Colour fields of the static hybrid gluon-quark-antiquark system

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Feynman Path Integral

$$Z = \int DA_\mu D\psi D\overline{\psi}\, e^{-S_E}$$

$$S = \int dx^4 \Big(\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a - \overline{\psi}\Big(i\gamma^\mu D_\mu - m\Big)\psi\Big)$$

Operator Mean Value

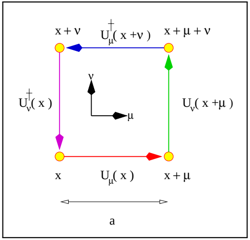
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int DA_\mu D\psi D\overline{\psi} \mathcal{O}\left(\psi, \overline{\psi}, A_\mu\right) e^{-S_E}$$

- We use the quenched approximation (neglect the fermionic part):
  - Lattice Gluon Action:

$$S_G = \beta \sum_x P_{\mu\nu}(x)$$

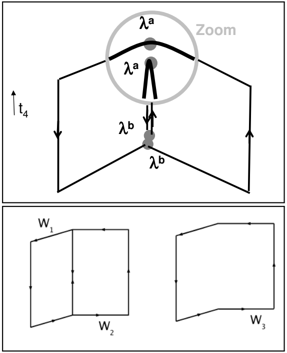
  - where the plaquette is given by

$$P_{\mu\nu}(x) = 1 - \frac{1}{3} \mathrm{Re}\,\mathrm{Tr}\left[U_\mu(x) U_\nu(x+\mu) U^\dagger_\mu(x+\nu) U^\dagger_\nu(x)\right]$$



The Wilson loop for the gluon-quark-antiquark is given by:

$$
\begin{aligned}
W_{gq\bar{q}} \quad = \quad & \frac{1}{16}\mathrm{Tr}\Big[U_4^\dagger(t-1,x)\cdots U_4^\dagger(0,x)\ \lambda^b \\
& U_4(0,x)\cdots U_4(t-1,x)\lambda^a\Big] \\
& \mathrm{Tr}\Big[U_{\mu_2}(t,x)\cdots U_{\mu_2}(t,x+(x_2-1)\hat{\mu}_2) \\
& U_4^\dagger(t-1,x+x_2\hat{\mu}_2)\cdots U_4^\dagger(0,x+x_2\hat{\mu}_2) \\
& U_{\mu_2}^\dagger(0,x+(x_2-1)\hat{\mu}_2)\cdots U_{\mu_2}^\dagger(0,x)\lambda^b \\
& U_{\mu_1}^\dagger(0,x-\hat{\mu}_1)\cdots U_{\mu_1}^\dagger(0,x-x_1\hat{\mu}_1) \\
& U_4(0,x-x_1\hat{\mu}_1)\cdots U_4(t-1,x-x_1\hat{\mu}_1) \\
& U_{\mu_1}(t,x-x_1\hat{\mu}_1)\cdots U_{\mu_1}(t,x-\hat{\mu}_1)\lambda^a\Big]\ .
\end{aligned}
$$



Using the Fiertz relation,

$$\sum_a \left(\frac{\lambda^a}{2}\right)_{ij}\left(\frac{\lambda^a}{2}\right)_{kl} = \frac{1}{2}\delta_{il}\delta_{jk} - \frac{1}{6}\delta_{ij}\delta_{kl}$$

we can prove that

$$W_{gq\bar{q}} = W_1 W_2 - \frac{1}{3}W_3$$

- Chromoelectric Field:

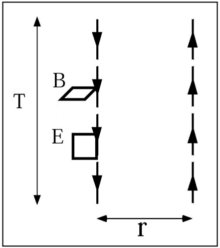
$$\langle E_i^2 \rangle \quad = \quad \langle P_{0i} \rangle - \frac{\langle W\, P_{0i} \rangle}{\langle W \rangle}$$

- Chromomagnetic Field:

$$\langle B_i^2 \rangle \quad = \quad \frac{\langle W\, P_{jk} \rangle}{\langle W \rangle} - \langle P_{jk} \rangle$$

where the jk indices of the plaquette complement the index i of the magnetic field, and where the plaquette is given by

$$P_{\mu\nu}(s) = 1 - \frac{1}{3}\mathrm{Re}\,\mathrm{Tr}\left[U_\mu(s)U_\nu(s+\mu)U_\mu^\dagger(s+\nu)U_\nu^\dagger(s)\right]$$



- The energy ($\mathcal{H}$) density:

$$\mathcal{H} = \frac{1}{2}\left(\langle E^2 \rangle + \langle B^2 \rangle\right)$$

- The lagrangian ($\mathcal{L}$) density:

$$\mathcal{L} = \frac{1}{2}\left(\langle E^2 \rangle - \langle B^2 \rangle\right)$$

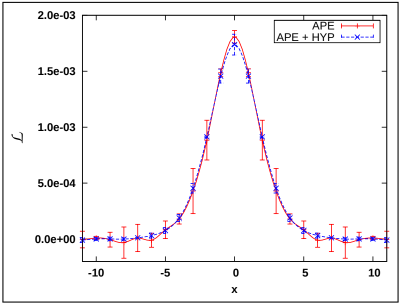
- In space: APE Smearing in the Wilson Loop: To increase the ground state overlap, the links are replaced by "fat links",

$$U_\mu(x) \quad \rightarrow \quad P_{SU(3)} \frac{1}{1+6w} \Big(U_\mu(x) + w \sum_{\mu \neq \nu} U_\nu(x)\, U_\mu(x+\nu)\, U_\nu^\dagger(x+\mu) \Big)$$

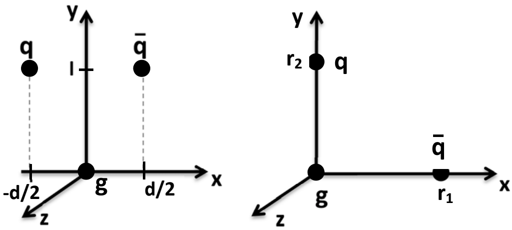
  We use $w = 0.2$ and iterate this procedure 25 times in the spatial direction.
- In Time: To achieve better accuracy in the flux tube, we apply the hypercubic blocking (HYP), [Hasenfratz and Knechtli, 2001], with

$$\alpha_1 = 0.75, \quad \alpha_2 = 0.6, \quad \alpha_3 = 0.3\ .$$

  These parameters are optimized to reduce large plaquette fluctuations.



- Results with 286 configurations with $24^3 \times 48$ and $\beta = 6.2$ generated with the version 6 of the MILC code, via a combination of Cabbibo-Mariani and overrelaxed updates;
- The results are presented in lattice spacing units of a, with $a = 0.07261(85)\, fm$ or $a^{-1} = 2718 \pm 32\, MeV$;
- Two geometries, U and L shape.



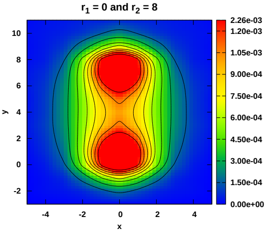
(a) U shape geometry. (b) L shape geometry.

- U geometry with:
  - $l = 8$ fixed;
  - $d = 0, 2, 4, 6, 8, 10, 12, 14, 16.$

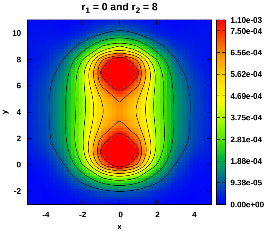
  for $d = 0$, the quark and the antiquark are superposed $\Rightarrow$ two gluon glueball case.

- L geometry with:
  - $r_2 = 8$ fixed;
  - $r_1 = 0.$

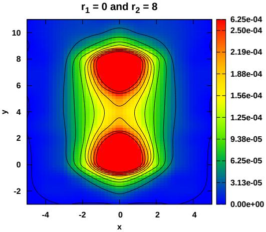
  In this case the gluon and the antiquark are superposed $\Rightarrow$ quark-antiquark case.



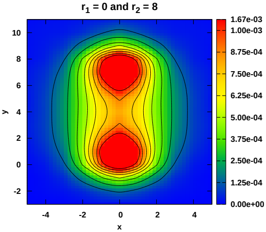
(a) $\left\langle E^2 \right\rangle$



(b) $-\left\langle B^2 \right\rangle$



(c) $\mathcal{H}$



(d) $\mathcal{L}$

Improved Results from [Cardoso et. al, 2009]

(a) $\left\langle E^2 \right\rangle$

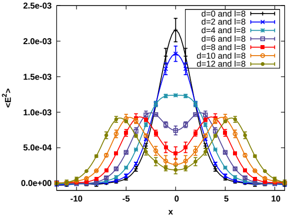
(b) $-\left\langle B^2 \right\rangle$

(c) $\mathcal{H}$

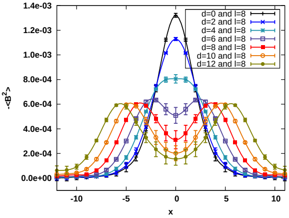
(d) $\mathcal{L}$

Improved Results from [Cardoso et. al, 2009]

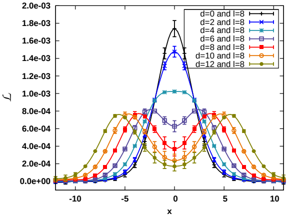
U geometry: profiles at $y = 4$ and $z = 0$



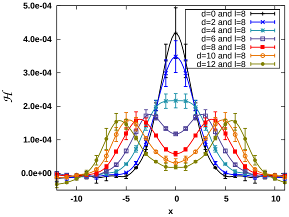
(a) $\left\langle E^2 \right\rangle$ at $y = 4$.



(b) $-\left\langle B^2 \right\rangle$ at $y = 4$.



(c) $\mathcal{L}$ at $y = 4$.

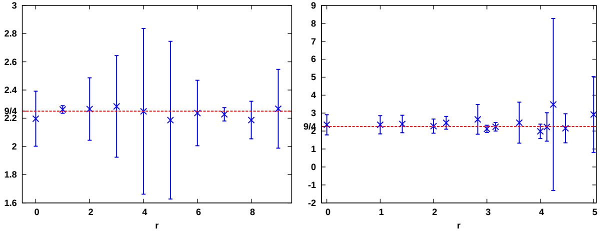


(d) $\mathcal{H}$ at $y = 4$.

Preliminary Results

- We can see the stretching and partial splitting of the flux tube in the equatorial plane $(y = 4)$ between the quark and the antiquark;

- We measure the quotient between the energy densities of the meson system and of the glueball system, in the mediatrix plane between the two particles ($x = 0$);



(a) $r = x$, $y = 4$ and $z = 0$. (b) $r = (x, z)$, $y = 4$.

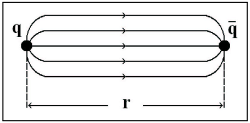
(a) - Improved Result from [Cardoso et. al, 2009] (b) - Preliminary Result

- These results are consistent with Casimir scaling, with a factor of $9/4$ between the energy density in the glueball and in the meson;
- This corresponds to the formation of an adjoint string;
- The results are compatible with an identical shape of the two flux tubes, but with a different density, and in this sense this agrees with the simple picture for the Casimir Scaling of [Semay, 2004].

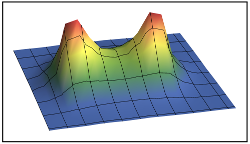
- Understanding how confinement arises from QCD is central problem of strong interaction physics.
- In 1970's, Nambu [Nambu, 1974], 't Hooft ['t Hooft, 1979] and Mandelstam [Mandelstam, 1976] proposed an interesting idea that quark confinement would be physically interpreted using the dual version of the superconductivity, the QCD vacuum state to behave like a magnetic superconductor.
- In the ordinary superconductor, Cooper-pair condensation leads to the Meissner effect, and the magnetic flux is excluded or squeezed like a quasi-one-dimensional tube as the Abrikosov vortex, where the magnetic flux is quantized topologically.
- There also is evidence for the dual superconductor picture from numerical simulations of QCD [Bali et al., 1996].
- If magnetic monopoles are condensed in the vacuum, then the electric sources are confined by electric flux tubes, as magnetic charges would be confined by Abrikosov-Nielsen-Olesen (ANO) vortices, [Abrikosov, 1957, Nielsen and Olesen, 1973], in an ordinary superconductor (Meissner effect).

Dual QCD

- The chromoelectric field originated by a $q\overline{q}$ pair is squeezed by Meissner effect into a dual Abrikosov flux tube, giving rise to the confining linear potential, the field is confined into flux tubes $\rightarrow$ QCD strings.



$$V_{q\overline{q}}\left(r\right)\rightarrow\sigma r$$



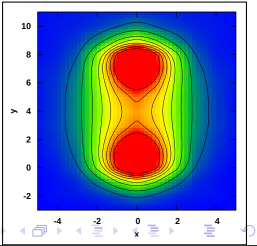
E^2

- Color confinement could be understood as the dual Meissner effect.
- In common superconductivity the magnetic field decays with $B\sim e^{-r/\lambda_L}$ and this could be interpreted in terms of an effective mass for the photon $m_\gamma=1/\lambda_L$. Some studies have point a similar behavior in QCD, [Di Giacomo and Panagopoulos, 1992, Baker et al., 1985, Bali, 1998];

- We tested two functions:

$$a\,e^{-2\mu r} \qquad\qquad a\,K_0^2\left(\mu r\right)$$

where, $\mu=\frac{1}{\lambda_L}$, λ_L is the penetration length and K_0 the modified Bessel function of order zero.



- μ as the dual gluon mass.

	$a\,e^{-2\,\mu\,r}$		$a\,K_0^2\,(\mu\,r)$	
	μ (GeV)	χ^2/dof	μ (GeV)	χ^2/dof
$E^2_{(1a)}\,(r)$	1.170 ± 0.228	1.069	0.805 ± 0.287	1.827
$\mathcal{L}_{(1a)}\,(r)$	1.170 ± 0.119	0.512	0.865 ± 0.188	1.203
$E^2_{(2a)}\,(r)$	1.231 ± 0.286	1.547	0.881 ± 0.334	2.084
$E^2_{(1b)}\,(r)$	1.210 ± 0.056	0.887	0.897 ± 0.085	1.185
$\mathcal{L}_{(1b)}\,(r)$	1.208 ± 0.068	0.560	0.909 ± 0.099	0.909
$E^2_{(2b)}\,(r)$	1.210 ± 0.063	1.162	0.889 ± 0.097	1.262
$\mathcal{L}_{(2b)}\,(r)$	1.191 ± 0.031	1.066	0.899 ± 0.048	1.106

Preliminary results for the dual gluon mass gauge invariant.

- $1a-$ two gluon glueball at $y = 4$ and $z = 0$ for $r = x \geq 2$;
- $2a-$ quark-antiquark at $y = 4$ and $z = 0$ for $r = x \geq 2$;
- $1b-$ two gluon glueball at $y = 4$ for $r = (x, z) \geq 2$;
- $2b-$ quark-antiquark at $y = 4$ for $r = (x, z) \geq 2$;

Mass, GeV	Reference	Estimation method
$\simeq 0.604$	[Baker et al., 1991]	Dual QCD Langrangian
$\simeq 0.900$	[Suganuma et al., 1998]	Lattice QCD, MA gauge
$\simeq 0.500$	[Suganuma et al., 1998]	Lattice QCD
$\simeq 0.500$	[Tanaka and Suganuma, 1999]	Lattice QCD
$\simeq 1.200$	[Suganuma et al., 2000]	Lattice QCD, MA gauge
$\simeq 1.100$	[Suganuma et al., 2002]	Lattice QCD, MA gauge
$\simeq 1.000$	[Suganuma and Ichie, 2003]	Lattice QCD, MA gauge
$\simeq 0.828$	[Kumar and Parthasarathy, 2004]	Lattice QCD
$\simeq 1.200$	[Suganuma et al., 2004a]	Lattice QCD
$\simeq 1.200$	[Suganuma et al., 2004b]	Lattice QCD

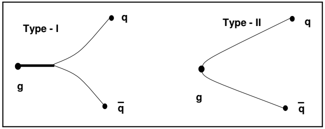
Review Gluon Mass: from literature

Mass, GeV	Reference	Estimation method
0.800	[Parisi and Petronzio, 1980]	$J/\psi \to \gamma X$
0.500 ± 0.200	[Cornwall, 1982]	Various
0.750	[Spiridonov and Chetyrkin, 1988]	$\Pi^{\text{e.m.}}_{\mu\nu}, \left\langle \mathrm{Tr} G^2_{\mu\nu} \right\rangle$
$0.687 - 0.985$	[Donnachie and Landshoff, 1989]	Pomeron parameters
0.800	[Hancock and Ross, 1993]	Pomeron slope
0.750	[Nikolaev et al., 1994]	Pomeron parameters
$1.500^{+1.2}_{-0.6}$	[Field, 1994]	PQCD at low scales (various)
1.460	[Kogan and Kovner, 1995]	QCD vacuum energy, $\left\langle \mathrm{Tr} G^2_{\mu\nu} \right\rangle$
$10^{-10} - 20$ MeV	[Yndurain, 1995]	QCD potential
0.570	[Liu and Wetzel, 1996]	$\Pi^{\text{e.m.}}_{\mu\nu}, \left\langle \mathrm{Tr} G^2_{\mu\nu} \right\rangle$
0.470	[Liu and Wetzel, 1996]	Glueball current, $\left\langle \mathrm{Tr} G^2_{\mu\nu} \right\rangle$
1.02 ± 0.10	[Leinweber et al., 1999]	Lattice QCD
$0.721^{+0.010}_{-0.009}{}^{+0.013}_{-0.068}$	[Field, 2002]	$J/\psi \to \gamma X$
$1.180^{+0.06}_{-0.06}{}^{+0.07}_{-0.28}$	[Field, 2002]	$\Upsilon \to \gamma X$
$\simeq \Lambda/2$	[Sauli, Talk Excited QCD 2010]	Pinch technique gluon propagator

Conclusion

- How can these two pictures, of one adjoint string and of two fundamental strings, with different total string tensions, match?

- This question is also related to the superconductivity model for confinement, is QCD similar to a Type-I or Type-II superconductor?



- Notice that in type Type-II superconductors the flux tubes repel each other while in Type-I superconductors they attract each other and tend to fuse in excited vortices.

- When the quark and the anti-quark are superposed, this corresponds to the formation of an adjoint string between the two gluon and agrees with Casimir Scaling measured by Bali [Bali, 2000].

- This can be interpreted with a type-II superconductor analogy for the confinement in QCD with repulsion of the fundamental strings and with the string tension of the first topological excitation of the string (the adjoint string) larger than the double of the fundamental string tension.

- We present a value for **dual gluon mass of ~ 1 GeV** which is **gauge independent**.