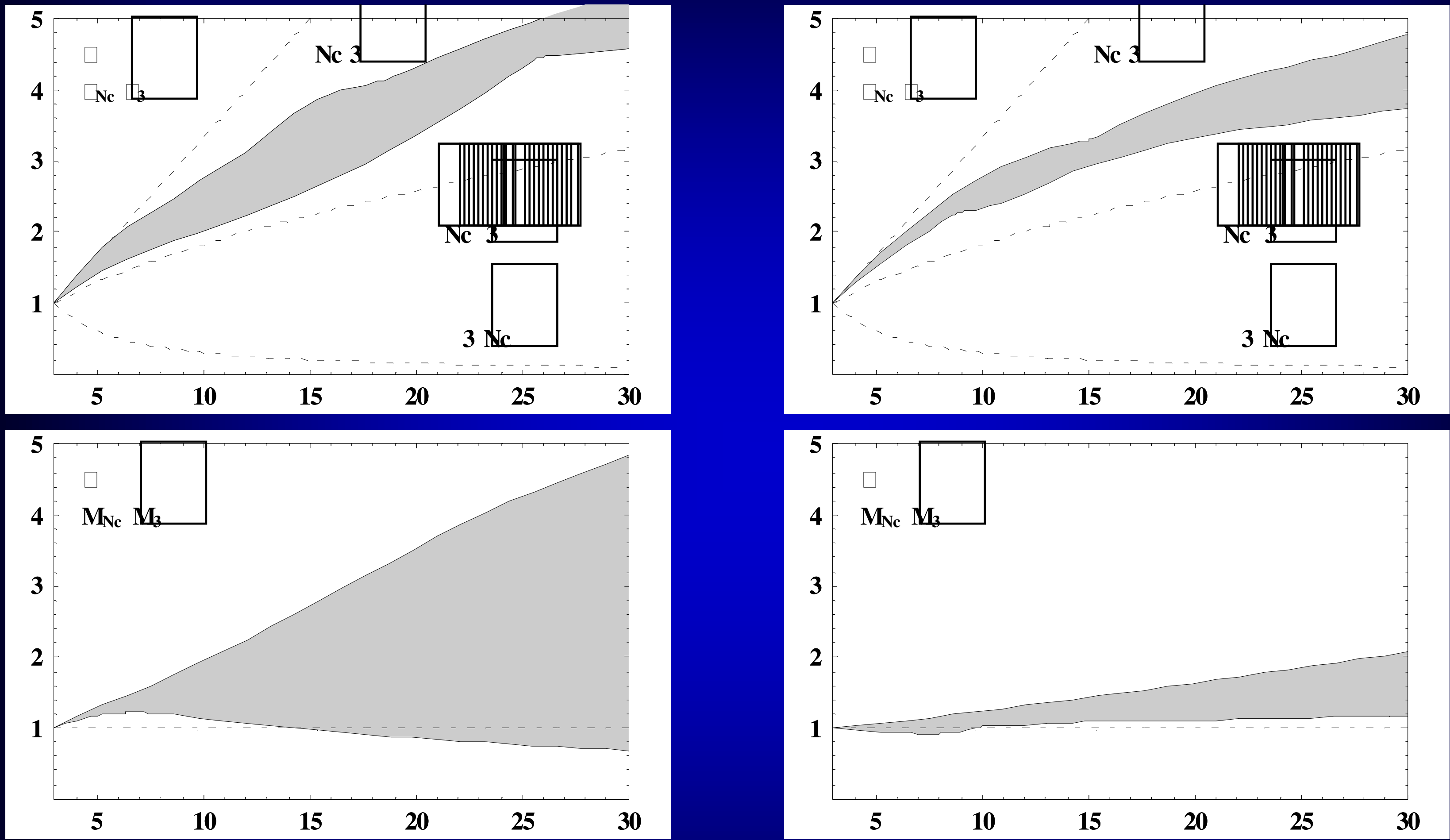
We use ChPT to evaluate the derivatives of t at the **Adler zero**
This is a low energy point → **ChPT perfectly justified**


Re $s_\sigma^{1/2}$ (MeV)
300 400 500 600 700 800 900 1000 1100 1200 1300
0 -100 -200 -300 -400 -500
Im $s_\sigma^{1/2}$ (MeV)
PDT estimate (2006)
PDT citations (2006)
Dobado, Pelaez 1996
Colangelo, Gasser, Leutwyler 2001
Pelaez 2004
Caprini, Colangelo, Leutwyler 2006
GM, Pelaez, Yndurain 2007
'87 result
'95 model result from guy who makes the review
Data after 2000 both scattering and production
Dispersive- model independent approaches
Chiral symmetry correct



Re $s_\sigma^{1/2}$ (MeV)
300
400
500
600
700
800
0
-50
-100
-150
-200
-250
-300
-350
Im $s_\sigma^{1/2}$ (MeV)
NA48/2 data only (A)
+ isospin breaking corrections
GMPY07 fit
+ isospin breaking corrections
CCL

The σ and the κ pole movement is NOT like that of $q\bar{q}$ states

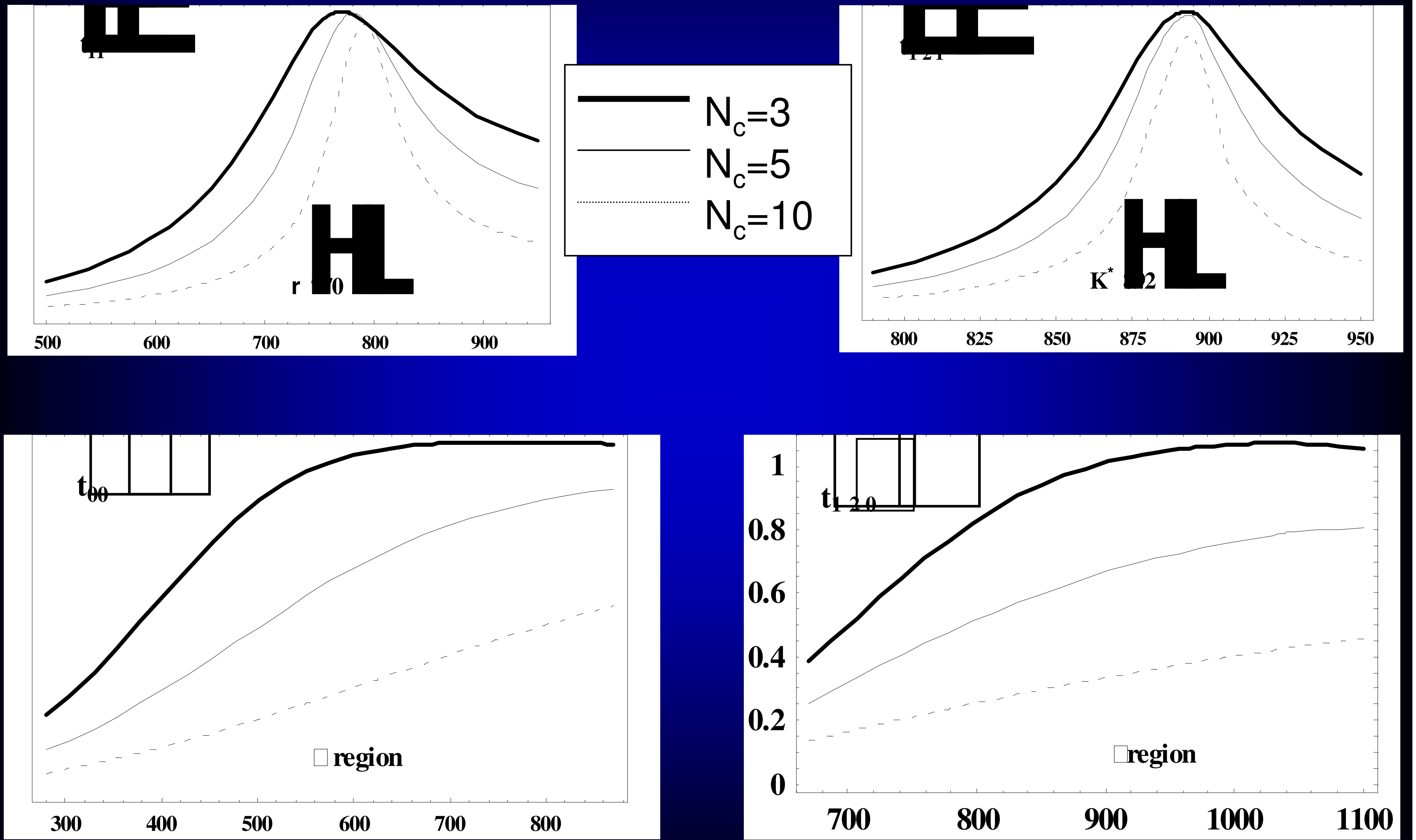


Large dependence on μ choice for Nc scaling, but

$$O(\sqrt{N_c}) < \Gamma < O(N_c)$$

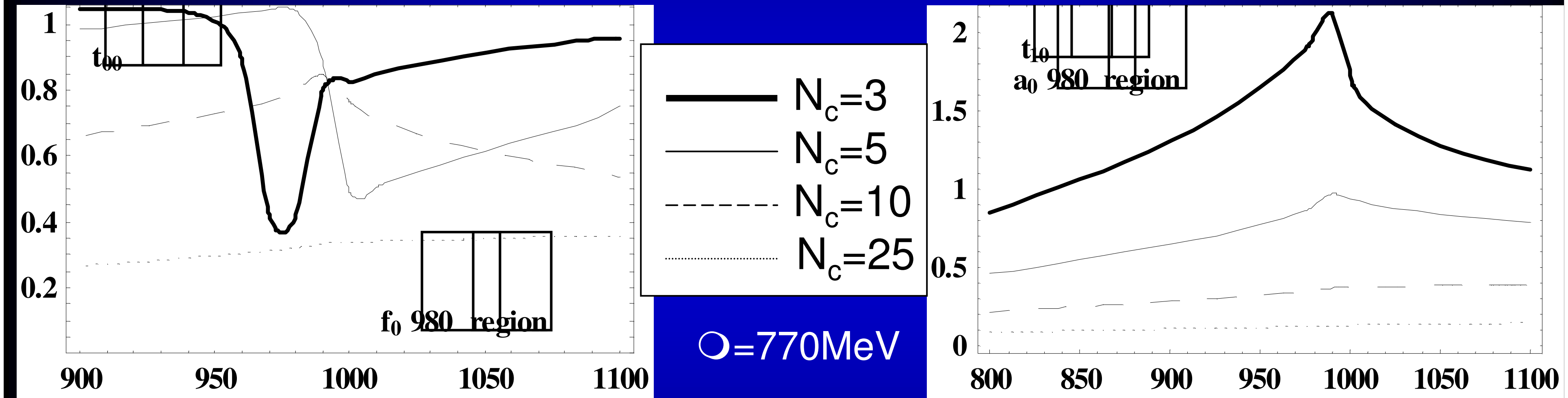
The width grows with Nc !!!

Another way of seeing it is with the modulus of the amplitude



What about the f_0 and the a_0 ?

Following the pole large Nc behavior is complicated
by the nearby thresholds, at least for relatively low Nc.
In addition, the $a_0(980)$ amplitude
on the real axis is more robust than the pole position



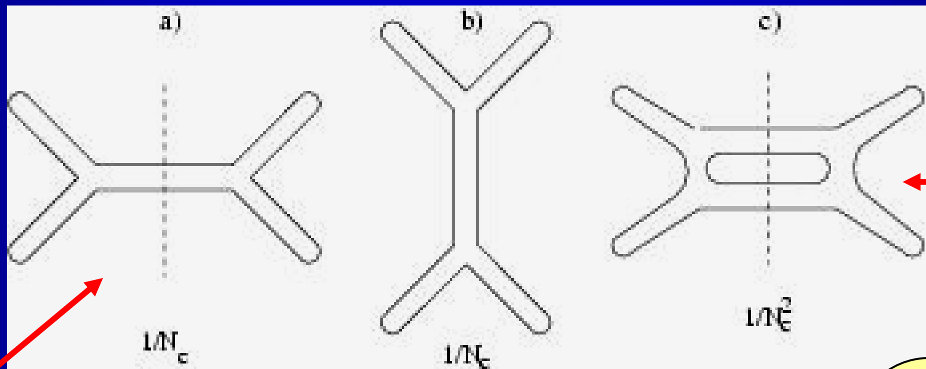
For sufficiently high Nc they also disappear in the continuum
like the σ and κ

- The main component of the σ and the κ, does not behave as qqbar state in the large Nc limit.

  What are they?

four quark=two meson
=glueball (if I=0 J=0)
in this counting

- <u>Possibility</u>: Some “four quark” or “two meson” states are predicted to become the Meson- meson continuum in the large Nc (R. Jaffe)



4 quark state
or glueball
Im t ~ $O(1/N_c^2)$

qqbar resonance
Im t ~ O(1) at peak
cannot be present
for σ and κ

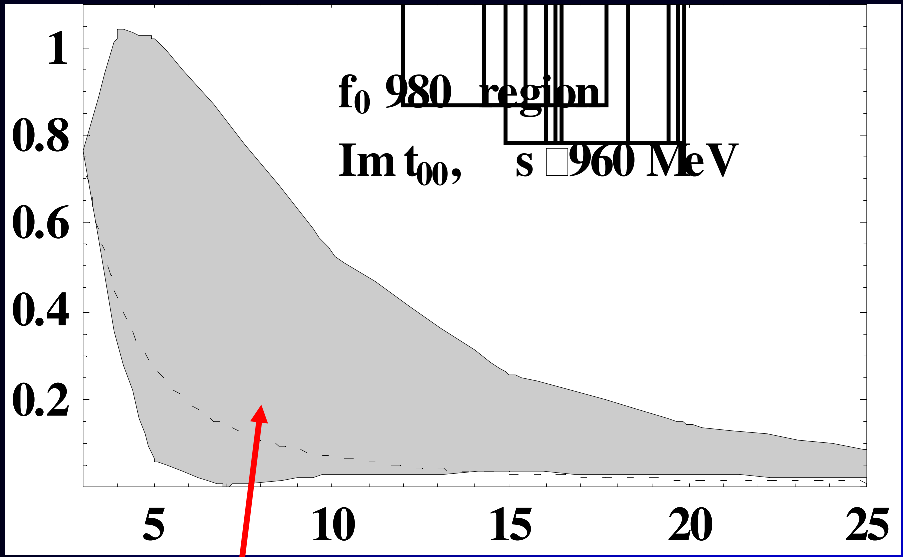
qqbar resonance t channel
contributes to Re t ~ O(1/Nc)
but
Im t = 0

For the σ and f_0
glueball interpretation
also possible.
Likely some mixing.
Not for κ or a_0

The imaginary part of amplitudes vanish consistently with Imt~$1/N_c^2$

$f_0(980)$: for all ❍

$a_0(980)$: for almost all ❍

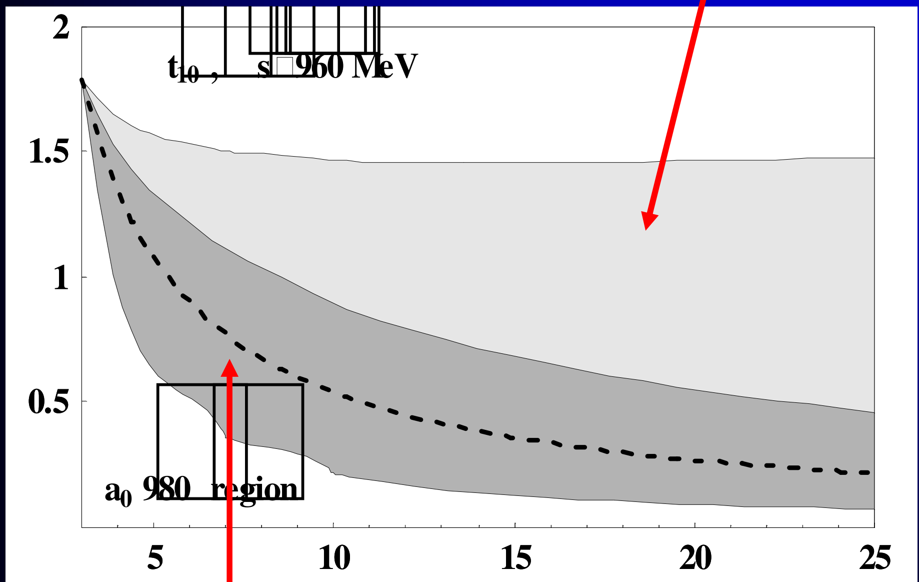


❍=0.5 to 1 GeV

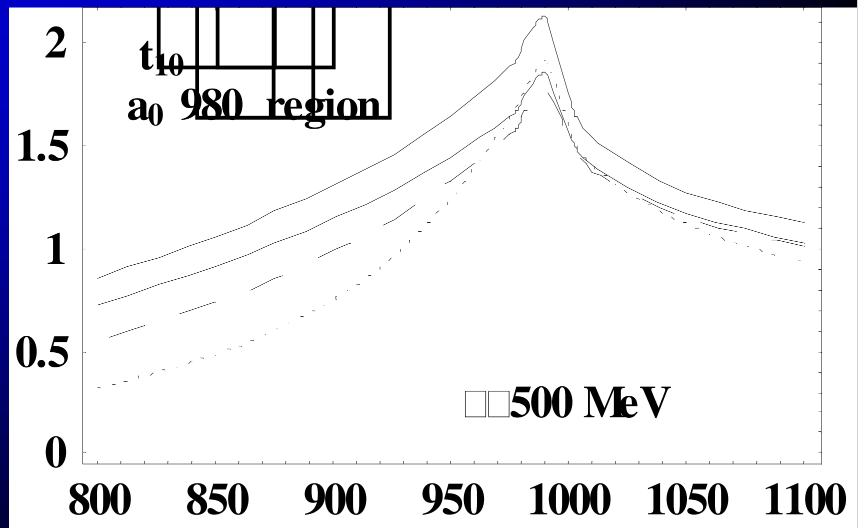
❍=0.50 to 0.55 GeV

in one corner of ❍ space,

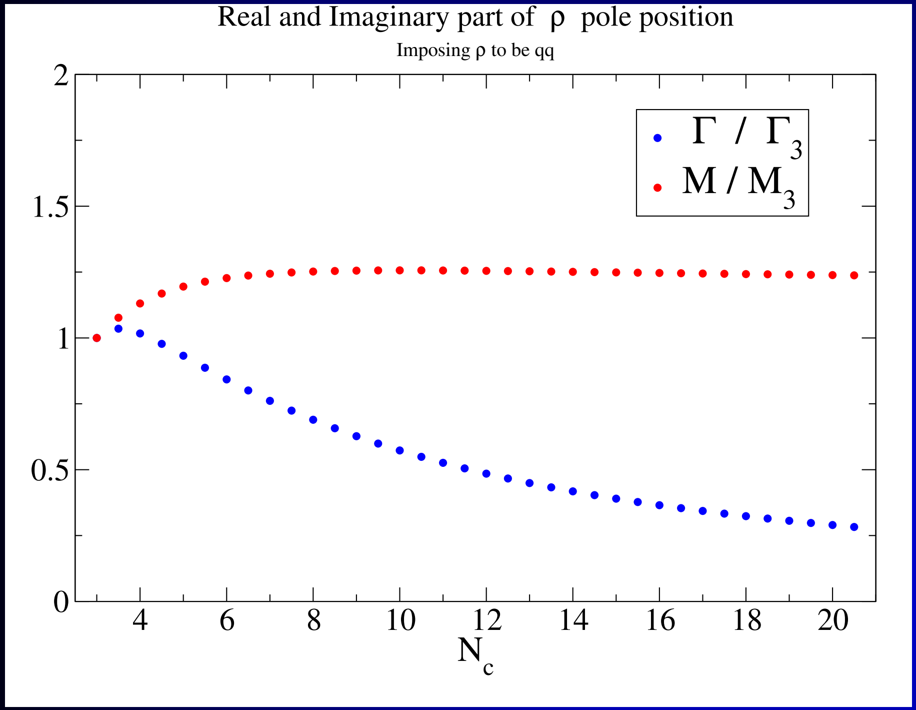
the $a_0(980)$ still behaves as qqbar

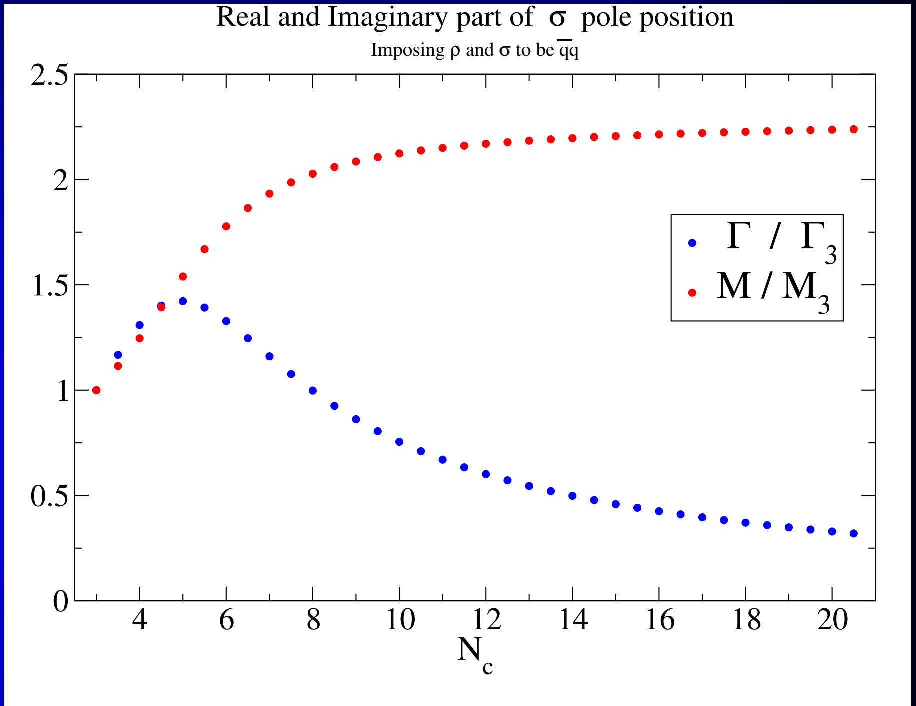


❍=0.55 to 1 GeV



The rho always comes out naturally as a quark-antiquark





The sigma cannot be made to behave as a quark-antiquark even by forcing it....

However....

Changing the quark mass = changing the pion mass

Actually, we study the pole dependence on $M_\pi^2 \propto m_q$

Including higher order ChPT corrections

O(p⁴) amplitudes written in terms of the μ scale independent LECs

$$\bar{l}_i = \frac{32\pi}{\gamma_i} l_i^r(\mu) - \log\left(\frac{m_\pi^2}{\mu^2}\right)$$

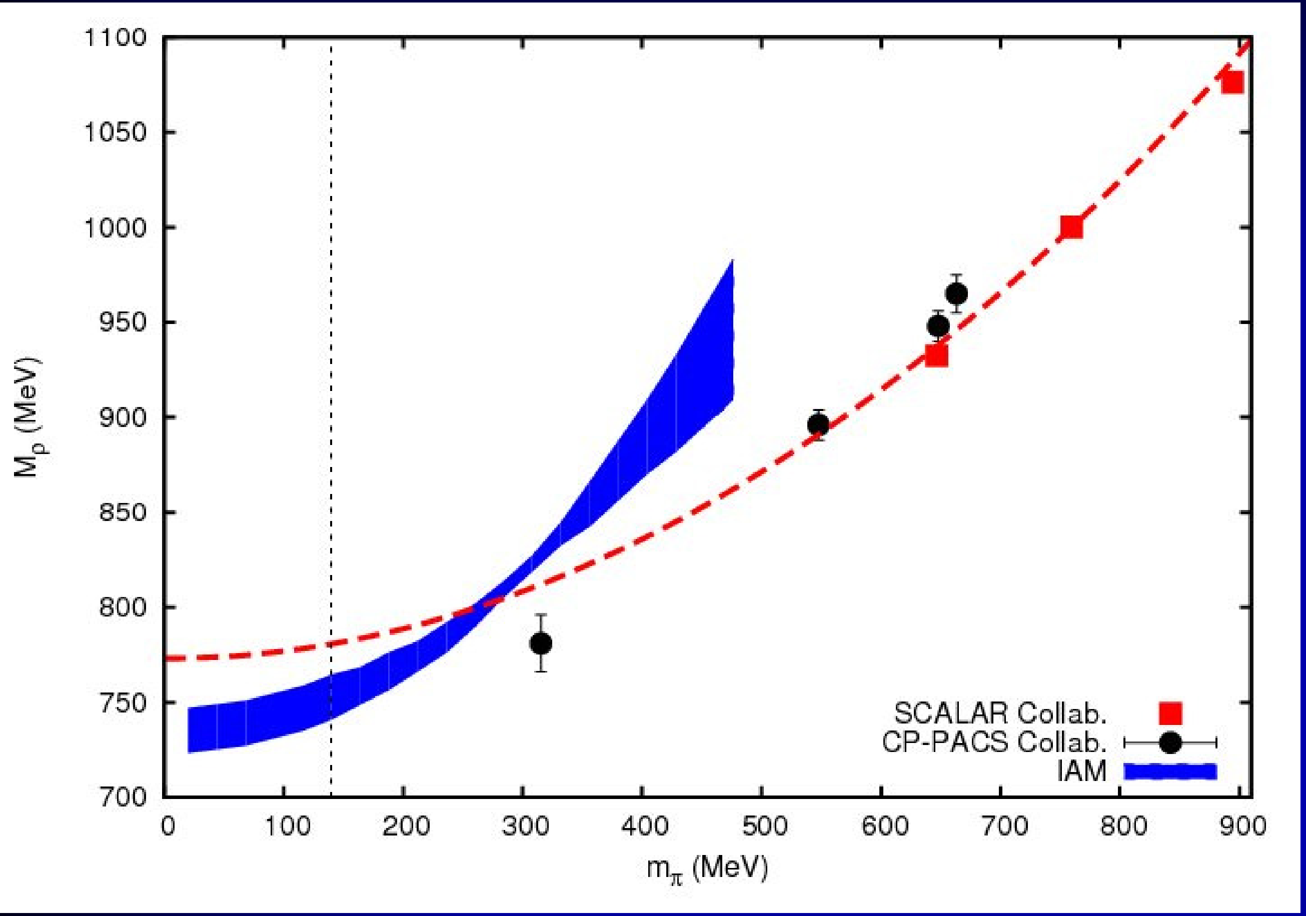
O(p⁴) amplitudes written in terms of the **physical** pion decay constant

$$f_\pi = f_0\left(1 + \frac{m_\pi^2}{16\pi^2 f_0^2}\bar{l}_4 + \cdots\right)$$

they depend on the pion mass

When changing m_π we have to change also f_π and the μ-independent LECs

Comparison with lattice results for the rho



CAUTION!!!

We give POLE MASS in complex plane

Lattice caveats:
Improved actions,
Lattice spacing...
Finite volume...
WIDTHLESS rho

CP-PACS. Aoiki et al. PRD**60** 114508 (1999)
SCALAR Collab, PRD70 (2004),034504.

The best would be to use ChPT on the lattice