Baby-steps beyond rainbow-ladder

Richard Williams

TU Darmstadt

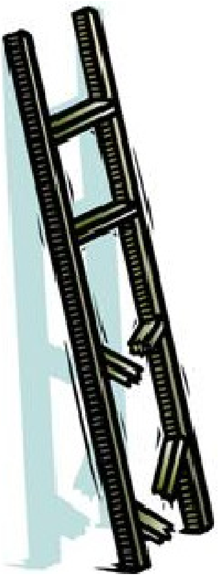
Excited QCD09 – Zakopane, 9/02/09

TECHNISCHE UNIVERSITÄT DARMSTADT

Christian S. Fischer, Dominik Nickel, RW, Eur.Phys.J.C,2008 [arXiv:0807.3486]

Christian S. Fischer, RW, Phys.Rev.D78:074006,2008 [arXiv:0808.3372].

Motivation



Desire:

- Poincaré covariant description of mesons
- formulated in the continuum
- description in terms of fundamental quantities of QCD

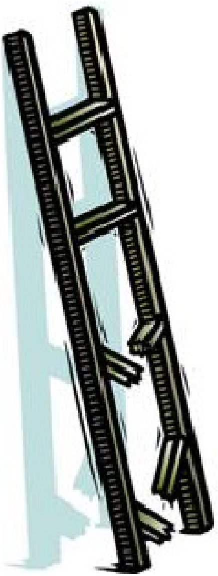
Natural framework:

- Bethe-Salpeter equations
- Schwinger-Dyson equations

Studied in detail for many years

- Rarely extended beyond simplest truncations
- Ad-hoc ‘improvements’ used.
- Severe approximations (e.g. M-N)

Motivation



Rainbow-Ladder:

- successful description of light-mesons subject to an apposite phenomenological ansatz for the interaction.

  e.g. Maris-Tandy model.

  [P. Maris, P. C. Tandy, PRC **60** (1999) 055214]

Lacking in many regards:

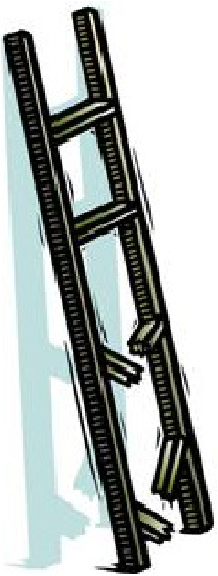
- no unquenching effects – pion cloud

  [A. W. Thomas, PPNP **61** (2008) 219]

- no η/η' splitting – $U_A(1)$ anomaly
- admits $(\bar{3})_c$coloured diquark bound-states – but useful in studying Baryons

  [A. Bender, C. Roberts, L.v Smekal, PRLB **380** (1996) 7]

Motivation



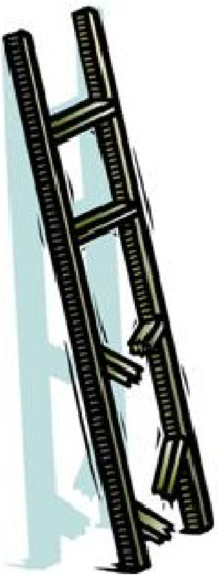
Moreover:

- Describes only pure $q\bar{q}$-states:
  - No flavour mixing
  - No decay channels
  - No exotics

Rainbow-Ladder provides ONLY

- $\gamma_\mu \otimes \gamma_\mu$ couplings
  - Simplicity of interaction means higher spin states of mesons are poorly represented.
  - No variety in attraction/repulsion.

Motivation



Goal:

- Consistent Green's function approach
  - Ghost/Gluon solutions of DSE
  - Quark-Gluon vertex beyond γ^μ
  - BSE kernel satisfying axWTI

"Break the ladder:"

- Unquenching effects
- Leading Yang-Mills corrections

Outline

Outline

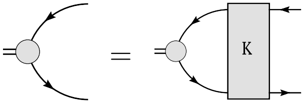
Bethe-Salpeter equations

Bound states:

- poles in $n \geq$ 3-point colour singlet Green's functions

$$\Gamma_H(p,P) = \frac{r_H \Gamma_h(p,P)}{P^2 + m_H^2} + \text{regular terms}$$

$\Gamma_h(p,P)$ solves homogeneous Bethe-Salpeter Equation:



Generalisation of Lippmann-Schwinger equation

Required inputs

- Quark propagator
- Gluon propagator
- Quark-Gluon vertex
- Scattering kernel K

Schwinger-Dyson equations

Basic objects are the propagators of the theory.

Quark

$$\left\langle \bar{\psi}^a \psi^b \right\rangle \equiv S_F^{ab}(p) = \delta^{ab} \frac{i\not{p} + M\left(p^2\right)}{p^2 + M^2\left(p^2\right)} Z_f(p^2)$$

Gluon†

$$\left\langle A_\mu^a A_\nu^b \right\rangle \equiv D_{\mu\nu}(p) = \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z\left(p^2\right)}{p^2}$$

Ghost

$$\left\langle \bar{c}^a c^b \right\rangle \equiv D_G^{ab}(p) = -\delta^{ab} \frac{G\left(p^2\right)}{p^2}$$

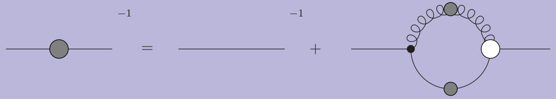
Each satisfy a SDE in terms of higher–Green's fns.

(†in Landau gauge)

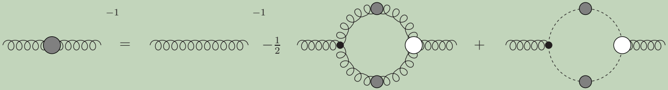
Schwinger-Dyson equations

Basic objects are the propagators of the theory.

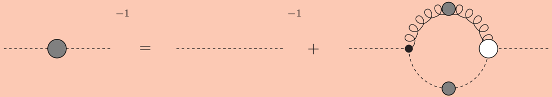
Quark



Gluon (truncated)



Ghost



Each satisfy a SDE in terms of higher–Green's fns.

(†in Landau gauge)

Bethe-Salpeter equations

Rainbow-Ladder truncation

Symmetries help constrain system

Axial-vector WTI

$$P_\mu \Gamma^a_{5\mu}(k;P) = S^{-1}(k_+)\frac{1}{2}\lambda^a_f i\gamma_5 + \frac{1}{2}\lambda^a_f i\gamma_5 S^{-1}(k_-) - M_\zeta i\Gamma^a_5(k;P) - i\Gamma^a_5(k;P)M_\zeta \, .$$

- Symmetry preserving truncation in DSE and BSE
  $\rightarrow$ preserve Goldstone character of the pion

BSE

$$\Gamma^{(\mu)}_{tu}(p;P) = \lambda(P^2) \int \frac{d^4k}{(2\pi)^4} K_{tu;rs}(p,k;P) \left[S(k_+)\Gamma^{(\mu)}(k;P)S(k_-) \right]_{sr}$$

Solve by introducing an eigenvalue $\lambda(P^2)$

Bethe-Salpeter equations
Rainbow-Ladder truncation

Replace quark-gluon vertex by tree-level form.

Quark-gluon vertex

$$\Gamma^{\mathrm{qg}}_{\nu}(p_1,p_2,p_3) \;=\; \gamma_\nu\, Z_2/\widetilde{Z}_3\;\Gamma^{YM}(p_3^2)$$

BSE Kernel constructed by considering AVWTI:

Quark scattering kernel

$$K_{tu;sr}(q,p;P) \;=\; \frac{g^2\,Z(k^2)\,\Gamma^{\mathrm{YM}}(k^2)\,Z_{1F}}{k^2}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\left[\frac{\lambda^a}{2}\gamma_\mu\right]_{ts}\left[\frac{\lambda^a}{2}\gamma_\nu\right]_{ru}$$

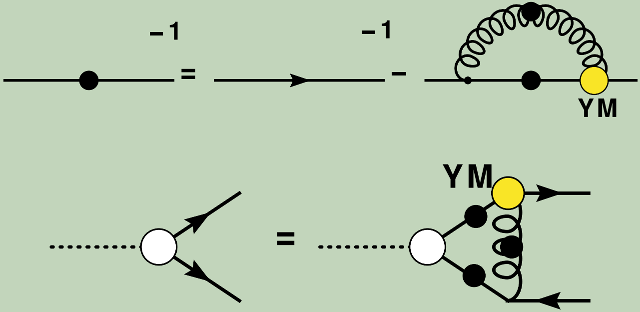
BSE obtained from quark SDE by the substitution:

$$\gamma^\mu S(k)\gamma^\nu \longrightarrow \gamma^\mu S(k_-)\Gamma^{(\rho)}_M(k;P)S(k_+)\gamma^\nu$$

Bethe-Salpeter equations

Rainbow-Ladder truncation

Pictorially



- Satisfies AV-WTI
- Reproduces:
  - masses of light pseudoscalar, vectors
  - leptonic decay constants
  - electromagnetic form factors, pion charge radius.

Beyond Rainbow-Ladder

Moving beyond Rainbow-Ladder:

- akin to looking for a pot of gold at the end of a *rainbow*

Technically very challenging:

- Coupled integral equations
- Must preserve symmetries
- Computationally involved:
  - Calculate input Green's functions
  - Solve normalisation conditions.
- Want:
  - Unquenching (quark-loops)
  - Yang-Mills corrections

Beyond Rainbow-Ladder

Be more humble and ask for some pi at the end of our rainbow.

- Arises from unquenching (pion-cloud)
- Hadronic contribution (decay widths?)
- additional tensor structure
  $\rightarrow$ beyond the rainbow

Challenging, but tractable within further simplifying approximations.

Beyond Rainbow-Ladder

Also mandatory to think about additional contributions from Yang-Mills sector.

- Use of *ab initio* quantities to determine:
  - gluon propagator
  - quark-gluon interaction

model determined dynamically

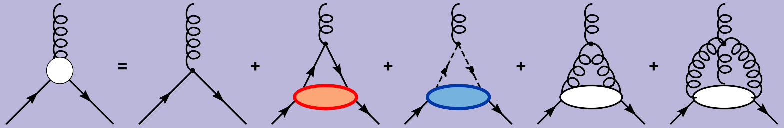
- non vector-vector couplings

  richer pattern of chiral symmetry breaking exhibited by meson masses

Outline

Quark-Gluon vertex

Quark-Gluon vertex



- Quark diagram Hadronic contributions

  [C. Fischer, D. Nickel, J. Wambach, PRD **76** (2007) 094009]

  [C. Fischer, D. Nickel, RW, arXiv:0807.3486, [hep-ph]]

  [C. Fischer, RW, arXiv:0808.3372, [hep-ph]]

- Ghost diagram Infrared leading

  [R. Alkofer, C. Fischer, F. Llanes-Estrada, MPL A **23**, 1105 (2008)]

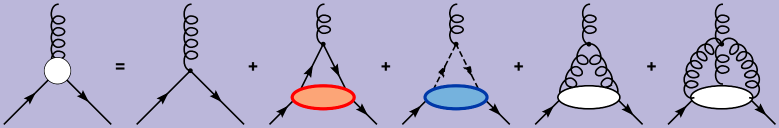
  [R. Alkofer, C. Fischer, F. Llanes-Estrada, K. Schwenzer, Annals of Physics, arXiv:0804.3042 [hep-ph]]

For all scales vanishing symmetrically, exhibit power law solutions

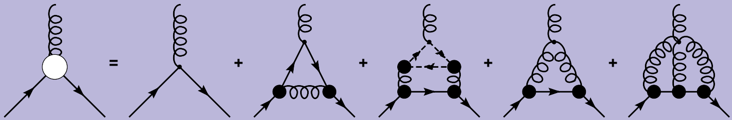
$$\Gamma^{n,m,l} \sim \left(p^2/\Lambda^2_{\mathrm{QCD}}\right)^{(n-m)\kappa - l/2}$$

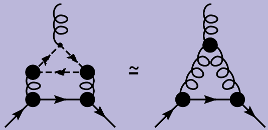
Quark-Gluon vertex

Quark-Gluon vertex



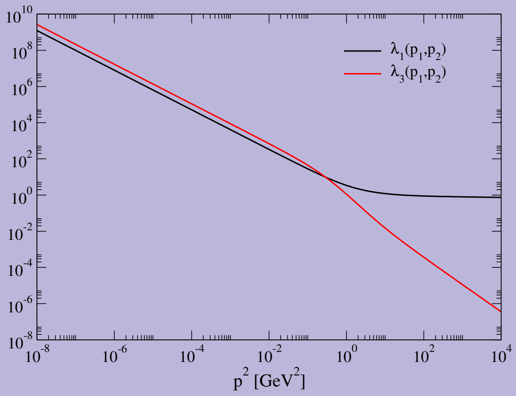
Quark-Gluon vertex – leading skeleton expansion





Quark-Gluon vertex: Numerical solutions

Vector and scalar dressing functions



[R. Alkofer, C. Fischer, F. Llanes-Estrada, K. Schwenzer, Annals of Physics, arXiv:0804.3042 [hep-ph]]

$$\Gamma^\mu(p_1,p_2) = \sum_{k=1}^{12} \lambda_k(p_1,p_2) L^\mu(p_1,p_2)$$

Outline

Hadronic Unquenching Effects (Pion Cloud)

First steps

1. Subsume all Yang-Mills corrections into a vertex dressing
   - form inspired by quark-gluon vertex calculations
   - scales left free (but constrained) for parameter fitting
2. Gluon propagator obtained from SDE solutions
3. Focus on and quantify hadronic effects.

For investigatory purposes we simplify the truncation further:

- Gives idea of necessary difficulty
- Allows techniques to be refined:
  - Solving quark in the complex plane (Euclidean space)
  - Normalisation condition for non-trivial Kernel.

Hadronic Quenching Effects (Pion Cloud)

Modelling the YM part

Yang-Mills part of quark-gluon interaction Γ_μ:

- $Z(k^2)\Gamma_{\rm YM} \sim \alpha(k^2)$: for large momenta
- $\Gamma_{\rm YM} \sim (k^2)^{-1/2-\kappa}$: IR soft-singularity in gluon momentum.

[R. Alkofer, C. Fischer, F. Llanes-Estrada, K. Schwenzer, Annals of Physics, arXiv:0804.3042 [hep-ph]]

- For consistency with axWTI, use $\Gamma_\mu \sim \Gamma_{\rm YM}\gamma_\mu$

Soft-Divergence

$$\Gamma_{\rm YM}(k^2) = \left(\frac{k^2}{k^2+d_2}\right)^{-1/2-\kappa} \times \left(\frac{d_1}{d_2+k^2} + \frac{k^2 d_3}{d_2^2+\left(k^2-d_2\right)^2} + \frac{k^2}{\Lambda_{\rm QCD}^2+k^2} \times \left[\frac{4\pi}{\beta_0\alpha_\mu}\left(\frac{1}{\log\left(\frac{k^2}{\Lambda_{\rm QCD}^2}\right)} - \frac{\Lambda_{\rm QCD}^2}{k^2-\Lambda_{\rm QCD}^2}\right)\right]^{-2\delta}\right)$$

Hadronic Quenching Effects (Pion Cloud)

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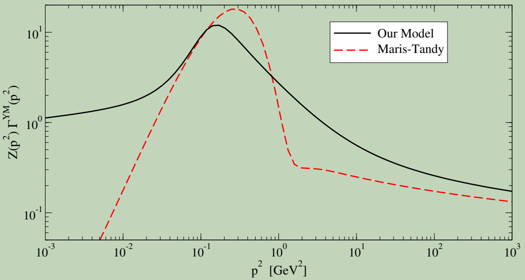
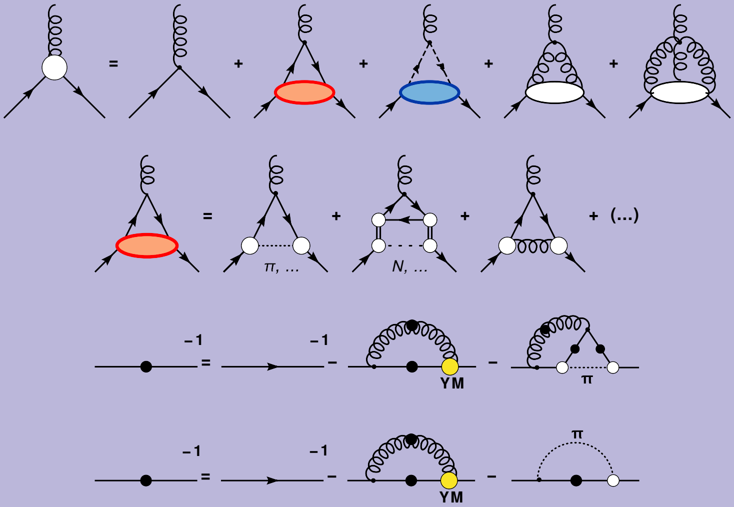
Soft-Divergence



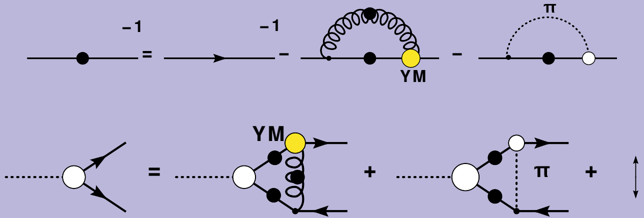
Quark-Gluon vertex - Hadronic unquenching

Truncation



Pion-cloud effects in light mesons

Coupled SDE/BSE system



- AxWTI satisfied in χ-limit:

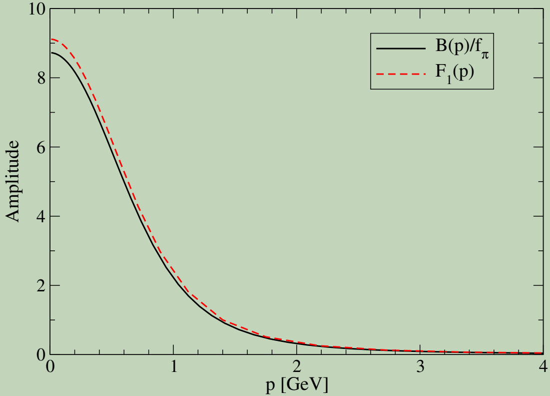
$$P_\mu \boldsymbol{\Gamma}^{a}_{5\mu}(k;P) = \boldsymbol{S}^{-1}(k_+)\frac{1}{2}\lambda^a_f i\gamma_5 + \frac{1}{2}\lambda^a_f i\gamma_5 \boldsymbol{S}^{-1}(k_-) - M_\zeta i\Gamma^a_5(k;P) - i\Gamma^a_5(k;P)M_\zeta \; .$$

- Generalised GMOR relation well-satisfied:

$$f_\pi m_\pi^2 = r_\pi \left(m_u(\mu^2) + m_d(\mu^2) \right) \; ,$$

Pion-cloud effects in light mesons

Simple off-shell prescription



$$\Gamma_\pi^j(p;P) = \tau^j \gamma_5 \frac{B_\chi(p^2)}{f_\pi}$$

Pion-cloud effects in light mesons

Normalisation

$$
\begin{aligned}
\delta^{ij} &= 2\frac{\partial}{\partial P^2}\mathrm{tr}\int\frac{d^4k}{(2\pi)^4} \\
&\Bigg[3\left(\overline{\Gamma}^i_\pi(k,-Q)S(k+P/2)\Gamma^j_\pi(k,Q)S(k-P/2)\right) \\
&+\int\frac{d^4q}{(2\pi)^4}[\overline{\chi}^i_\pi]_{sr}(q,-Q)K^{\mathrm{pion}}_{tu;rs}(q,k;P)[\chi^j_\pi]_{ut}(k,Q)\Bigg] ,
\end{aligned}
$$

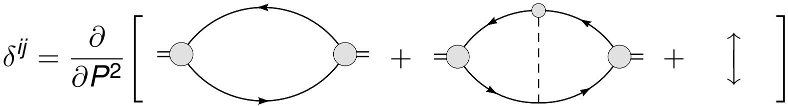
Canonical condition:

Demand residue of bound-state in inhomogeneous Bethe-Salpeter equation is equal to unity.

Pion-cloud effects in light mesons

Normalisation

$$\delta^{ij} = 2\frac{\partial}{\partial P^2}\mathrm{tr}\int\frac{d^4k}{(2\pi)^4}$$
$$\Bigg[3\left(\overline{\Gamma}^i_\pi(k,-Q)S(k+P/2)\Gamma^j_\pi(k,Q)S(k-P/2)\right)$$
$$+\int\frac{d^4q}{(2\pi)^4}[\overline{\chi}^i_\pi]_{sr}(q,-Q)K^{\mathrm{pion}}_{tu;rs}(q,k;P)[\chi^j_\pi]_{ut}(k,Q)\Bigg],$$



Results

Spectrum of light mesons

	Maris-Tandy w/o pi	Maris-Tandy incl. pi	Our Model w/o pi	Our Model incl. pi	Experiment
M_π	140	138†	125	138†	138
f_π	104	93† (90)	102	93† (90)	93
M_σ	746	598	638	485	400 – 1200
M_ρ	821	720	795	703	776
f_ρ	160	167 (167)	159	162 (165)	156
M_{a_1}	979	913	941	873	1230
M_{b_1}	820	750	879	806	1230

Results

Spectrum of light mesons

	Maris-Tandy		Our Model		Experiment
	w/o pi	incl. pi	w/o pi	incl. pi	
M_π	140	$138^\dagger$	125	$138^\dagger$	138
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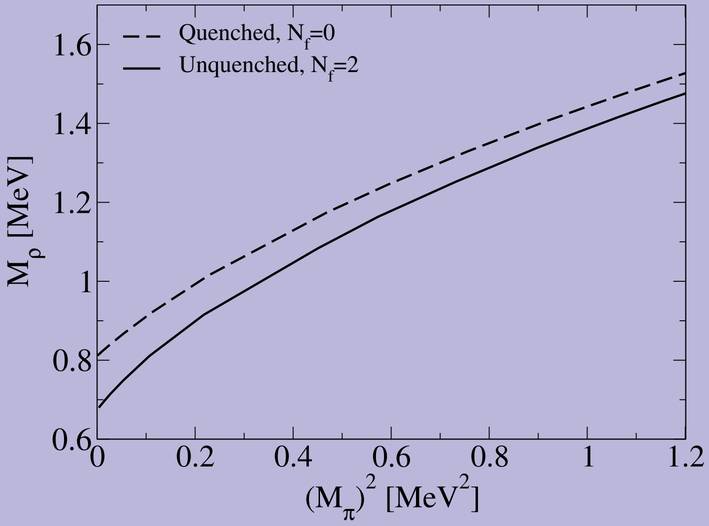
f_ρ	160	167 (167)	159	162 (165)	156
M_{a_1}	979	913	941	873	1230
M_{b_1}	820	750	879	806	1230
M_η			493	497	548
$M_{\eta'}$			949	963	948

Yang-Mills part of vertex too simple.

Vector vs. Pseudoscalar mass

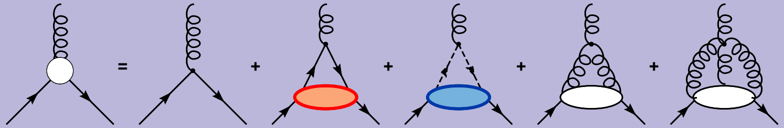
Mass plots



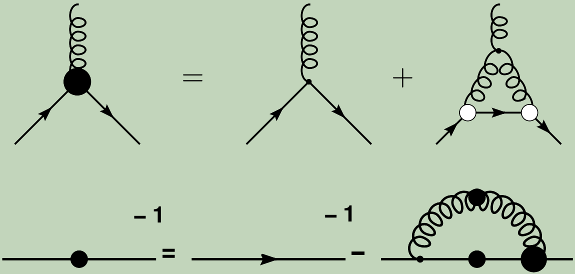
Outline

Gluonic corrections

Quark-Gluon vertex



Consider the approximated system:



Gluonic corrections

Challenges

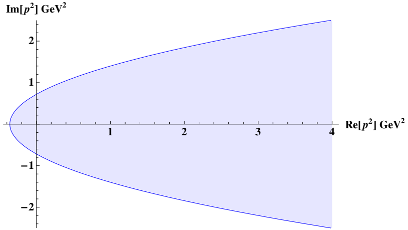
Solving vertex SDE for real Euclidean Momenta

- Nowadays ROUTINE.

  all basic QCD vertices tackled to date within some approximation

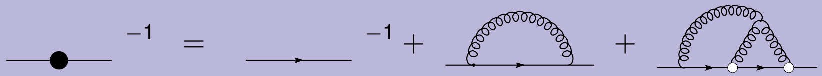
Bound-states in Euclidean space → $P^2 = -M^2$.

- Need both quark propagator and Quark-Gluon vertex for $\mathbb{C}$-momenta
- Only a (very) technical difficulty – surmounted.

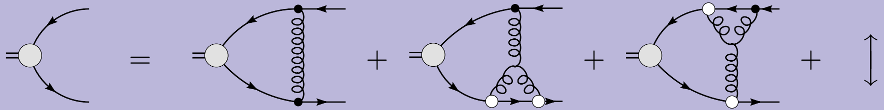


Gluonic corrections

Bethe-Salpeter equation



Cutting propagators/substitution rules give BSE Kernel:



- We *really* calculate these two-loop diagrams
- No Munczek-Nemirovsky
- Axial-Vector WTI preserving truncation.

Gluonic corrections
Simple exploratory model

- Replace three-gluon vertex with tree-level form

$$\Gamma^{(0)abc}_{\mu\nu\rho}(p,k,q) = g\, f^{abc} \left(\delta_{\mu\nu}\,(p-q)_\rho + \delta_{\nu\rho}\,(q-k)_\mu + \delta_{\rho\mu}\,(k-p)_\nu\right)$$

- Replace internal quark-gluon vertices with γ_μ.
- Replace gluon with some integrated strength

$$Z(p^2) = \frac{g^2}{4\pi}\frac{\pi D}{\omega^2} p^4 \exp\left(-p^2/\omega^2\right)$$

NOT representative of what we expect in nature:

- think of effective gluon interaction compensating for lack of internally dressed vertices.
- no ultraviolet support - renormalisation trivial.

Gluonic corrections

Simple exploratory model

- Replace three-gluon vertex with tree-level form

$$\Gamma^{(0)abc}_{\mu\nu\rho}(p,k,q) = g\, f^{abc} \left(\delta_{\mu\nu}\,(p-q)_\rho + \delta_{\nu\rho}\,(q-k)_\mu + \delta_{\rho\mu}\,(k-p)_\nu \right)$$

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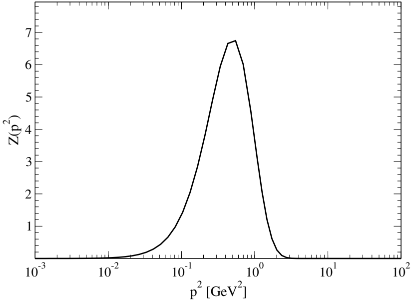
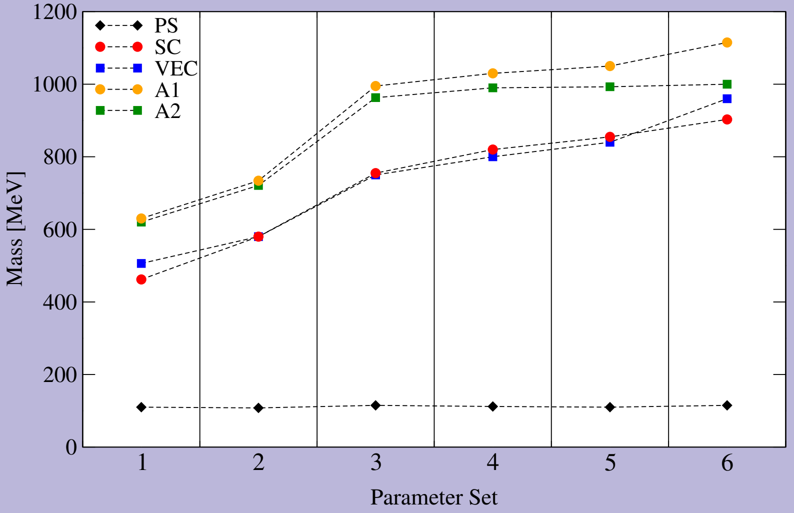


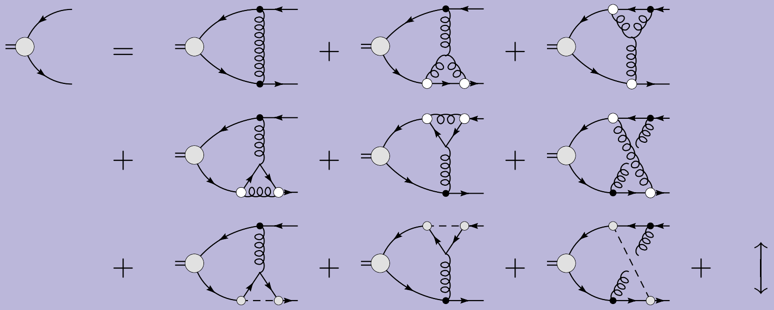
Gluonic corrections

Simple exploratory model – Results



Outline

Outlook



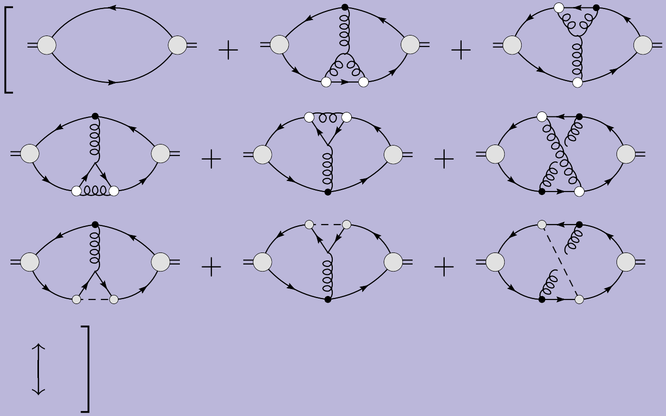
- Unquenching: Improve pion off-shell prescription
- Inputs: Employ solutions from SDE solutions
- Results: Meson spectrum, EM form factors

Seven down - two to go. But we must still solve . . .

Outlook

Normalisation

$$\delta^{ij} = \frac{\partial}{\partial P^2}\Big[\cdots \Big]$$



Need in order to determine leptonic decay constants

Conclusions

Summary

Quark-Gluon vertex **critical** object. Contains

- Hadronic unquenching effects
- Yang-Mills corrections

Demonstrated that effects from the pion-cloud:

- are generally attractive
- generate effects of right size
- can be successfully modelled in a simple model

Presented progress on state-of-the-art calculations:

- incorporation of leading corrections to vertex
- full two-loop calculations - no kinematic restrictions.