The Phase Structure of Dense QCD from Chiral Models

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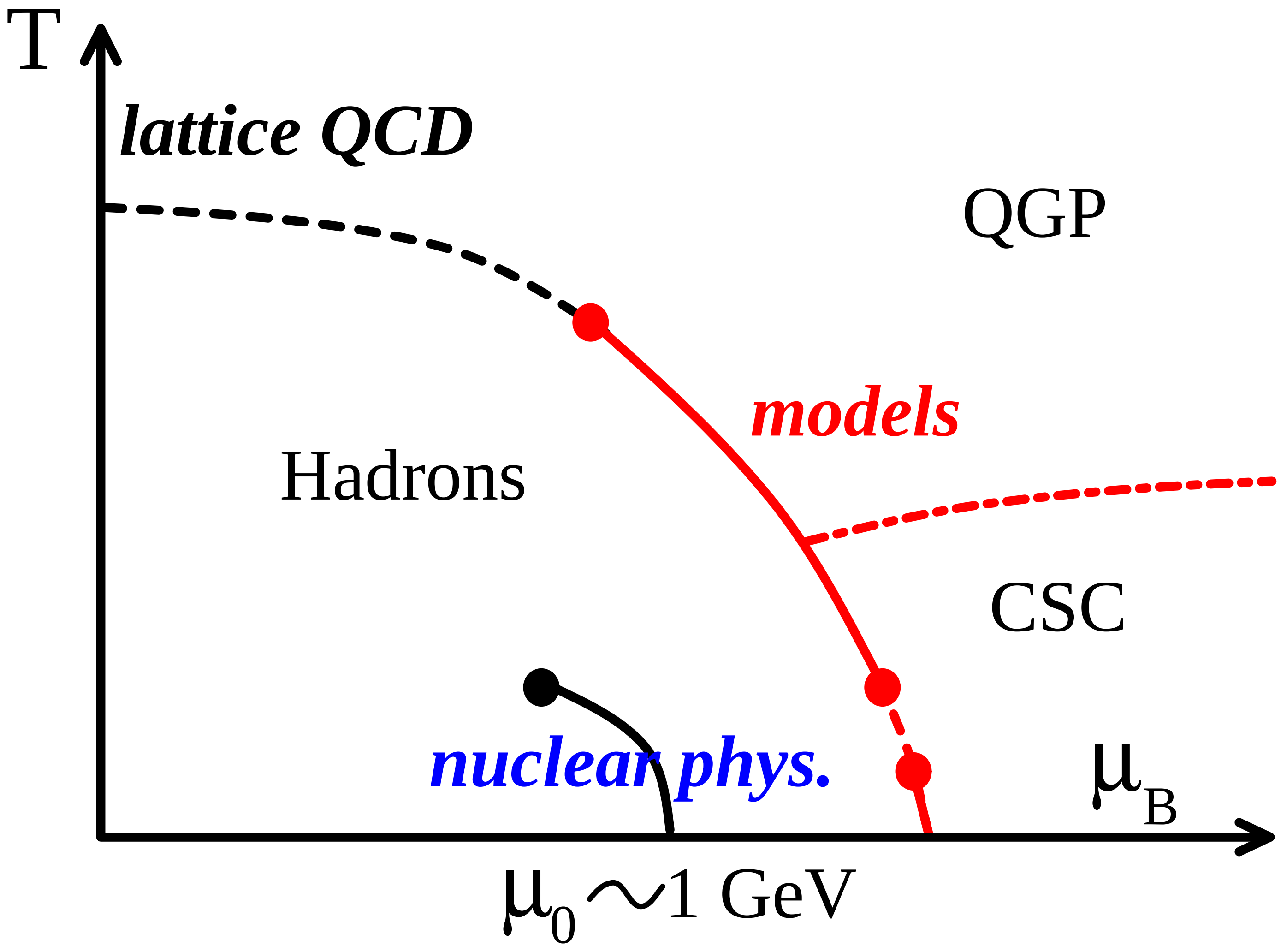
references

L. McLerran, K. Redlich and CS, Nucl. Phys. A **824**, 86 (2009);
M. Harada, CS and S. Takemoto, Phys. Rev. D **81**, 016009 (2010);
A. Andronic *et al.*, arXiv:0911.4806 [hep-ph].

Phases of QCD at finite temperature and density

- **what we know and what we don't know**



order of phase transition?   chiral and deconfinement?   critical point(s)?

- **strong interaction: spontaneous χSB and color confinement**
  - 2 order parameters: $\underbrace{\text{chiral condensate } \langle \bar{q}q \rangle}_{\text{quark dynamics}}$ & $\underbrace{\text{Polyakov loop } \langle \Phi \rangle}_{\text{gluon dynamics}}$

    ∗ left-right mixing: $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle$

    ∗ free energy of a static quark: $\langle \Phi \rangle = e^{-F_q/T}$

  - NJL model with Polyakov loops (PNJL model) [Fukushima (03), Ratti et al. (05)]

    ∗ quark (NJL) and gluon (potential $\mathcal{U}(\Phi)$) minimally coupled via covariant derivative: $\mathcal{L} = \mathcal{L}_{\text{NJL}}(\psi, \Phi[A_0]) + \mathcal{U}(\Phi)$

$$\mathcal{L}_{\text{kin}} = \bar{\psi} i \not{D} \psi \,, \quad D^\mu = \partial^\mu - i A^\mu \,, \quad A^\mu = \delta_{\mu 0} A^0$$

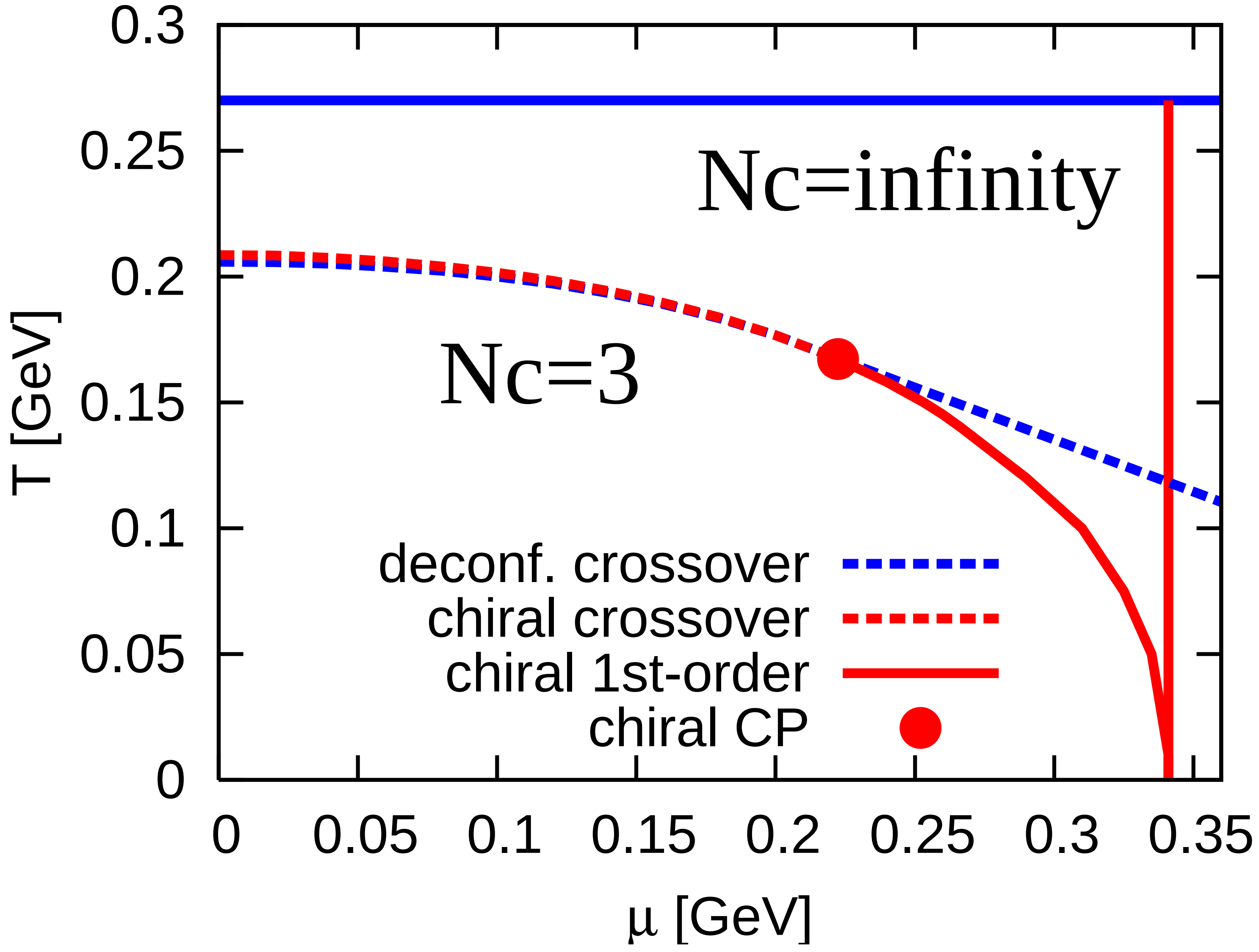
    ∗ model mimics confinement at low temperature $\langle \Phi \rangle \sim 0$

$$\Omega \sim \langle \Phi \rangle \cdot [\text{1-quark states} + \text{2-quark states}] + \text{3-quark}$$

$\Rightarrow$ 1- and 2-quark states are thermodynamically irrelevant.

- **chiral & deconf. trs for $N_c = 3$ & ∞ from a PNJL**

[McLerran-Redlich-CS (08)]



- **“transition” from hadronic to quarkyonic phase for $N_c = 3$?**

quarkyonic phase $\overset{???}{\sim}$ chirally restored & confined phase

– large N_c limit: clear distinction of 2 different confined-phases
baryon number density $\langle N_B \rangle = 0$ (mesonic) and $\neq 0$ (quarkyonic)

– finite N_c: no clear definition
but it may separate meson dominant from baryon dominant region
$\Rightarrow$ quarkyonic “transition” as meson-baryon “transition”

[A. Andronic *et al.* (09)]

– enhancement of baryon number susceptibility in chiral models

baryons near chiral symmetry restoration?

– standard LSM (naive): DχSB generates masses $\quad m_N \overset{\sigma \to 0}{\longrightarrow} 0$

– parity doublet model (mirror): DχSB generates mass difference
$m_{N_+} \overset{\sigma \to 0}{\longrightarrow} m_{N_-} = m_0 \neq 0$ [Detar-Kunihiro (89)]

anomaly matching? w/o Lorentz invariance

– no WZW term, massless excitations...

Dense baryonic matter in chiral models

- **nuclear matter: known properties**
  - binding energy: $E/A(\rho_0) - m_N = -16$ MeV
  - saturation density: $\rho_0 = 0.16$ fm^{-3}
  - incompressibility: $K = 9\rho_0^2 \partial^2(E/A)/\partial\rho^2|_{\rho=\rho_0} = 200\text{-}400$ MeV

- **standard LSM vs. parity doublet model**
  - LSM: no stable ground state corr. to nuclear matter saturation

    [Kerman-Miller (74)]

    no direct interaction among N_+-N_--π at tree
    cf. vacuum decay width $\Gamma(N_-(1535) \to N_+\pi)^{(\mathrm{exp})} = 70$ MeV
  - PDM: possible to describe saturation [Zschiesche-Tolos-Schaffner-Bielich-Pisarski (07)]

- **cold nuclear matter in SU(2) parity model** [Zschiesche et al. (07)]
  - 2 nucleon fields

$$\begin{array}{ll} \psi_{1L} : (1/2, 0) & \psi_{1R} : (0, 1/2) \\ \psi_{2L} : (0, 1/2) & \psi_{2R} : (1/2, 0) \end{array}$$

  - Lagrangian

$$\begin{aligned} \mathcal{L} &= \bar{\psi}_1 i\partial\!\!\!/ \psi_1 + \bar{\psi}_2 i\partial\!\!\!/ \psi_2 + m_0 \left(\bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2 \right) \\ &\quad + a\bar{\psi}_1 (\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) \psi_1 + b\bar{\psi}_2 (\sigma - i\gamma_5 \vec{\tau} \cdot \vec{\pi}) \psi_2 \\ &\quad - g_\omega \bar{\psi}_1 \omega\!\!\!/ \psi_1 - g_\omega \bar{\psi}_2 \omega\!\!\!/ \psi_2 + \mathcal{L}_M \, , \\ \mathcal{L}_M &= \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \pi \partial^\mu \pi \\ &\quad - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + (g_4)^4 (\omega_\mu \omega^\mu)^2 \\ &\quad + \frac{1}{2} \bar{\mu}^2 (\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2)^2 + \epsilon \sigma \end{aligned}$$

  - masses: $m_\pm = \frac{1}{2} \left[\sqrt{(a+b)^2 \sigma^2 + 4m_0^2} \mp (a-b)\sigma \right]$
  - parameters ($g_4 = 0$): $m_0 = 790\,\text{MeV}$, $m_\sigma = 371\,\text{MeV}$, $g_\omega = 6.79$

• thermodynamics of SU(2) parity model [CS (2010)]

– mean-field approx.: $\langle\sigma\rangle$ and $\langle\omega\rangle$ from $\frac{\partial\Omega}{\partial\sigma} = \frac{\partial\Omega}{\partial\omega} = 0$

– baryon and meson number densities:

$$\rho_B(T,\mu_B) = \sum_{i=\pm} d_i \int \frac{d^3p}{(2\pi)^3} f(T,\mu;m_i)\,,$$

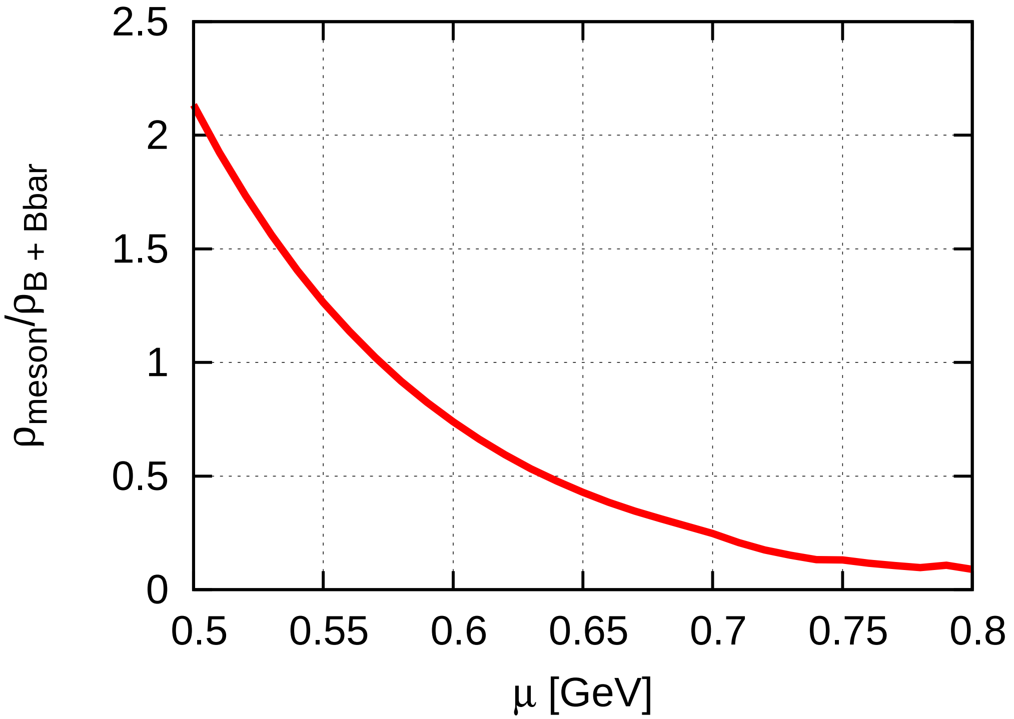
$$\rho_{\bar{B}}(T,\mu_B) = \sum_{i=\pm} d_i \int \frac{d^3p}{(2\pi)^3} \bar{f}(T,\mu;m_i)\,,$$

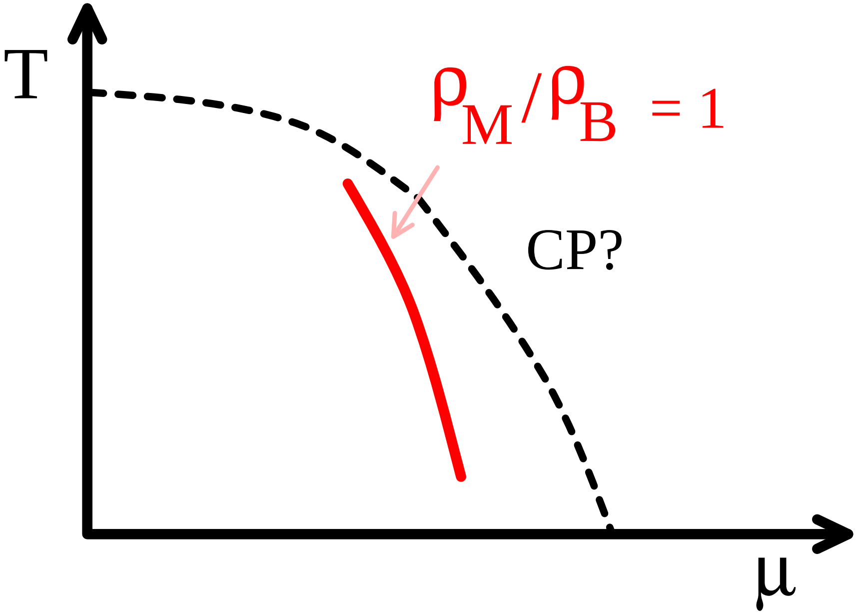
$$\rho_M(T) = \sum_{i=\sigma,\pi,\omega} d_i \int \frac{d^3p}{(2\pi)^3} b(T;m_i)$$

– in-medium meson masses

$$m_\sigma^2 = \frac{\partial^2\Omega}{\partial\sigma^2}\,, \quad m_\pi^2 = \frac{\partial^2\Omega}{\partial\pi^2}$$

– EoS at finite temperature ($T = 100$ MeV)

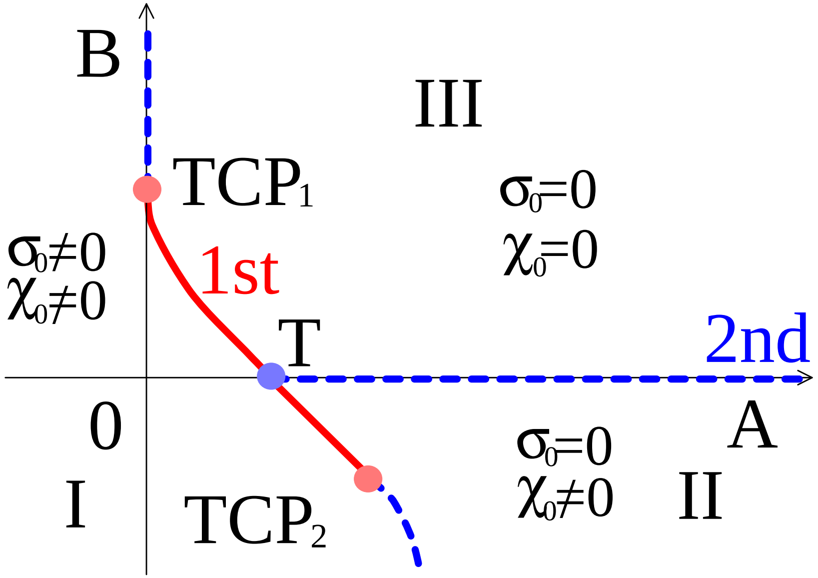


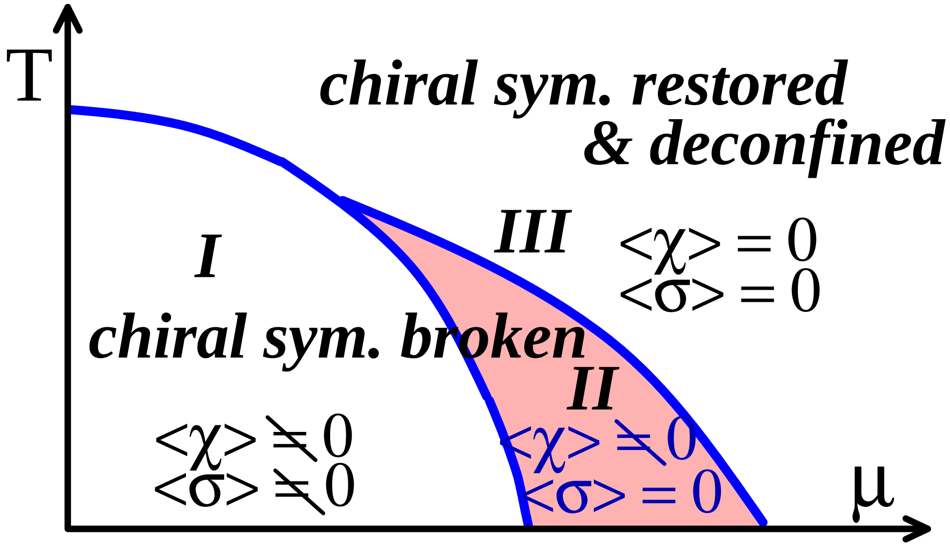


- $*$ chiral crossover $\mu_\chi \sim 0.7$ GeV
- $*$ $\rho_{\rm meson}/\rho_{\rm baryon} \sim 1$ or $s_{\rm meson}/s \sim 1$ at $\mu \sim 0.55$ GeV
- $*$ from meson dominant to baryon dominant

• 2 phases with broken symmetry: distinguished by n_B

[Harada-CS-Takemoto (09)]

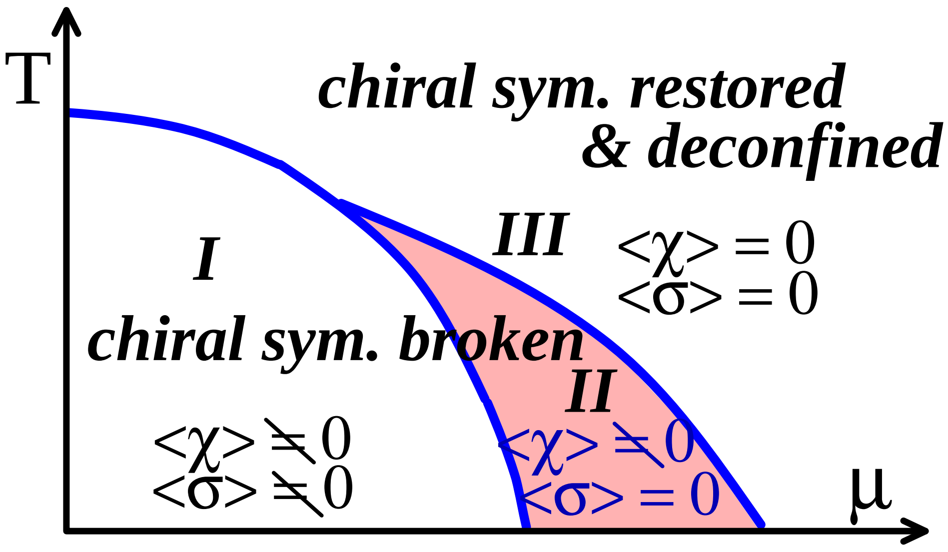


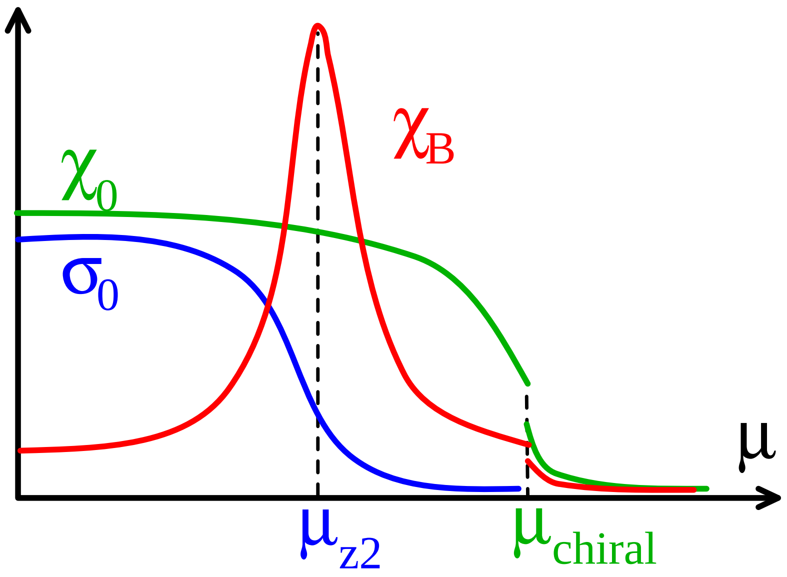


- symmetry breaking: $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V \times Z_{N_f} \to SU(N_f)_V$
- order parameters: 2-quark state $\sigma \sim \bar{q}q$ and 4-quark state $\chi \sim (\bar{q}q)^2 + \bar{q}\bar{q}\text{-}qq$
- 3 phases from a Ginzburg-Landau potential $(V = A\sigma^2 + B\chi^2 + \cdots)$

- **2 phases with broken symmetry: distinguished by n_B**

[Harada-CS-Takemoto (09)]





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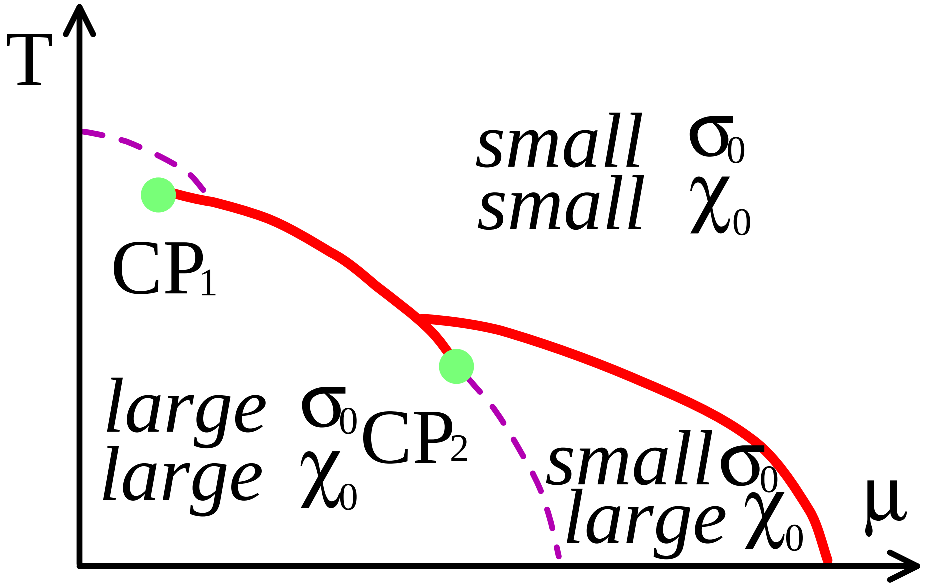
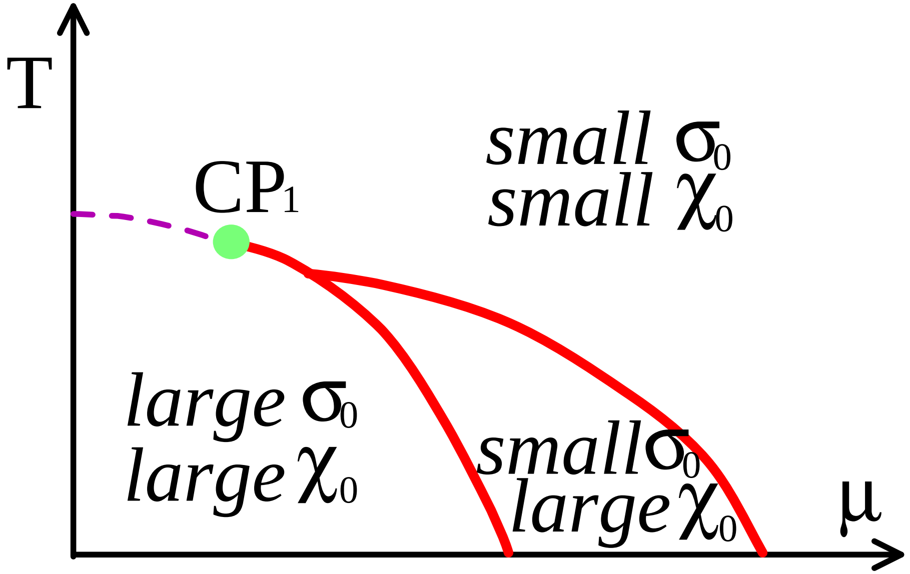
- order parameters: 2-quark state $\sigma \sim \bar{q}q$ and 4-quark state $\chi \sim (\bar{q}q)^2 + \bar{q}\bar{q}\text{-}qq$
- 3 phases from a Ginzburg-Landau potential
  I-II: χ_B max. $(\sigma \to 0)$
  II-III: χ_B no much change (no Yukawa term $\bar{N}N\chi$ in phase II)
- χ_B max. along Z_2 restoration line
  can be interpreted as “quarkyonic transition”: baryons more activated

- **hypothetical phase diagram in T-μ plane (w/ explicit breaking)**





– 2 order parameters: σ (2-quark) and χ (4-quark)
$\Rightarrow$ 2 phase transitions: restoration of Z_2 center and chiral symmetries

– multiple critical points:
CP1 and CP2 belong to the same universality class
$\Leftrightarrow$ different universality from anomaly induced CP [Hatsuda et al. (06-07)]
$\because U(1)_B$ is broken in CFL phase.

- **hadron mass spectra**

phase I: $\sigma_0 \neq 0\,, \chi_0 \neq 0$	phase II: $\sigma_0 = 0\,, \chi_0 \neq 0$
$SU(2)_V$	$SU(2)_V \times (Z_2)_A$
$m_S \neq 0\,, m_P = 0$ $m_V \neq m_A$	$m_S \neq m_P \neq 0$ $m_V \neq m_A$
$m_{N^+} \neq 0$	(i) naive: $\begin{cases} m_{N^+} = 0 \text{ (ground state)} \\ m_{N'^+} = m_{N'^-} \neq 0 \\ \quad \text{(excited states)} \end{cases}$ (ii) mirror: $\begin{cases} m_{N^+} = m_{N^-} \neq 0 \\ \quad \text{(all states)} \end{cases}$

phase I: $\sigma_0 \neq 0\,, \chi_0 \neq 0$	phase II: $\sigma_0 = 0\,, \chi_0 \neq 0$
$SU(3)_V$	$SU(3)_V \times (Z_3)_A$
$m_S \neq 0\,, m_P = 0$ $m_V \neq m_A$	$m_S = m_P \neq 0$ $m_V \neq m_A$
$m_{N^+} \neq 0$	(i) naive: $m_{N^+} \neq 0$ (ii) mirror: $m_{N^+} = m_{N^-} \neq 0$

$N_f = 2+1$: (u,d sector) $m_S \neq m_P$ (s sector) $m_S \simeq m_P$

Summary and prospects

- **dense nuclear matter and its modeling**
  - saturation properties ⇒ parity doublet model
  - meson-baryon “transition”
  - $SU(N_f)_L \times SU(N_f)_R \times Z_{N_f}$ in dense matter
    ⇒ a model for 2- and 4-quark states
  - enhancement of χ_B associated with Z_{N_f} symmetry restoration
    “quarkyonic transition”: baryons are more activated
  - 2 domains in χ-broken phase?
- **origin of hadron masses?**
  - trace anomaly and hadron mass generation?
    cf. $\langle G_{\mu\nu}G^{\mu\nu}\rangle_{T_\chi} \neq 0$
  - naive vs. mirror? sign of axial-couplings