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SUPERIOR
TÉCNICO

Pedro Bicudo

CFTP, IST, Lisboa

*work partly done with M. Cardoso, N. Cardoso,
F. Llanes-Estrada, T. Van Cauteren*



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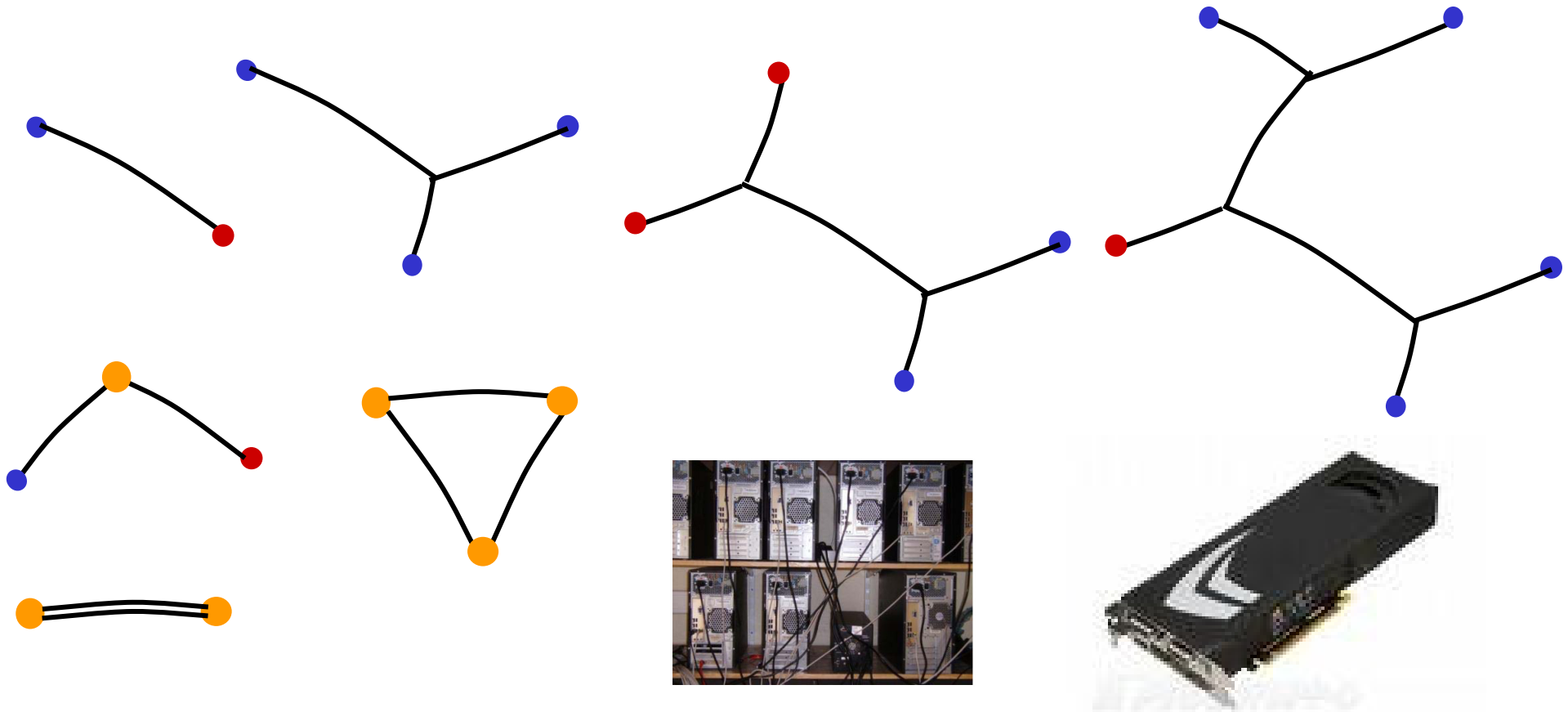
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*work partly done with M. Cardoso, N. Cardoso,
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- Motivation for confining + chiral quark models
- Extracting the quark mass from excited baryons
- Meson large degeneracy and gluon excitations
- Can the Coulomb potential produce Chiral SB?
- The finite T string tension, the finite current mass, and the Chiral symmetry and confinement crossovers

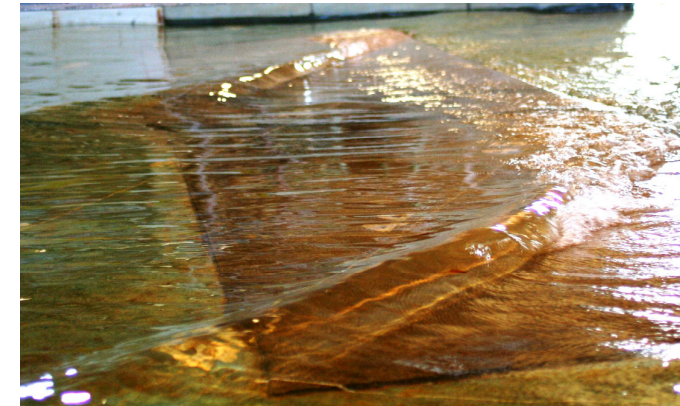
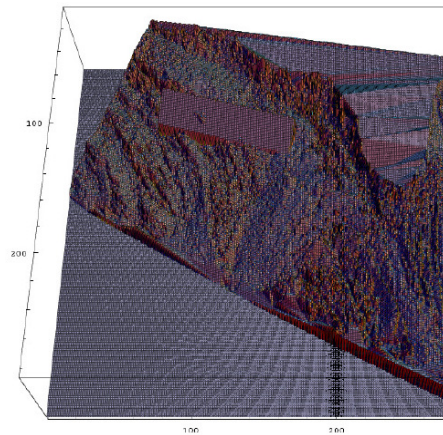
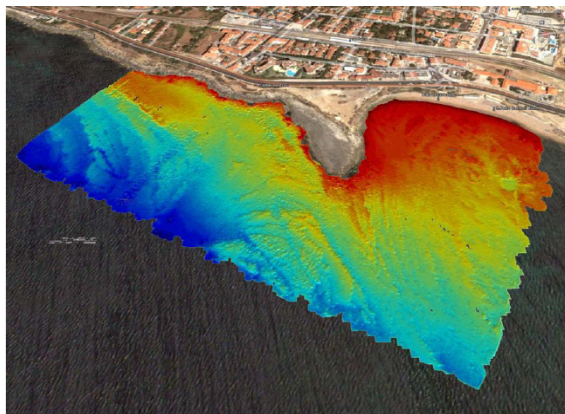
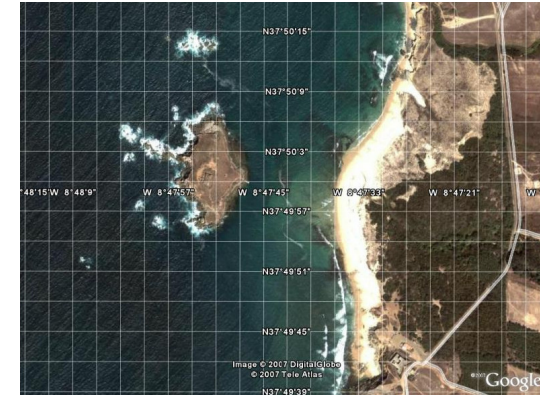
Excited Spectra and QCD Phase Diagram in Confining and Chiral S Quark Model

For ++ discussions, we also apply quark-gluon models and **Lattice QCD**, using pc clusters and graphics boards, to **all sorts of exotics**, **molecules**



Excited Spectra and QCD Phase Diagram in Confining and Chiral S Quark Model

LOL, and some of us also develop surf technology too!



Excited Spectra and QCD Phase Diagram in Confining and Chiral S Quark Model

The *modern χ Quark Model with chiral symmetry breaking* was developed in the 1980's for **light quarks**, to address the low π mass problem, and to microscopically compute correct **hadron-hadron interactions**. The model is closer to QCD, to which it relates with diagram truncations either in the Coulomb gauge or in the Balitsky gauge. Moreover the Quark Model remains confining and able to estimate any hadronic mass or reaction.

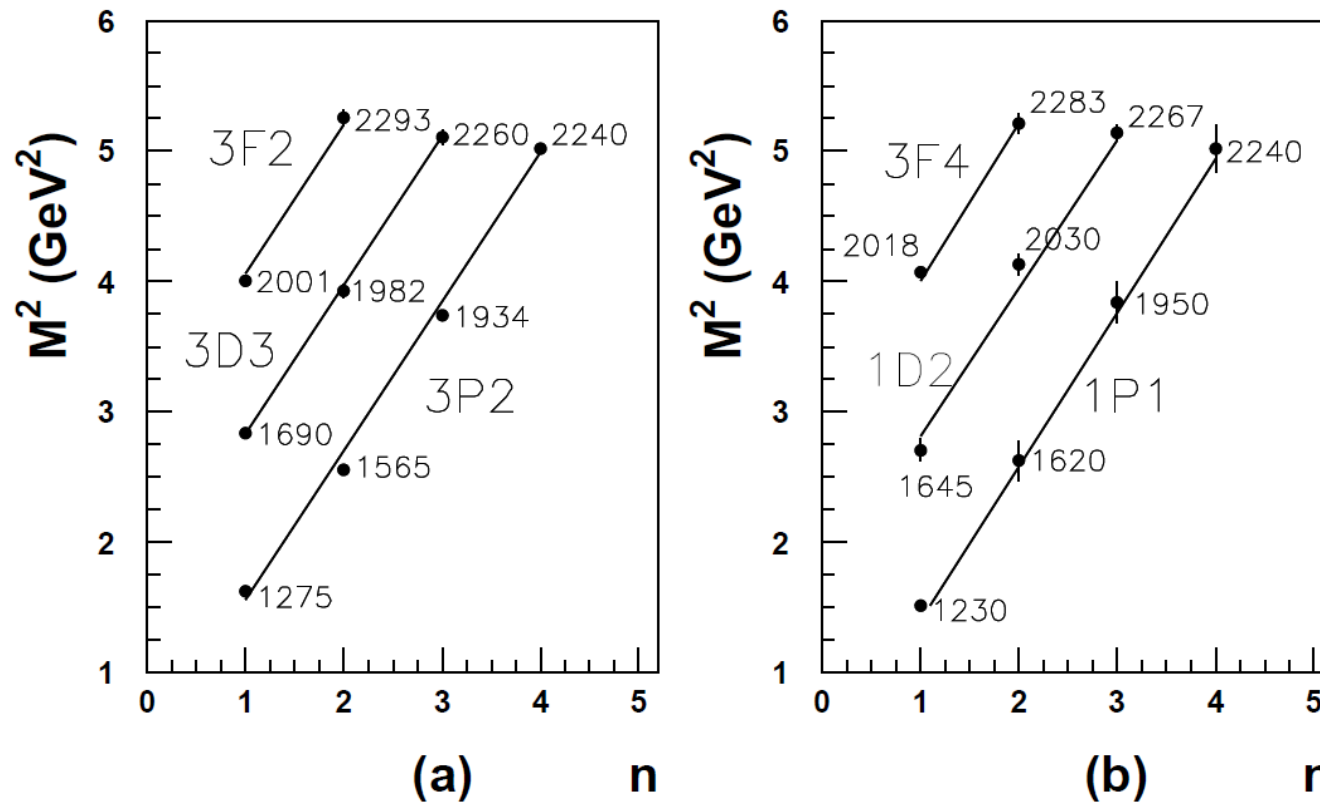
Thus the confining and chiral symmetric quark model are possibly the most adequate framework, apart from Lattice QCD which is much harder to solve, and apart from QCD which is unsolved, to address two phenomena.

Our 1st main motivation is to understand the light excited Meson and Baryon spectra to be studied at CBELSA, CLAS ... Fair, LHC, RHIC...

We are able to study the very excited light hadrons, since the χ QM is confining and chiral symmetric. The study of excited hadrons has been strongly pushed in particular by Leonid in the 2000's.

Excited Spectra and QCD Phase Diagram in Confining and Chiral S Quark Model

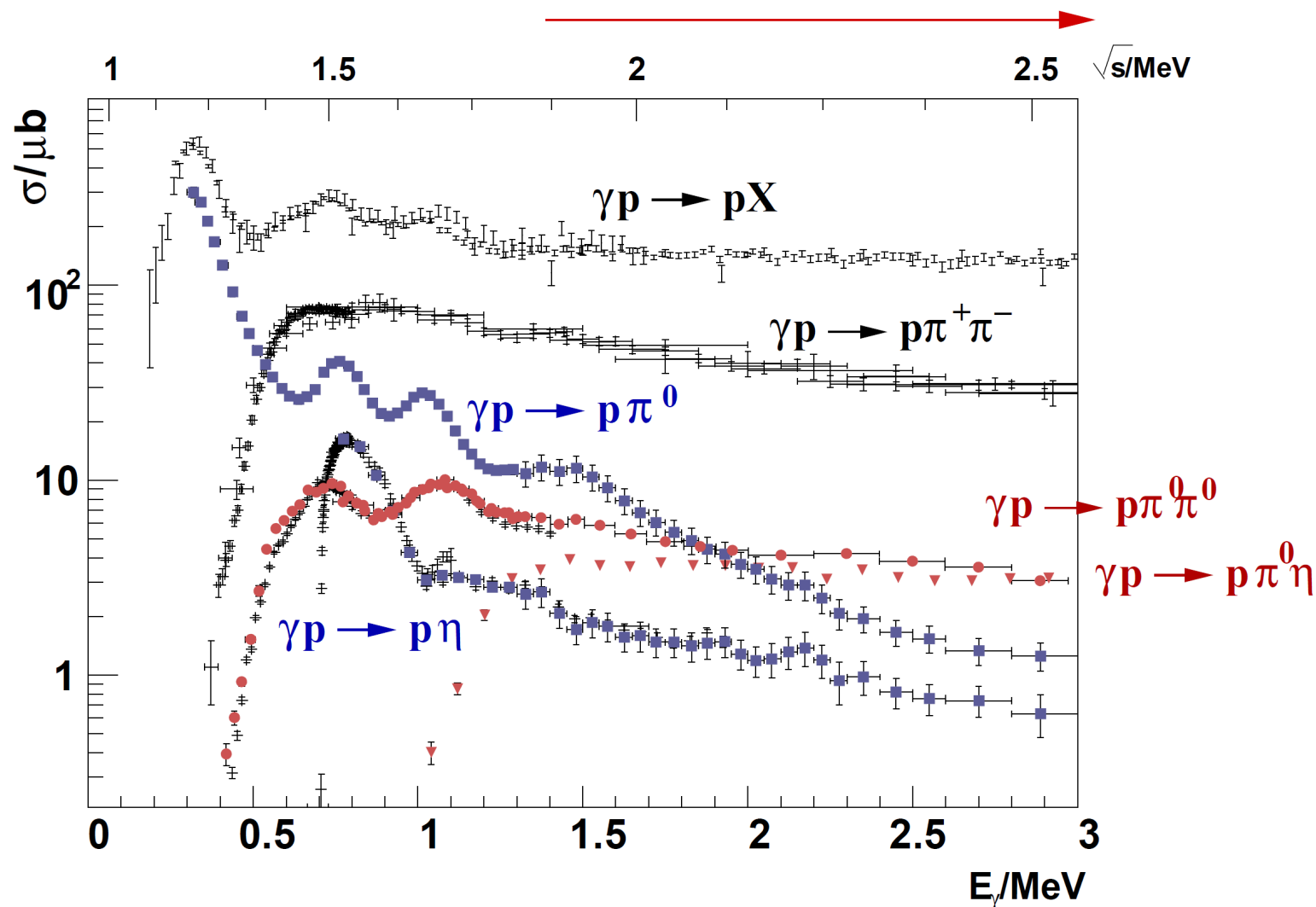
LEAR CB data, thanks to David Bugg et al



Regge trajectories of some of the $I = 0, C = +1$ states

Excited Spectra and QCD Phase Diagram in Confining and Chiral S Quark Model

CBELSA data, thanks to Ulrike Thoma et al



The 2nd main motivation is to contribute to understand the QCD phase diagram, for finite T and μ , to be studied at LHC, RHIC and FAIR.

Using *modern* quark models for **light quarks** we address chiral symmetry breaking, or quark mass generation, at finite T

Recently we used the bottomonium and charmonium as good prototypes to study finite T quark-antiquark potentials, in that case had to solve the Schrödinger equation with static lattice QCD potentials, since

$m_b, m_c \gg T_{QCD}$ and $m_c \gg T_c$

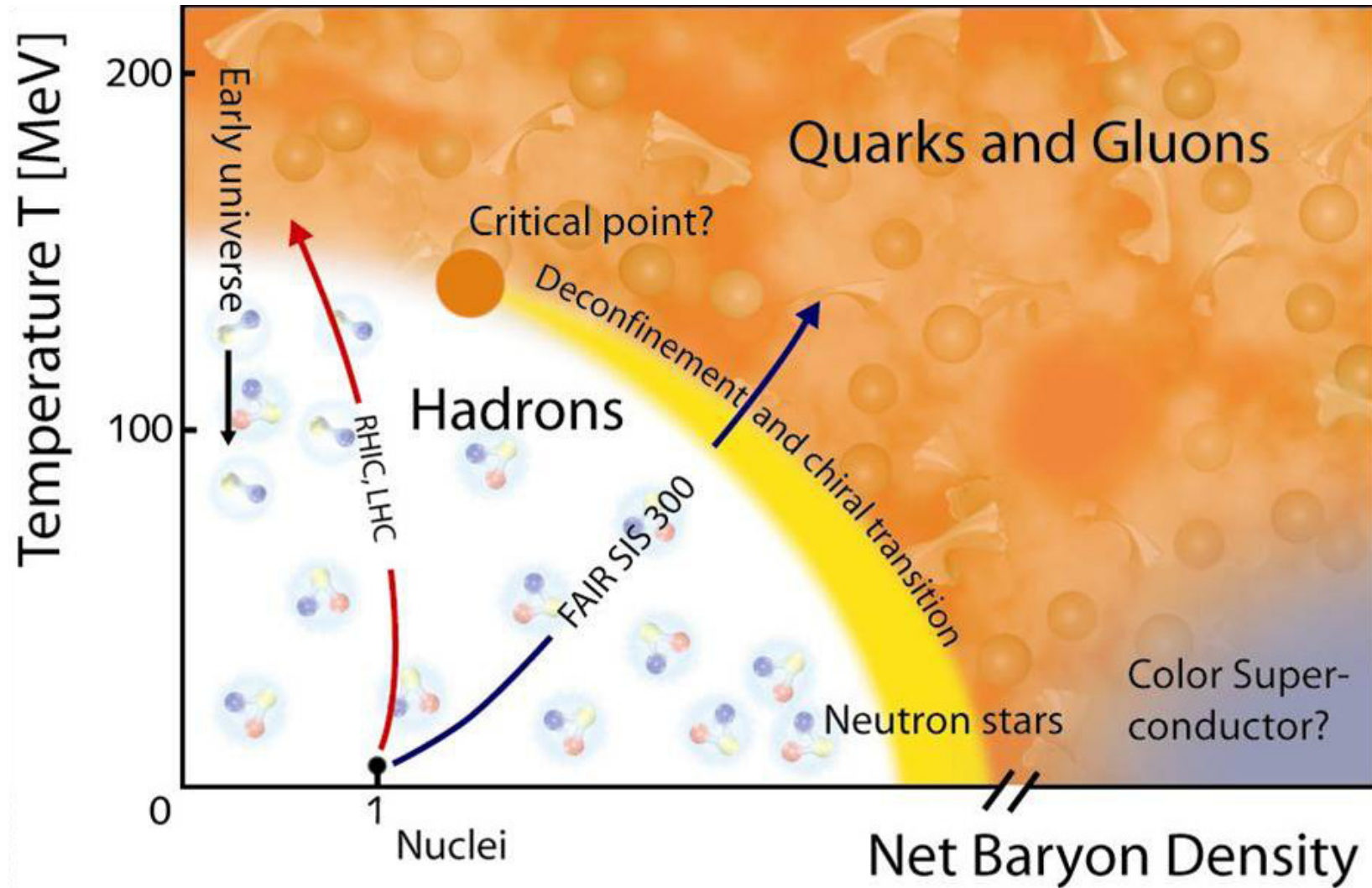
Allowed us to neglect in the quark sector, temperature effects, spontaneous chiral symmetry breaking, relativistic effects and coupled channels.

Here we address the light quarks at finite T and in the future we may,

- compute the spectrum of any hadron at finite T and μ
- compute the interaction of any hadron-hadron at finite T and μ

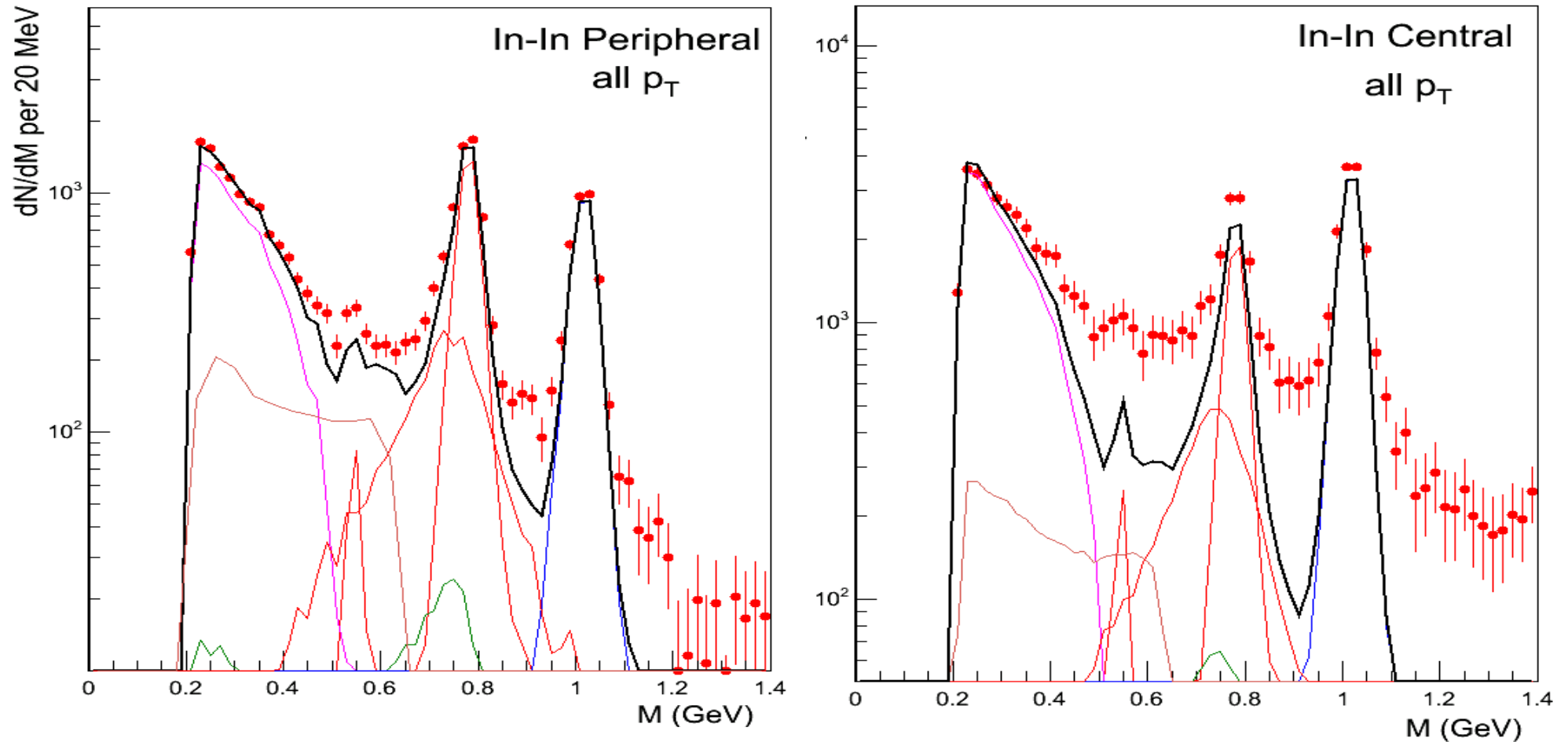
Excited Spectra and QCD Phase Diagram in Confining and Chiral S Quark Model

Illustration, thanks to FAIR



Excited Spectra and QCD Phase Diagram in Confining and Chiral S Quark Model

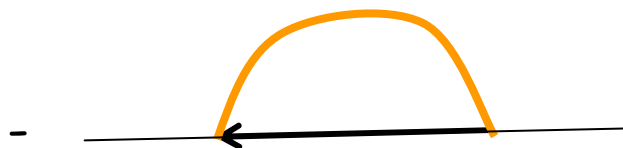
NA60 data, thanks to Carlos Lourenço et al



Excited Spectra and QCD Phase Diagram in Confining and Chiral S Quark Model

The mass gap equation at the ladder/rainbow truncation of Coulomb Gauge QCD in equal time reads,

$$\overleftarrow{-1} = \overleftarrow{\frac{-1}{0}}$$



$$0 = u_s^\dagger(k) \left\{ k\hat{k} \cdot \boldsymbol{\alpha} + m_0\beta - \int \frac{dw'}{2\pi} \frac{d^3\mathbf{k}'}{(2\pi)^3} i\tilde{V}(k-k') \right. \\ \left. \sum_{s'} \left[\frac{u(k')_{s'} u^\dagger(k')_{s'}}{w' - E(k') + i\epsilon} - \frac{v(k')_{s'} v^\dagger(k')_{s'}}{-w' - E(k') + i\epsilon} \right] \right\} v_{s''}(k)$$

$$E(k) = u_s^\dagger(k) \left\{ k\hat{k} \cdot \boldsymbol{\alpha} + m_0\beta - \int \frac{dw'}{2\pi} \frac{d^3\mathbf{k}'}{(2\pi)^3} i\tilde{V}(k-k') \right. \\ \left. \sum_{s'} \left[\frac{u(k')_{s'} u^\dagger(k')_{s'}}{w' - E(k') + i\epsilon} - \frac{v(k')_{s'} v^\dagger(k')_{s'}}{-w' - E(k') + i\epsilon} \right] \right\} u_s(k)$$

... this is tricky and nasty due to the cancelling infrared divergences already mentioned by Eric... but once it is solved, one then one just has to solve the Bethe Salpeter equation to get the whole hadronic spectra for mesons, baryons, glueballs, hybrids, multiquarks, constituted by light quarks, heavy quarks, gluons, and also to compute hadron-hadron interactions

Extracting the quark mass from excited baryons

PRL **103**, 092003 (2009)

The chiral symmetry restoration, or chiral symmetry insensitivity, in the excited hadron spectrum, can be understood in the light quark limit, (opposite to the heavy quark limit of Isgur and Wise). When

$$m(k) / k \rightarrow 0$$

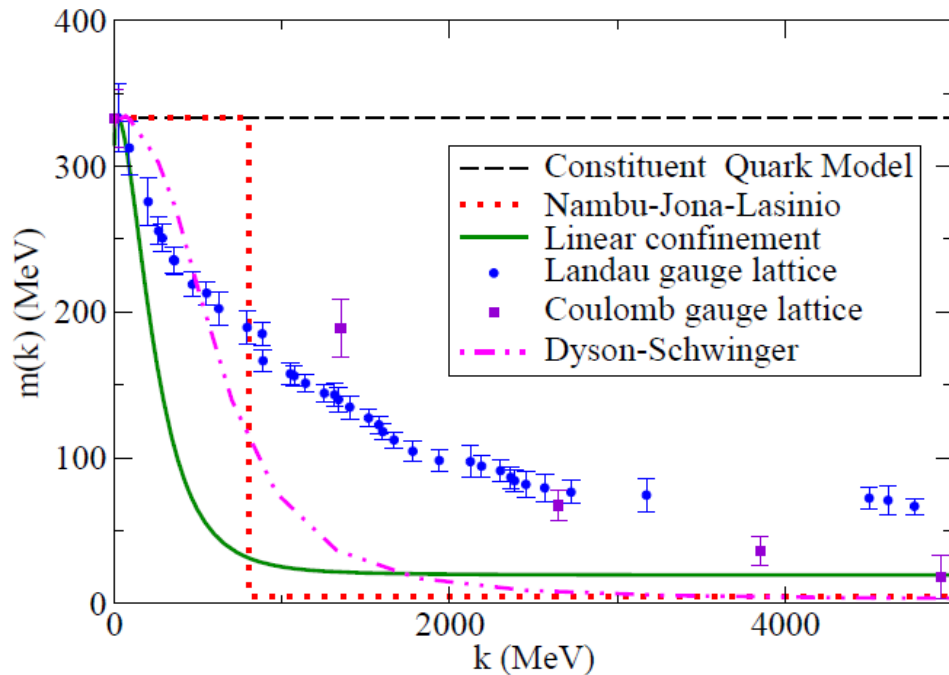
but before this limit is reached, a breaking of the chiral symmetry remains and the $m(k)/k$ becomes a good expansion parameter, appearing in any remaining breaking of chiral symmetry, in particular in the splittings of parity quasi-doublets. **We propose to**

Experimentally determine the power-law behaviour of the mass difference between parity partners and their spin j :

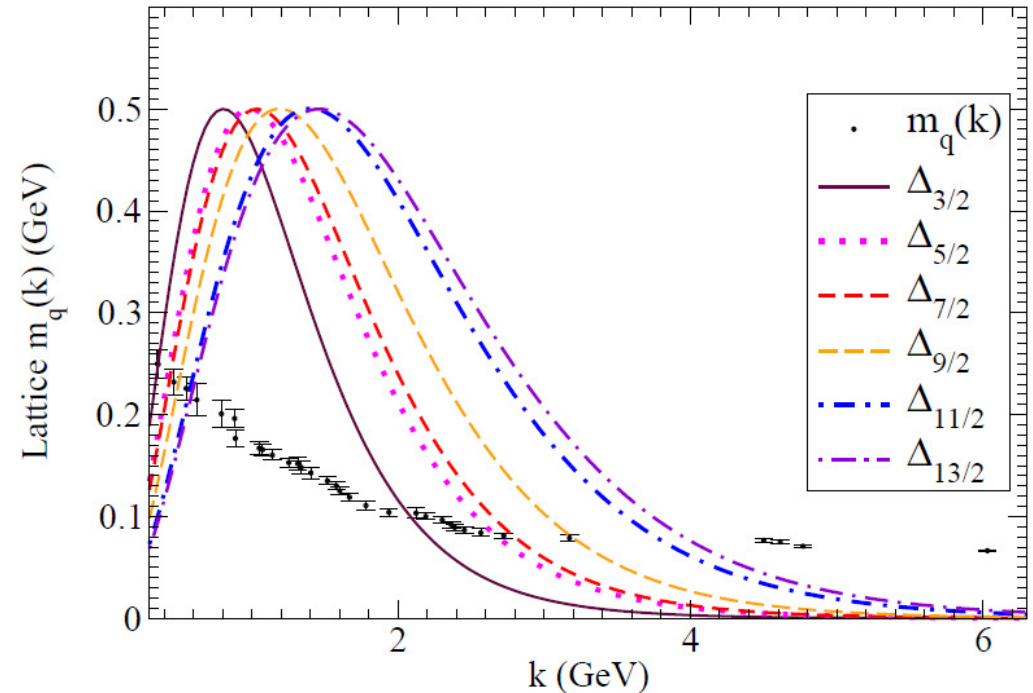
$$|M^+ - M^-| \propto j^{-i}$$

$$\longrightarrow m\left(\frac{g}{\sqrt{\alpha}}\sqrt{j}\right) \propto j^{-i+1} \text{ or } m(k) \propto k^{-2i+2}$$

Extracting the quark mass from excited baryons



The quark mass runs with momentum, and is enhanced in the infrared



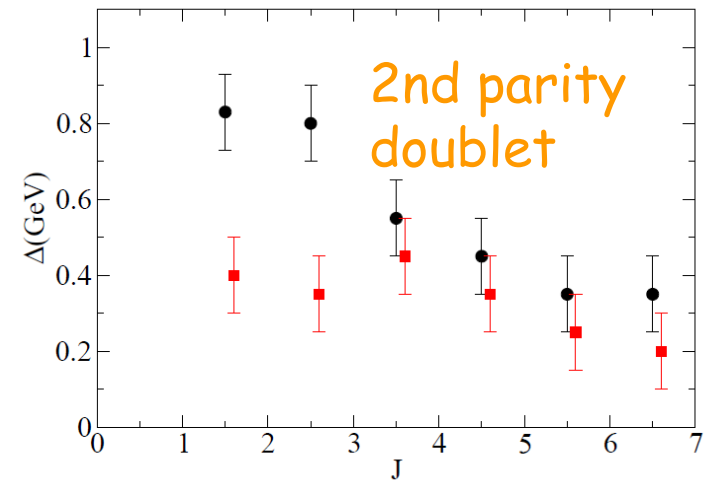
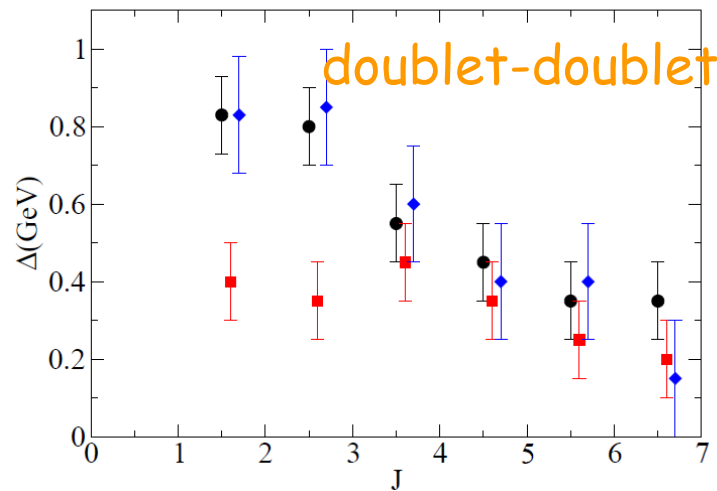
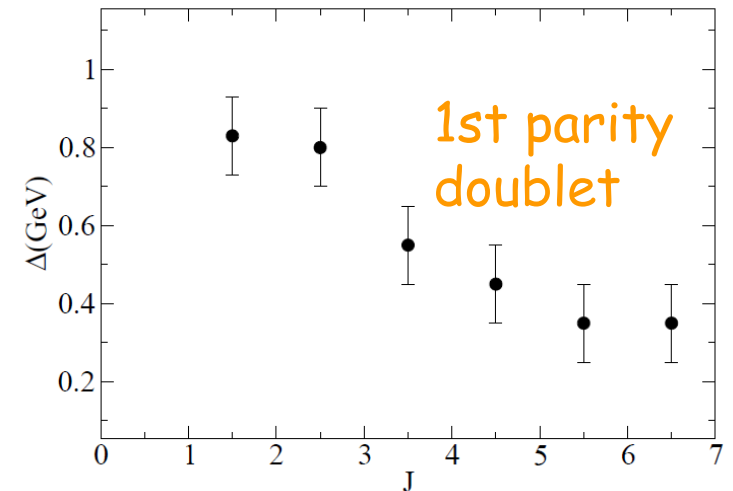
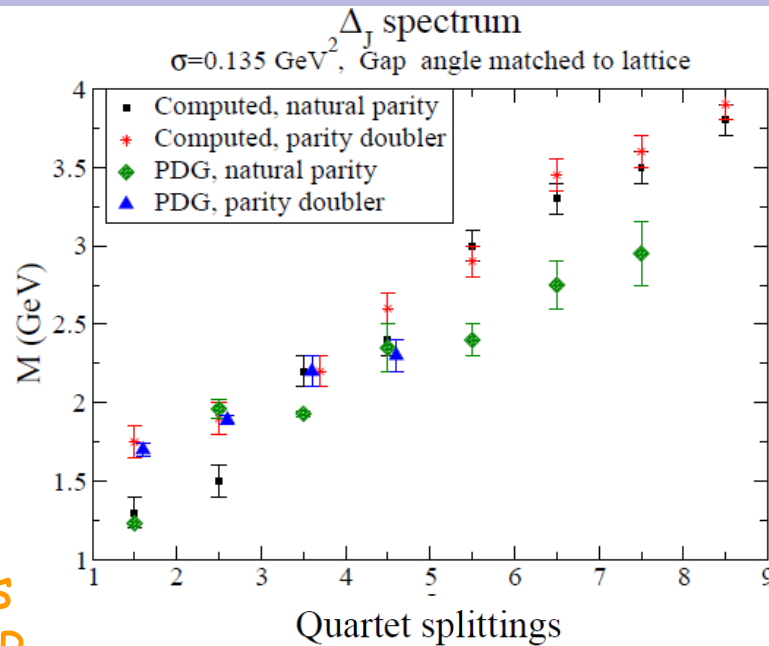
The average momentum in a baryon increases with the respective J

We propose that the running quark mass can be measured from the spectrum of excited baryons, say at **ELSA, JLAB, FAIR, LHC, RHIC...**

Extracting the quark mass from excited baryons

We compute for the **1st time** a baryon spectrum with chiral symmetry and confinement!
 We have **12 dim eigenvalue integral equations**
1 year of coding :P

For the Δ s we show that we have a nearly degenerate quartet, with two parity doublets



Meson large degeneracy and gluon excitations

PHYSICAL REVIEW D **81**, 014011 (2010)

PHYSICAL REVIEW D **76**, 094005 (2007)

In the meson sector there is a large degeneracy, similar to the one of QED where there is a principal quantum number

$$j + n \simeq \alpha_0 + \alpha M^2,$$
$$\alpha \simeq (2\pi\sigma)^{-1} = 0.84 \text{ GeV}^{-2}$$

Well... the j / n degeneracy is very puzzling for the χ QM ... but adding string degrees of freedom, and using the **einbein approximation** to the confining potential, as Fabien and Vincent usually do, we can show that the masses of the excited mesons approximately follow

$$M \simeq \sqrt{2\pi\sigma(\mathcal{N}_{q\bar{q}} + \mathcal{N}_g)},$$

so we find a principal quantum number!

$$\mathcal{N}_{q\bar{q}} + \mathcal{N}_g = 2n_{q\bar{q}} + j_{q\bar{q}} + 2n_g + l_g + 6$$

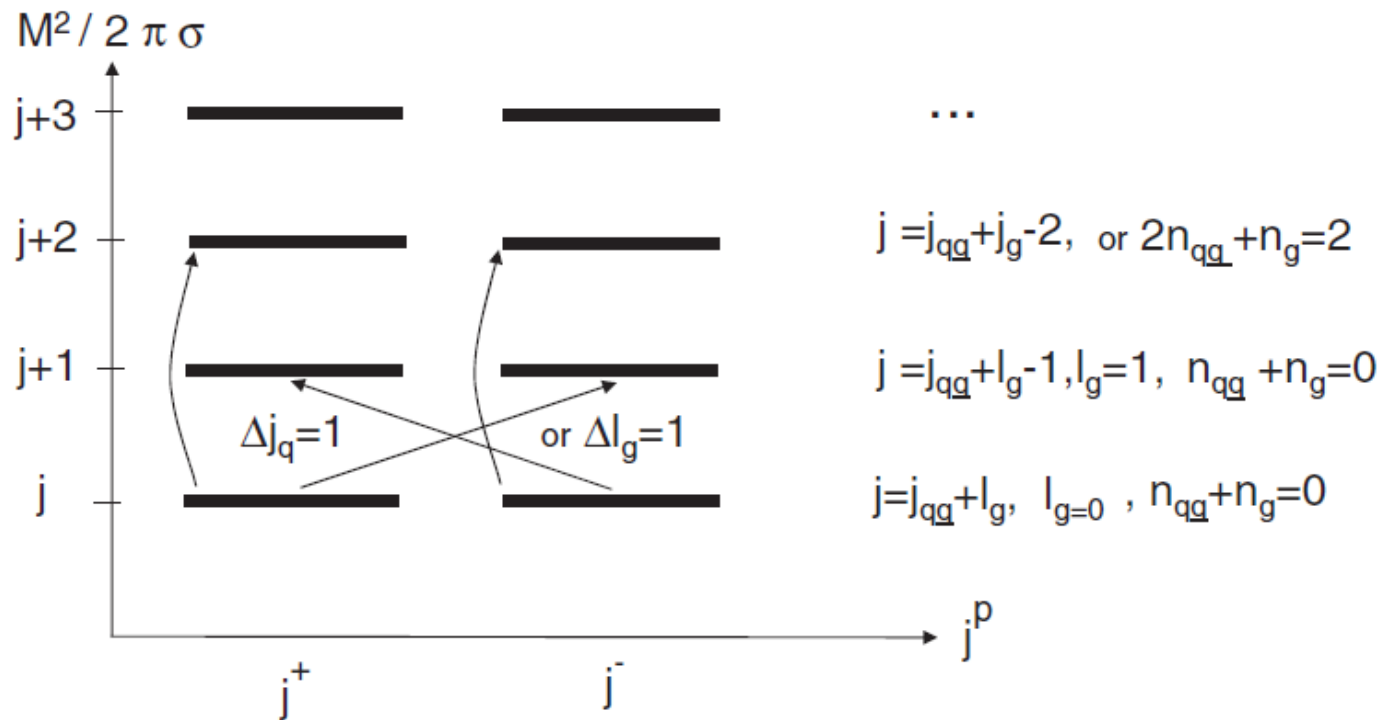
Meson large degeneracy and gluon excitations

moreover, combining the two spectra, we find that the principal $N_{qq} + N_g$ quantum number simplifies to

$j + n$

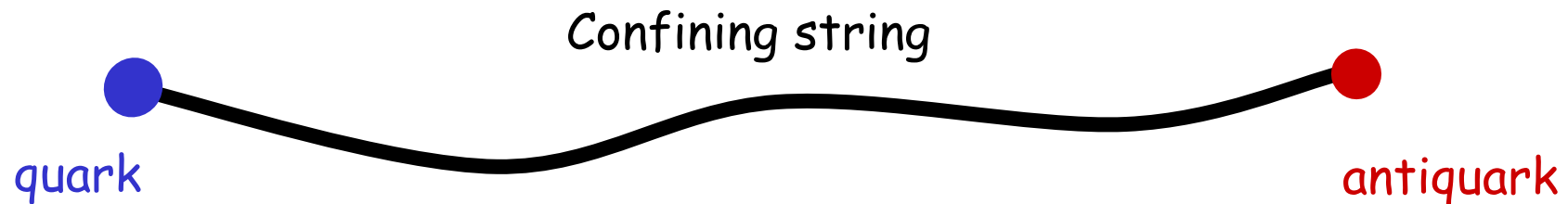
where j is the total angular momentum,

and n indexes excitations with the same quantum numbers



Fits of the finite T string tension from the Lattice QCD energy F1

The **confinement**, modelled by a string, is dominant at moderate distances



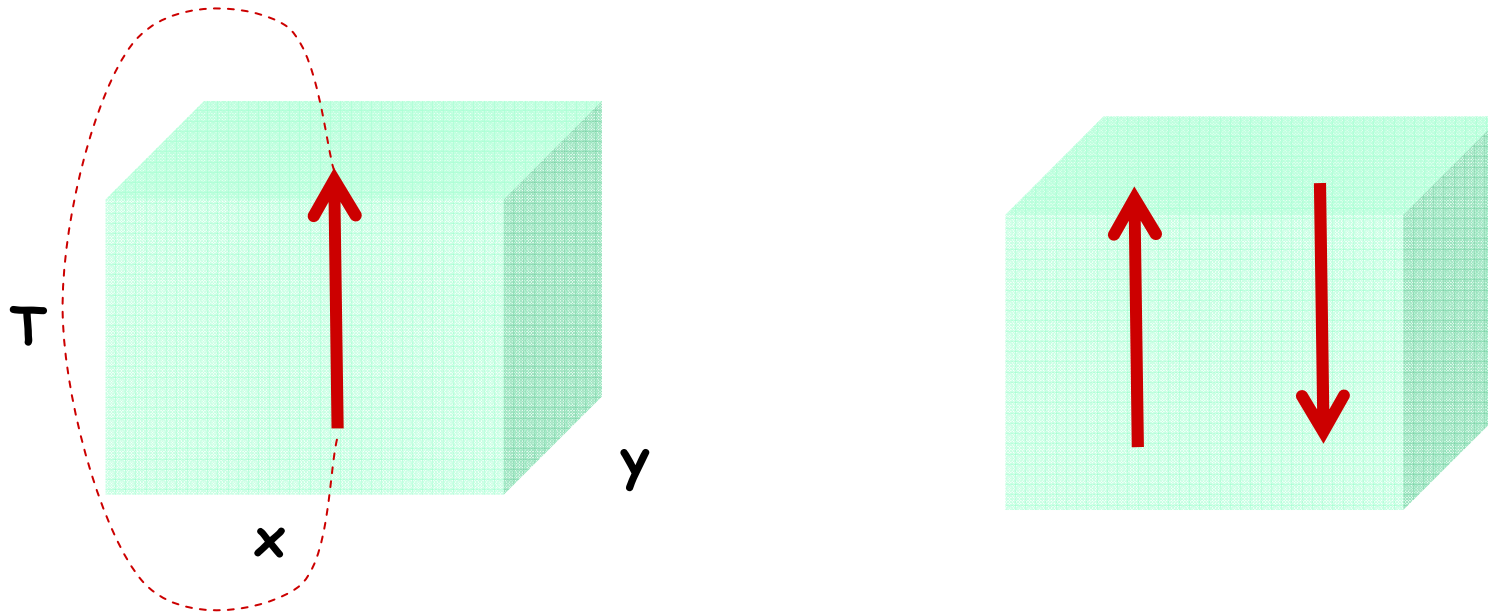
σ is the string tension

$$V(r) \text{ ----> } \frac{\pi}{12 r} + V_0 + \sigma r$$

preliminary

At short distances we have the Luscher or Nambu-Gotto Coulomb due to the **string vibration** + the **OGE** coulomb, however the Coulomb is not important for chiral symmetry breaking, thus we will focus on the **linear confinement**

Fits of the finite T string tension from the Lattice QCD energy F1



preliminary

The **Polyakov loop** is a gluonic path, closed in the imaginary time t_4 / inverse temperature T direction in QCD discretized in a periodic boundary euclidian Lattice. It measures the free energy F of **one** or **more** static quarks

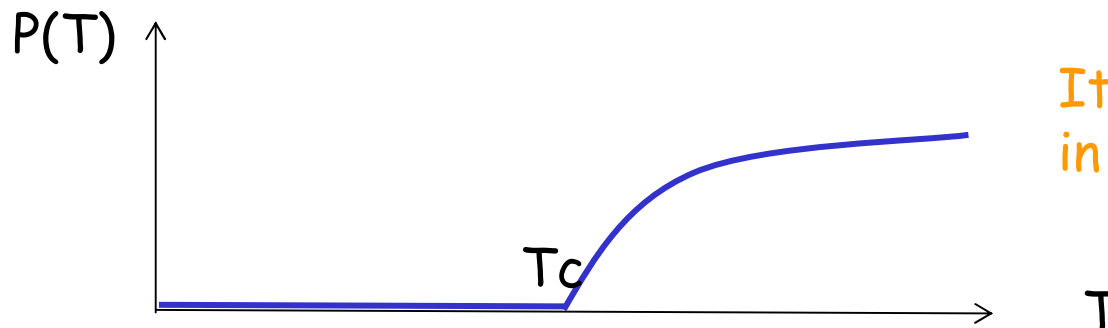
$$P = \mathcal{N} \text{Exp}[- F / T]$$

Fits of the finite T string tension from the Lattice QCD energy F1

END
OF THE
UNIVERSE



If we consider a single solitary lonely quark in the universe, in the confining phase, his string will travel as far as needed to look for a partner antiquark, resulting in an infinite energy F . Thus the 1 quark **Polyakov loop P** is a frequently used **order parameter** for deconfinement.



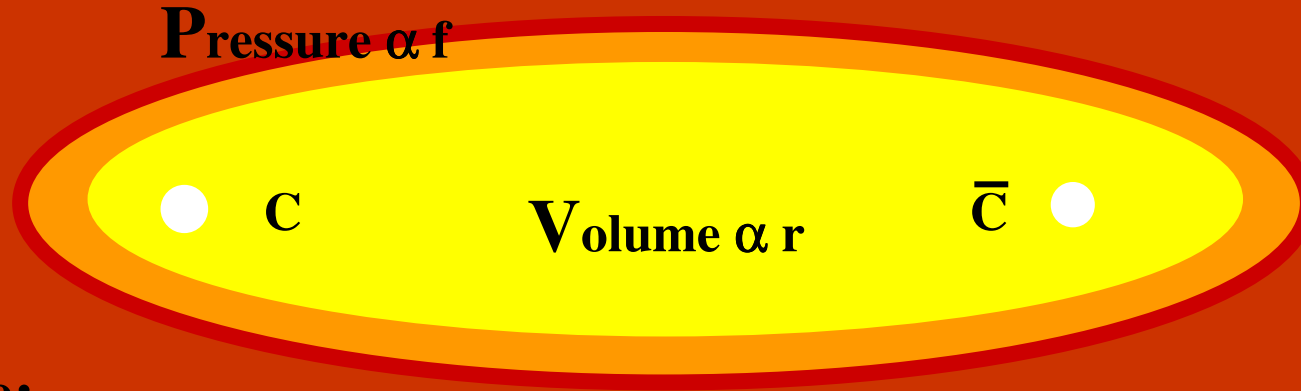
It is a phase transition
in pure gauge QCD!

preliminary

However, since we are interested in approaching the **deconfinement transition** from below T_c , we prefer here to use the string tension σ as the **order parameter**, computed in the quark-antiquark colour singlet Polyakov loop **P**.

Fits of the finite T string tension from the Lattice QCD energy F1

Pressure $\propto f$



With
Temperature:

preliminary

Potential: $V(r) = - f d r$

Free Energy: $F_1(r) = - f d r - S d T$ *OK for isothermic*

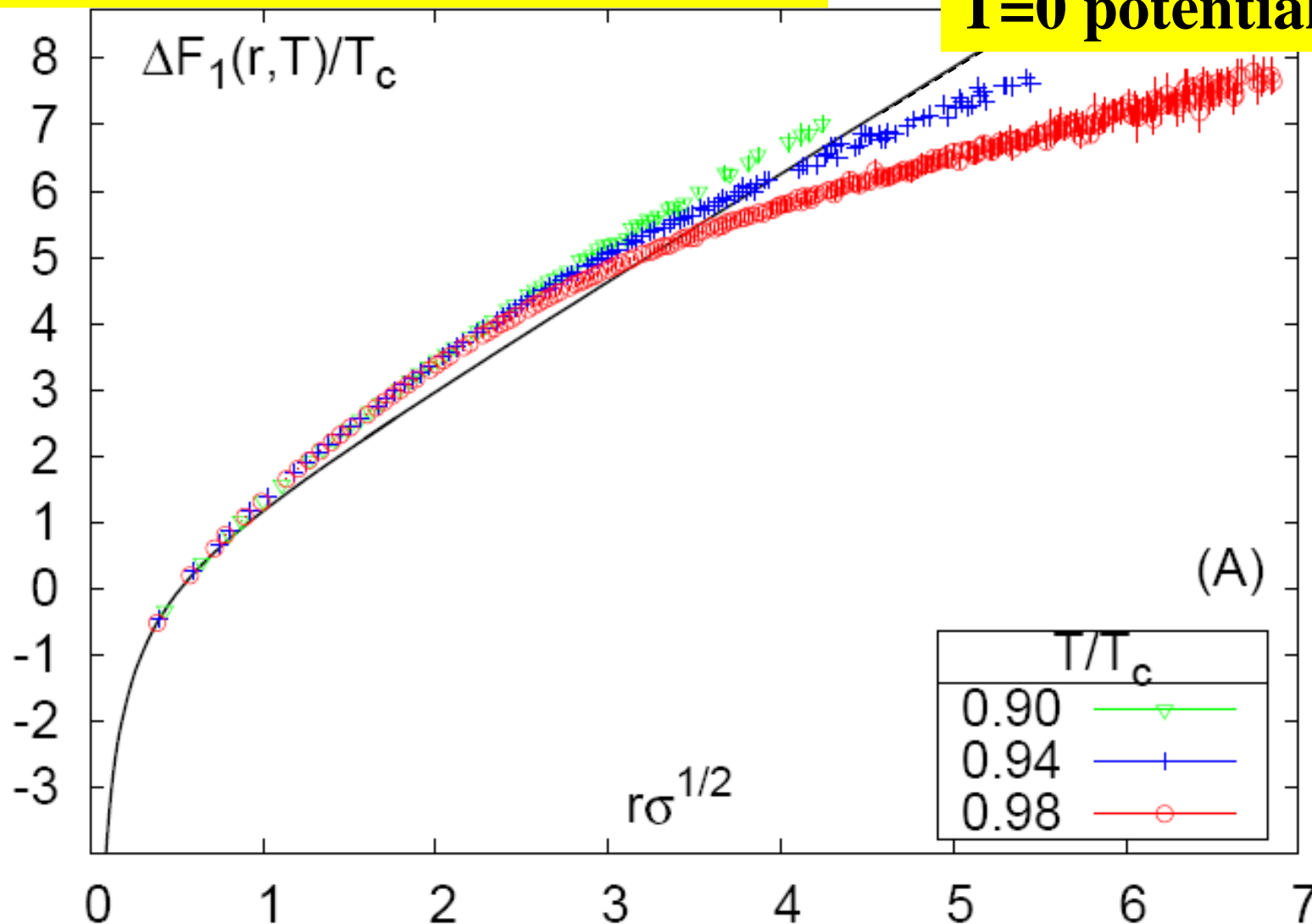
Internal Energy: $E_1(r) = - f d r + T d S$ *OK for adiabatic*

Fits of the finite T string tension from the Lattice QCD energy F1

Lattice QCD data, thanks to Olaf Kaczmarek et al.

T=0 potential V

**Free
Energy
F1**



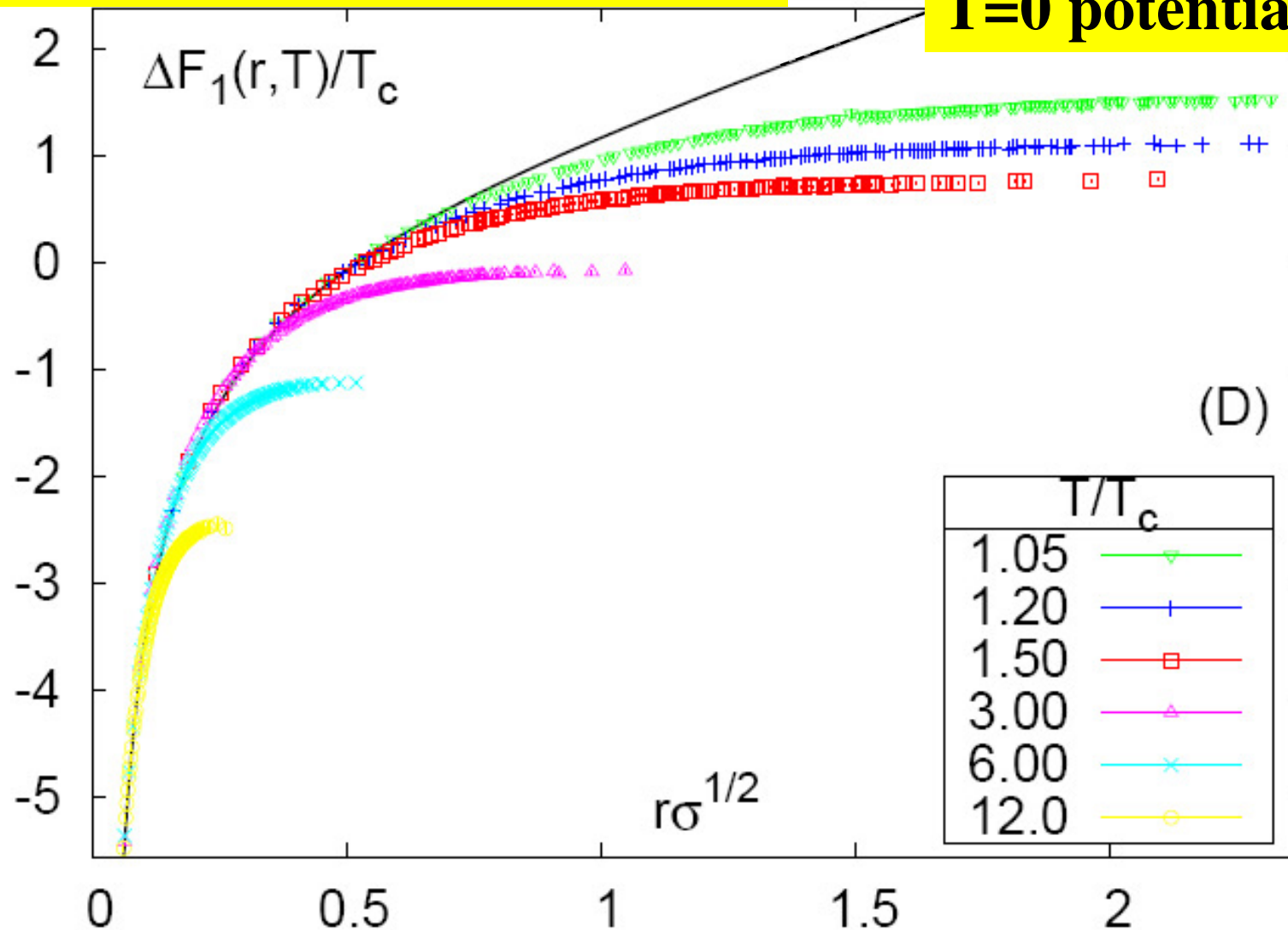
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Fits of the finite T string tension from the Lattice QCD energy F1

Lattice QCD data, thanks to Olaf Kaczmarek et al.

T=0 potential V

**Free
Energy
F1**



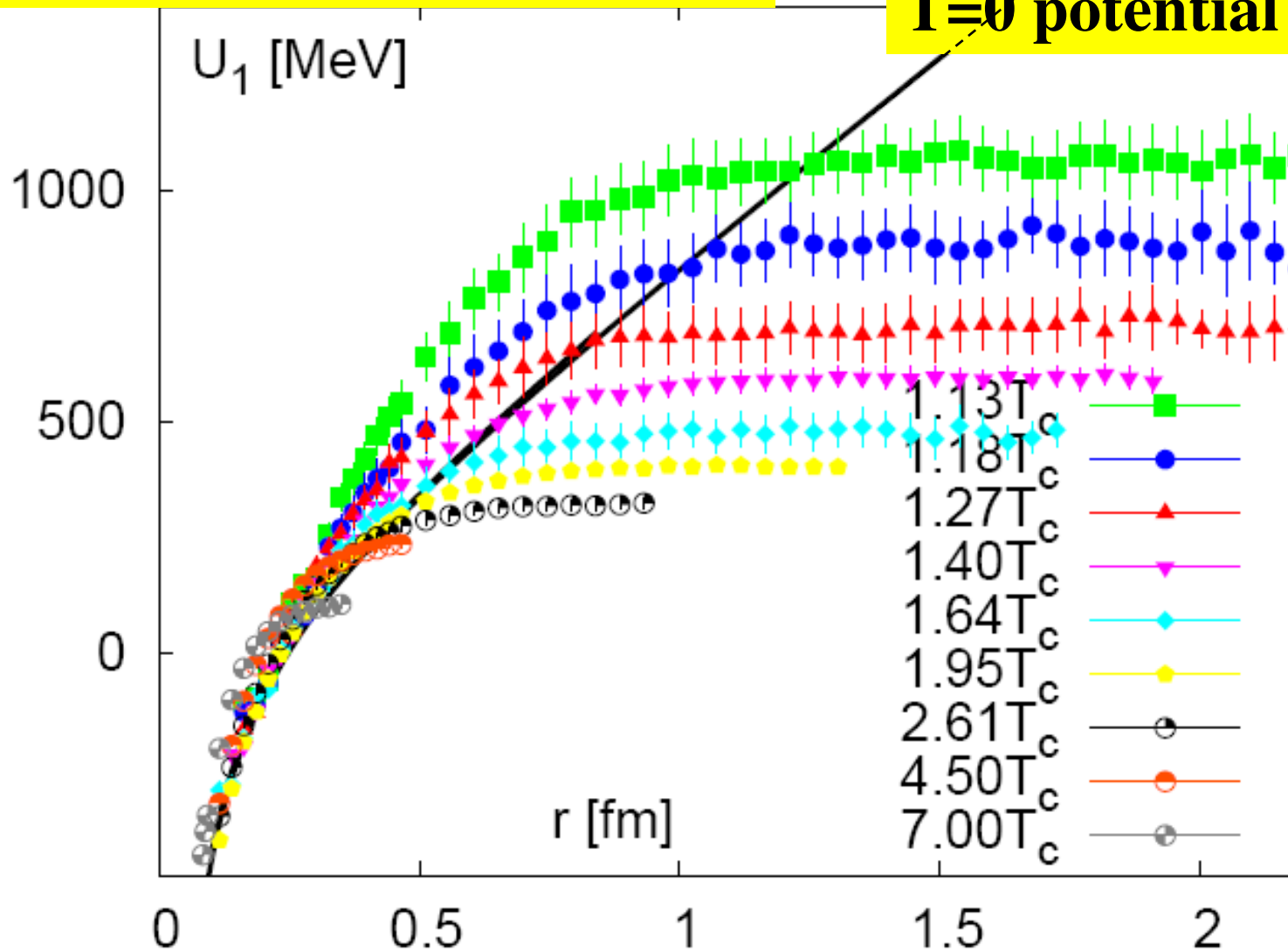
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Fits of the finite T string tension from the Lattice QCD energy F1

Lattice QCD data, thanks to Olaf Kaczmarek et al.

T=0 potential V

Internal
Energy
U1

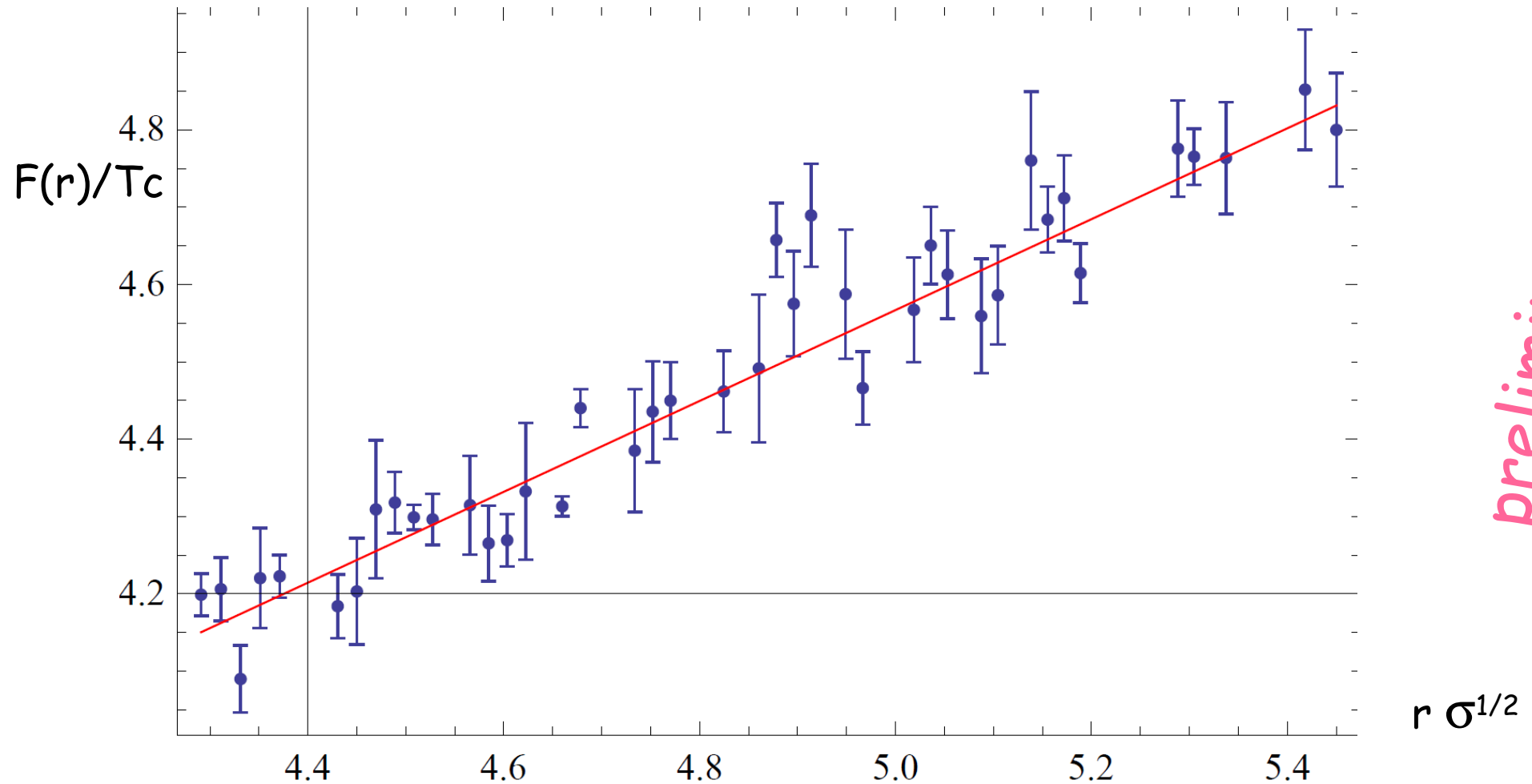


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Fits of the finite T string tension from the Lattice QCD energy F1

Linear fit of the longer distance part of the free energy F.

We cut the short distance in such a way that $\chi^2/\text{dof} \sim 1$. σ is the slope.

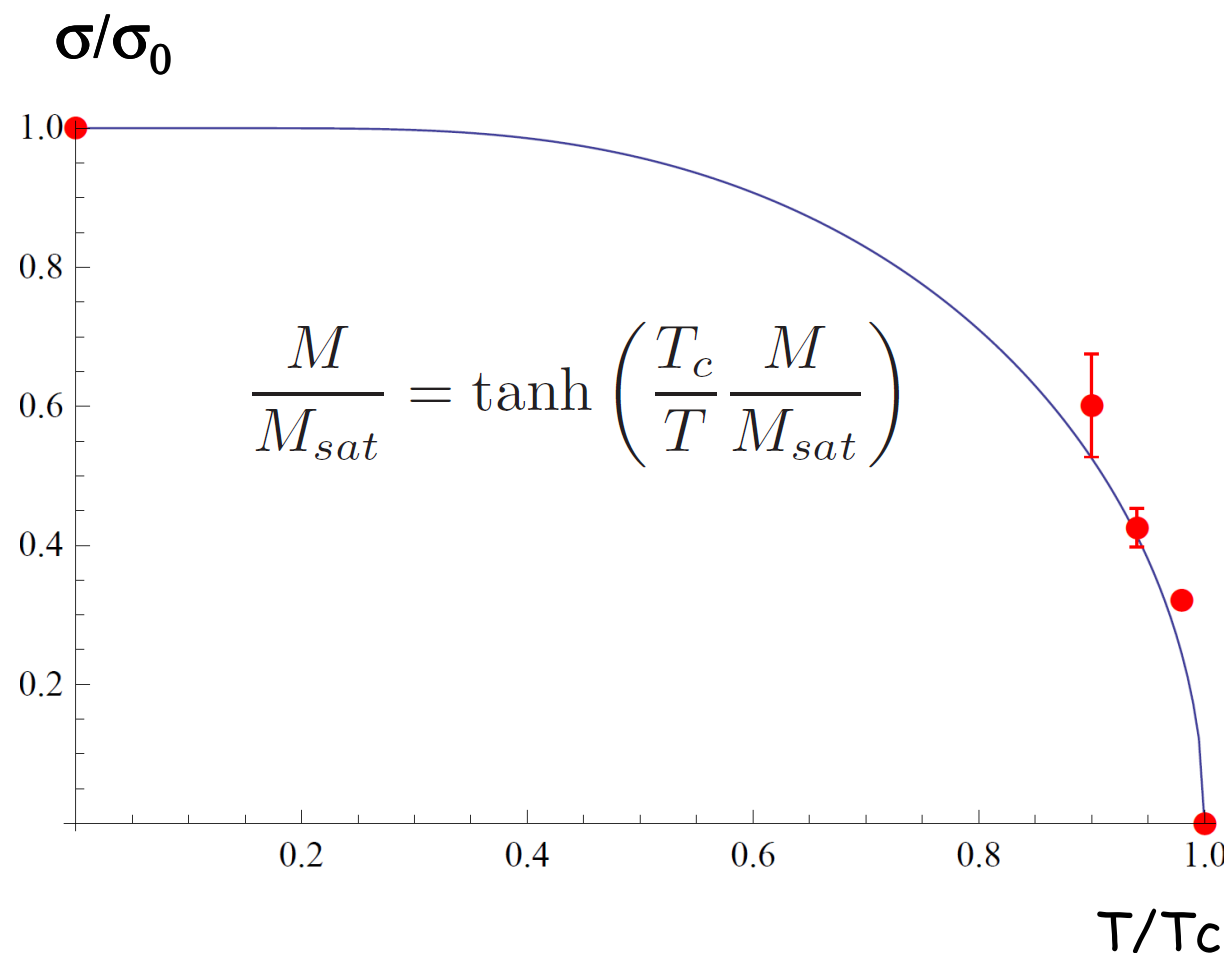


Fits of the finite T string tension from the Lattice QCD energy F1

Comparing the string tensions at T=0, with the cond mat magnetization curve

The magnetization curve of a magnetic material is a textbook curve well modelled by the **statistics of spin 1/2 systems**.

Here we show that the same curve also models the σ string tension and the **deconfinement curve!**



preliminary

What is the importance of the Coulomb potential in χ SB?

PHYSICAL REVIEW D **79**, 094030 (2009)

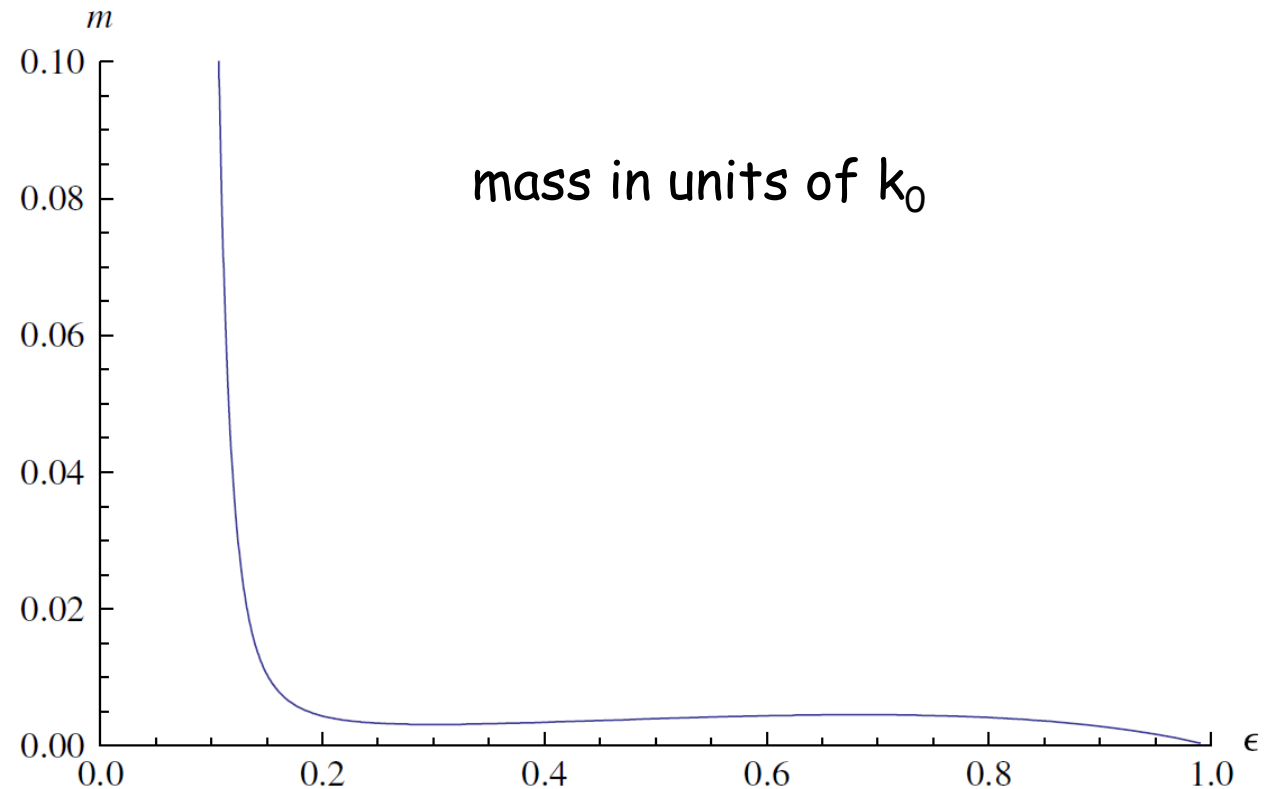
One may wonder if, even when there is deconfinement, the remaining Coulomb-like potential may still lead to chiral symmetry breaking, thus we solve the mass gap equation for a Coulomb-like potential,

$$V(\mathbf{r}) = -\alpha \frac{k_0^\epsilon}{r^{1+\epsilon}}$$

and indeed mass is generated.

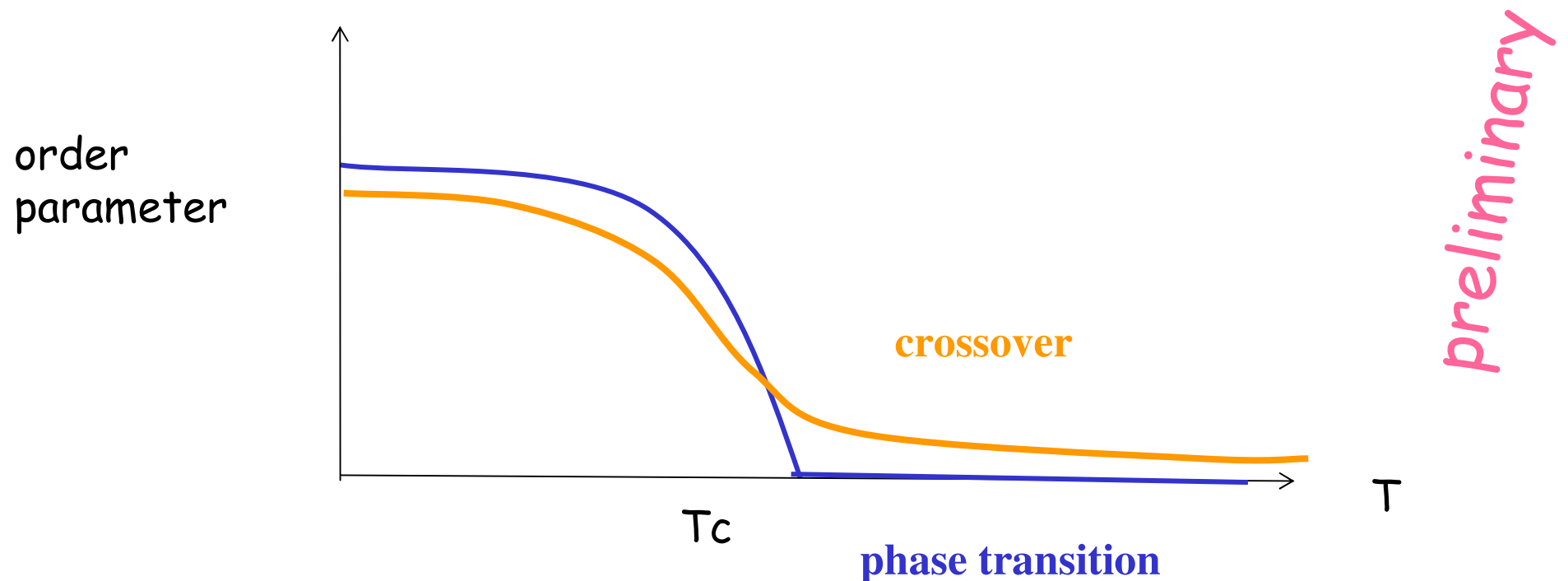
However the mass is UV divergent, and has to be renormalized,

in the end we get only the current mass m_0



The mass gap equation with finite T and finite quark mass

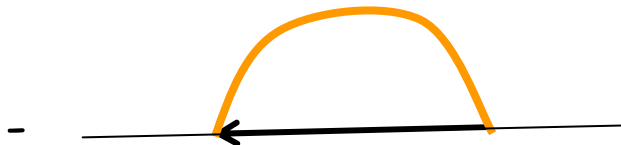
Now, the critical point occurs when the phase transition changes to a crossover, and the crossover in QCD is produced by the **finite current quark mass m_0** , since it affects the order parameters P or σ , and $m(0)$ or $\langle qq \rangle$



The mass gap equation with finite T and finite quark mass

The mass gap equation at the ladder/rainbow truncation of Coulomb Gauge QCD in equal time reads,

$$\overleftarrow{-1} = \overleftarrow{\frac{-1}{0}}$$



$$0 = u_s^\dagger(k) \left\{ k\hat{k} \cdot \alpha + m_0\beta - \int \frac{dw'}{2\pi} \frac{d^3\mathbf{k}'}{(2\pi)^3} i\tilde{V}(k-k') \right.$$

$$\left. \sum_{s'} \left[\frac{u(k')_{s'} u^\dagger(k')_{s'}}{w' - E(k') + i\epsilon} - \frac{v(k')_{s'} v^\dagger(k')_{s'}}{-w' - E(k') + i\epsilon} \right] \right\} v_{s''}(k)$$

$$E(k) = u_s^\dagger(k) \left\{ k\hat{k} \cdot \alpha + m_0\beta - \int \frac{dw'}{2\pi} \frac{d^3\mathbf{k}'}{(2\pi)^3} i\tilde{V}(k-k') \right.$$

$$\left. \sum_{s'} \left[\frac{u(k')_{s'} u^\dagger(k')_{s'}}{w' - E(k') + i\epsilon} - \frac{v(k')_{s'} v^\dagger(k')_{s'}}{-w' - E(k') + i\epsilon} \right] \right\} u_s(k)$$

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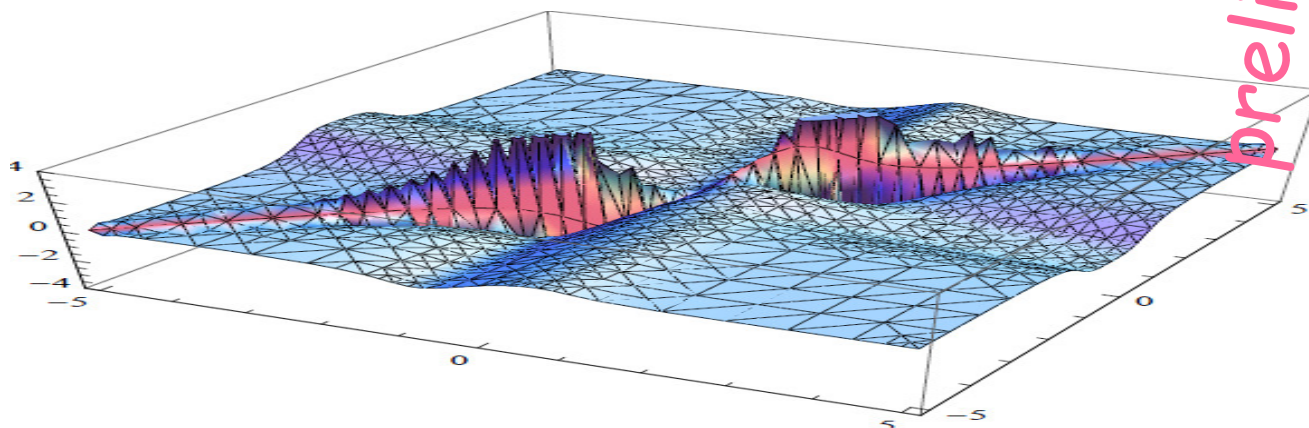
At finite T, one only has to change the string tension to the finite T string tension s , and also to replace the integral in w by a sum in Matsubara Frequencies. Both are equivalent to a reduction in the string tension, $\sigma \rightarrow \sigma^*$ and thus all we have to do is to solve the mass gap equation in units of σ^* .

The mass gap equation with finite T and finite quark mass

The mass gap equation for the running mass $m(p)$ is a non-linear integral equation with a nasty cancellation of **Infrared** divergences.

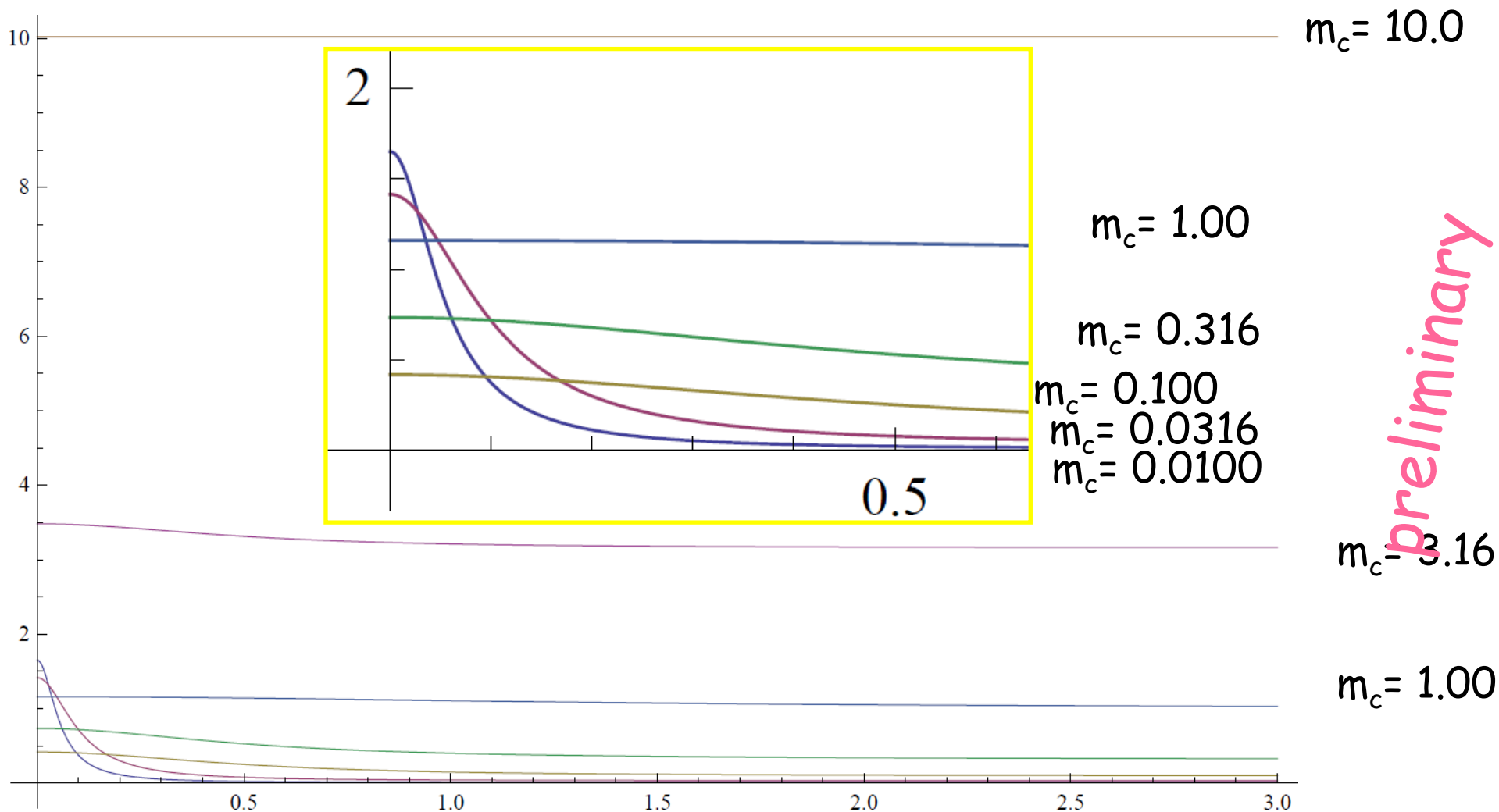
We devise a **new method** with a **rational ansatz**, and with **relaxation**, to get a maximum precision in the IR where the equation is nearly almost unstable.

$$m(p) = m_0 + \frac{\sigma}{p^3} \int_0^\infty \frac{dk}{2\pi} \frac{4p^2 k^2}{((p-k)^2 + \mu^2)((p+k)^2 + \mu^2)} \frac{m(k)p - m(p)k}{\sqrt{k^2 + m(k)^2}} + \left[\frac{2pk}{(p+k)^2 + \mu^2} + \frac{1}{2} \log \frac{(p-k)^2 + \mu^2}{(p+k)^2 + \mu^2} \right] \frac{-m(p)k}{\sqrt{k^2 + m(k)^2}}$$



The mass gap equation with finite T and finite quark mass

Solution of the mass gap equation at T=0, in units of $\sigma^2=0.19 \text{ GeV}^2=1$

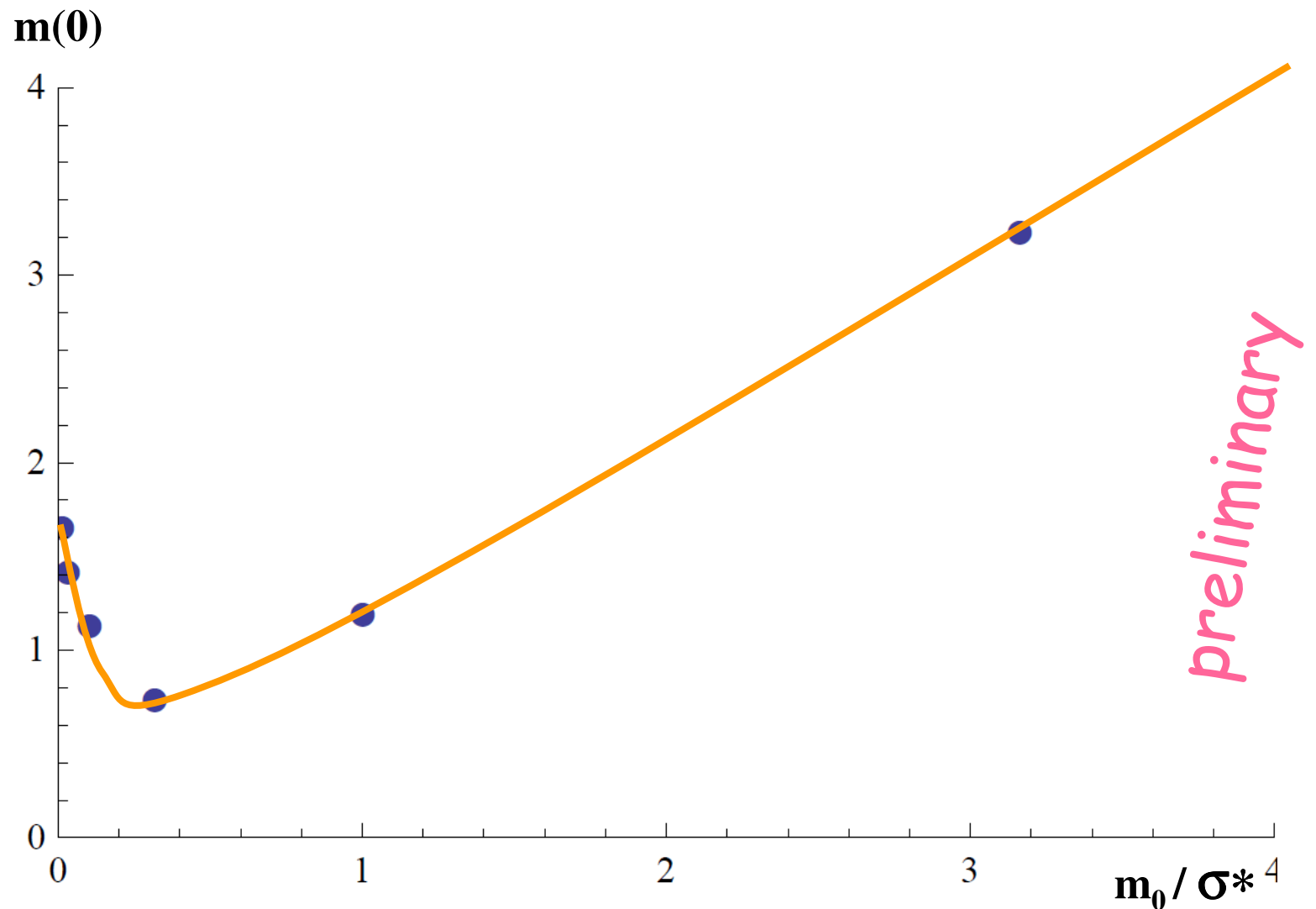


The mass gap equation with finite T and finite quark mass

the mass gap

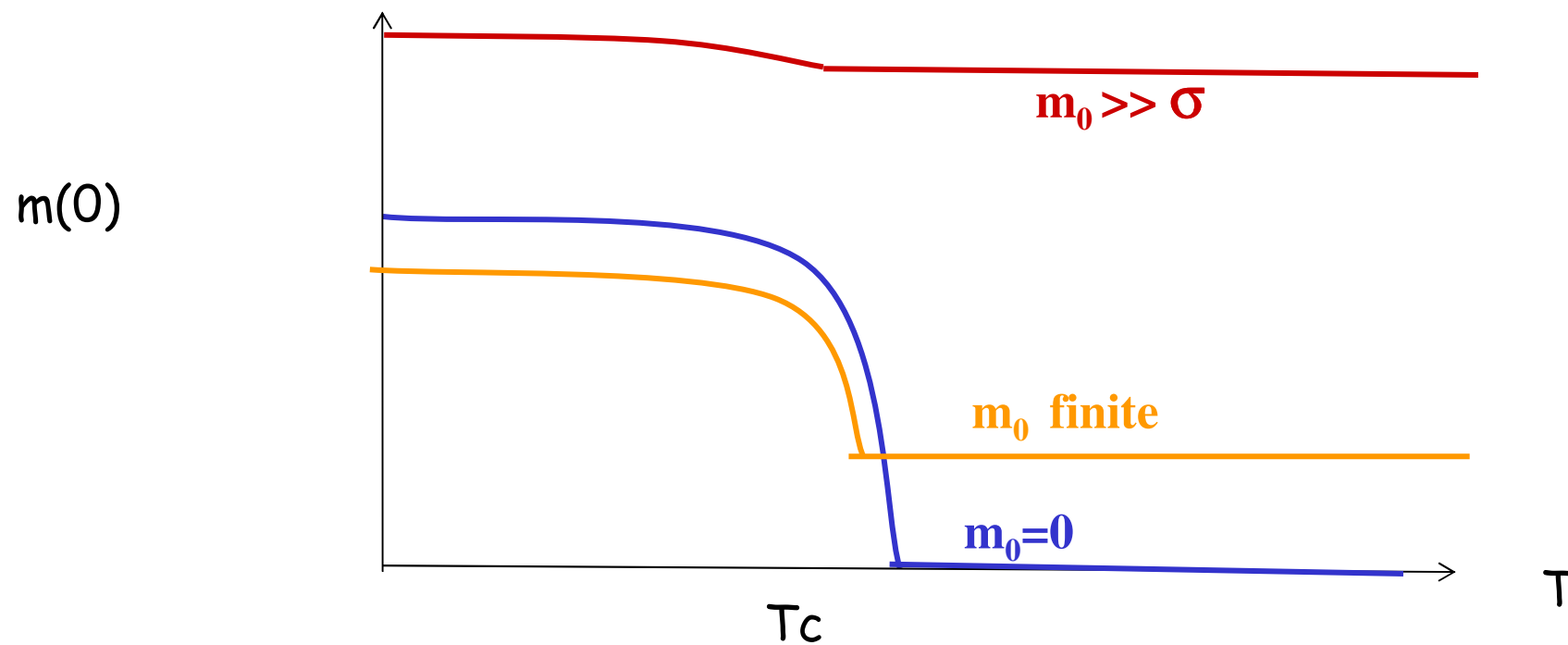
Thus we get for the mass gap $m(0)$ as a function of m_0 / σ^* this curve.

At finite T we just need to use the corresponding σ^*



The quark mass and Chiral symmetry and confinement crossovers

Thus at vanishing m_0 we have a chiral symmetry phase transition,
and at finite m_0 we have a crossover,
that gets weaker and weaker when m_0 increases:



Preliminary

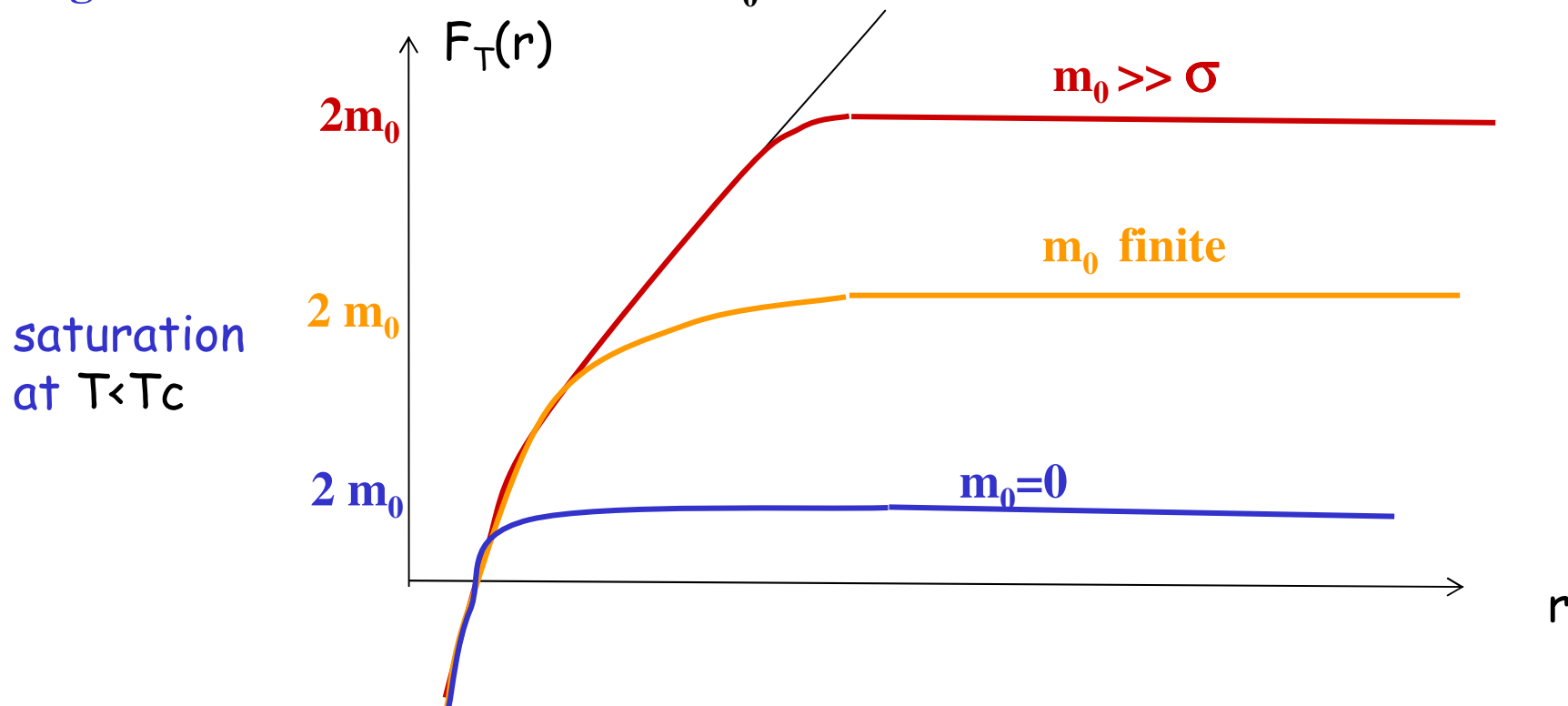
The quark mass and Chiral symmetry and confinement crossovers

In what concerns confinement, the linear confining quark-antiquark potential saturates when it reaches the energy for the creation of a quark-antiquark pair

Thus at infinite m_0 we have a confining phase transition,

and at finite m_0 we have a crossover,

that gets weaker and weaker when m_0 decreases:



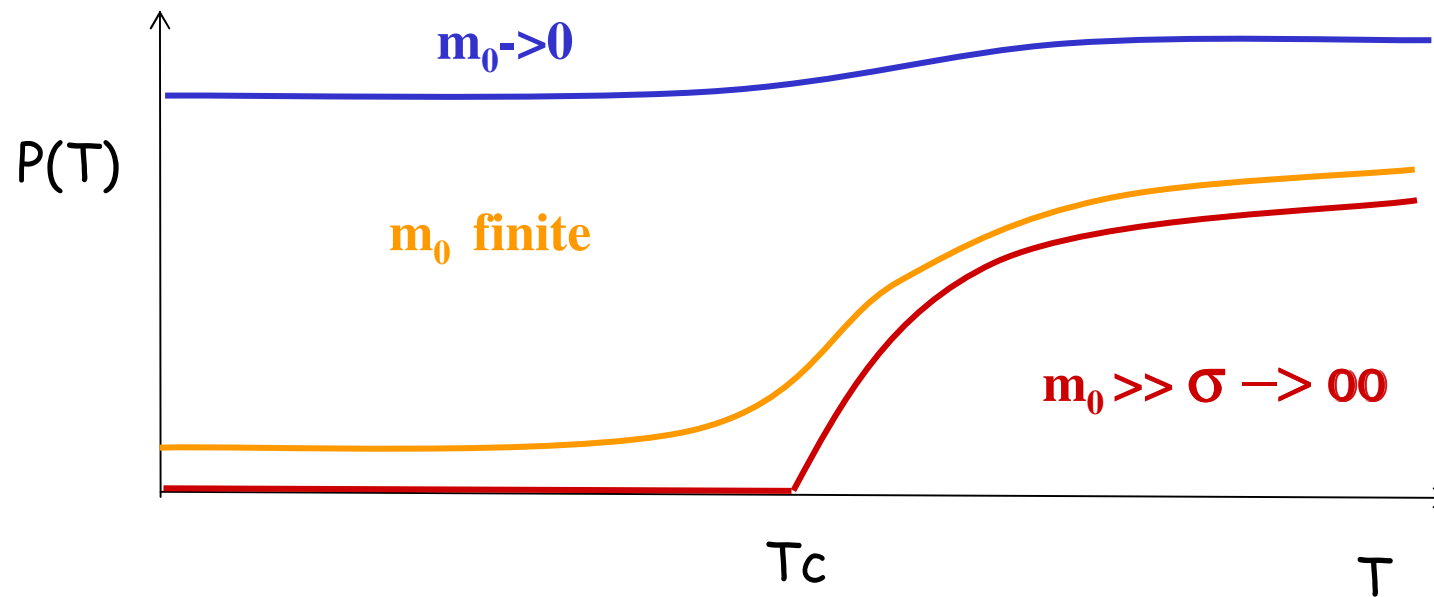
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The quark mass and Chiral symmetry and confinement crossovers

The Polyakov loop,

$$P = \mathcal{N} \text{Exp}[- F(\infty) / T]$$

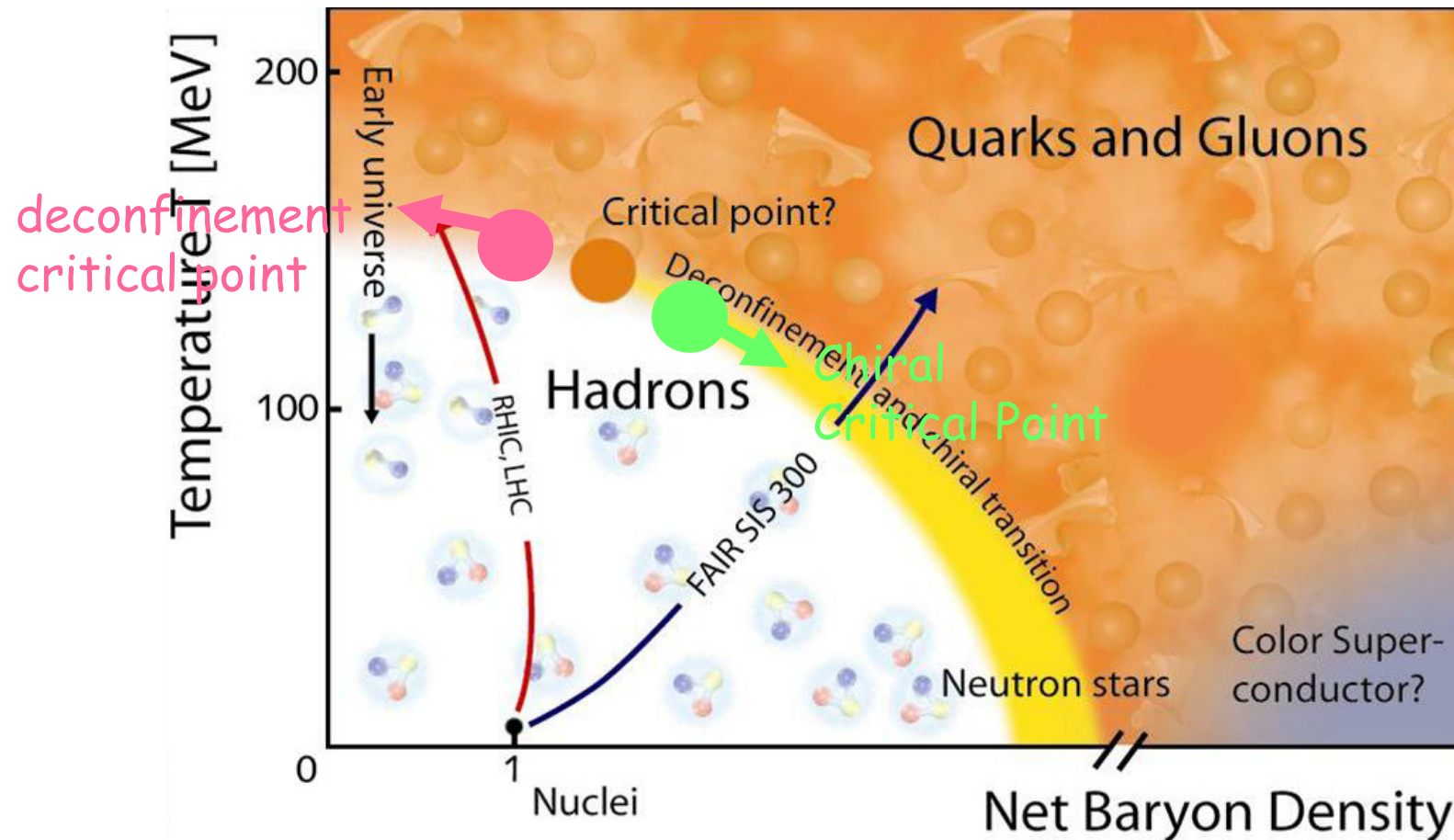
then shows a crossover for finite quark mass, with the dependence opposite of the one for the mass gap,



preliminary

The quark mass and Chiral symmetry and confinement crossovers

Back to the QCD phase diagram, is the mass m_0 is either small (depicted here), or large the critical points always separate, there will be identical only by coincidence.



preliminary

Conclusion & Outlook

- We review our recent results on excited mesons and baryons. The study of excited baryons at CBELSA and CLAS @ JLAB, or in LATTICE QCD **may lead to the first experimental evidence of the running quark mass $m(k)$, enhanced in the infrared.**
- While the quark sector alone is not sufficient to account for the large degeneracy reported by David Bugg for the CB at LEAR, CERN, including string degrees of freedom may explain this novel **principal quantum number.**
- We compute the dynamically generated quark mass $m(p)$, solving the mass gap equation both for finite current quark masses mc and for **finite T .**
- We show how the current quark mass changes both the **confinement** and the **chiral symmetry** phase transitions into **crossovers.**
- Since the finite current quark mass affects in opposite ways the confinement and the chiral symmetry, we conjecture in **finite T** and μ there will be not one but two critical points, where the crossovers are separated from the phase transitions.
- In the future we will move on to study light hadrons at **finite T** and μ .

preliminary