

Electromagnetic Probes in Heavy-Ion Collisions II

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- 1 Electromagnetic probes and vector mesons
 - Relation to chiral symmetry
- 2 Elementary vacuum cross sections: hadrons $\rightarrow \ell^+\ell^-$
 - chiral symmetry constraints
 - Electrodynamics of pions and ρ mesons (VMD model)
 - Dalitz decays of hadron resonances
- 3 Dileptons in pp and pA collisions at SIS energies
 - The Transport Model GiBUU
 - Baryon-resonance model at SIS energies
 - Dileptons in pp and pNb reactions at HADES
- 4 Conclusions and Outlook

Why Electromagnetic Probes?

- γ, l^\pm : only e. m. interactions
- whole matter evolution

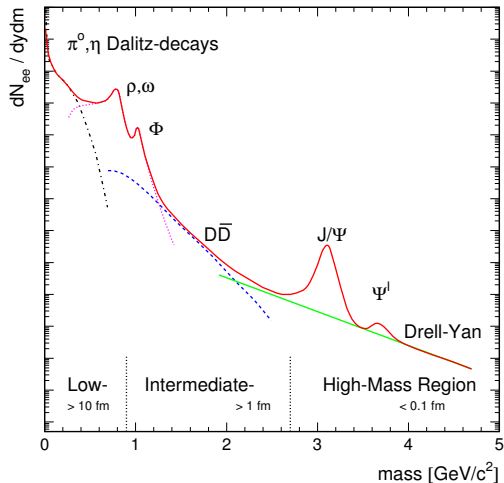
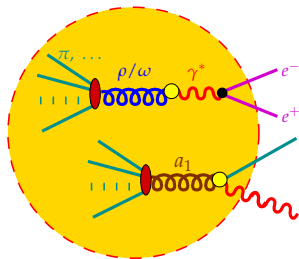


Fig. by A. Drees (from [RW00])

Vector Mesons and electromagnetic Probes

- **photon** and **dilepton** thermal emission rates given by **same** electromagnetic-current-correlation function ($J_\mu = \sum_f Q_f \bar{\psi}_f \gamma_\mu \psi_f$)

$$\Pi_{\mu\nu}^<(q) = \int d^4x \exp(iq \cdot x) \langle J_\mu(0) J_\nu(x) \rangle_T = -2n_B(q_0) \text{Im} \Pi_{\mu\nu}^{(\text{ret})}(q)$$

$$q_0 \frac{dN_\gamma}{d^4x d^3\vec{q}} = -\frac{\alpha_{\text{em}}}{2\pi^2} g^{\mu\nu} \text{Im} \Pi_{\mu\nu}^{(\text{ret})}(q) \Big|_{q_0=|\vec{q}|} f_B(p_0)$$

$$\frac{dN_{e^+e^-}}{d^4x d^4k} = -g_{\mu\nu} \frac{\alpha^2}{3q^2 \pi^3} \text{Im} \Pi_{\mu\nu}^{(\text{ret})}(q) \Big|_{q^2=M_{e^+e^-}^2} f_B(p_0)$$

- **Caveat:** NOT manifestly Lorentz covariant \Leftrightarrow **heat-bath rest frame!**
- to lowest order in α : $4\pi\alpha\Pi_{\mu\nu} \simeq \Sigma_{\mu\nu}^{(\gamma)}$
- derivable from underlying thermodynamic potential, $\Omega!$

Vector Mesons and chiral symmetry

- **vector** and **axial-vector** mesons \leftrightarrow respective current correlators

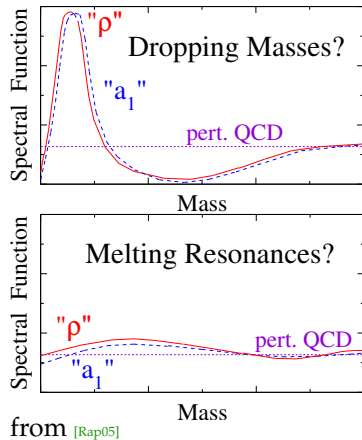
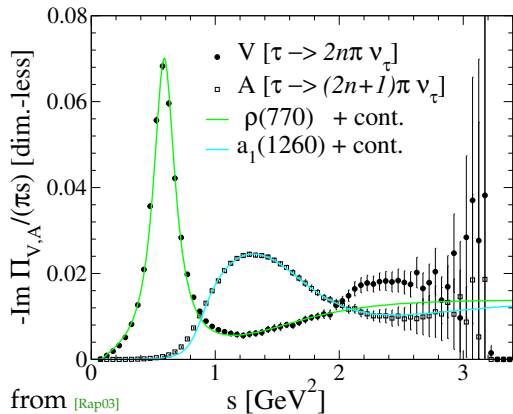
$$\Pi_{V/A}^{\mu\nu}(p) := \int d^4x \exp(ipx) \left\langle J_{V/A}^\nu(0) J_{V/A}^\mu(x) \right\rangle_{\text{ret}}$$

- Ward-Takahashi Identities of χ symmetry \Rightarrow **Weinberg-sum rules**

$$f_\pi^2 = - \int_0^\infty \frac{dp_0^2}{\pi p_0^2} [\text{Im } \Pi_V(p_0, 0) - \text{Im } \Pi_A(p_0, 0)]$$

- spectral functions of vector (e.g. ρ) and axial vector (e.g. a_1) directly related to **order parameter of chiral symmetry!**

Vector Mesons and chiral symmetry



Chiral-symmetry constraints

- different realizations of **chiral symmetry**
- equivalent only on shell (“**low-energy theorems**”)
- model-independent conclusions only in **low-temperature/density limit** (chiral perturbation theory) or from **lattice-QCD calculations**
- QCD sum rules (see Lect. I):
allow for dropping-mass or melting-resonance scenario
- use **phenomenological hadronic many-body theory** (HMBT) to assess medium modifications of vector mesons
 - build models with **hadrons** as effective degrees of freedom
 - based on **(chiral) symmetries**
 - constrained by data on cross sections, branching ratios,... in vacuum
 - in-medium properties assessed by **many-body (thermal) field theory**

Example: vector-meson dominance model

- early model for **electromagnetic interaction** of charged pions

[Sak60, KLZ67, GS68, Her92, Hee00]

- QED like U(1)-gauge model with massive vector meson for ρ_0 and π^\pm
- Stückelberg: introduce auxiliary scalar field for free vector mesons:

$$\mathcal{L}_\rho = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{1}{2}m^2V_\mu V^\mu + \frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) + m\varphi\partial_\mu V^\mu$$

- gauge invariant under local transformation

$$\delta V_\mu(x) = \partial_\mu\chi(x), \quad \delta\varphi = m\chi(x)$$

- usual way of gauge fixing using gauge condition

$$\partial_\mu V^\mu = -\xi m\varphi$$

- effective Lagrangian of free ρ meson, Stückelberg and FP ghosts

$$\begin{aligned}\mathcal{L}_{\rho,\text{gf}} = & -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{m}{2}V_\mu V^\mu - \frac{1}{2\xi}(\partial_\mu V^\mu)^2 + \frac{1}{2}(\partial_\mu\varphi)(\partial_\mu\varphi) - \frac{\xi m^2}{2}\varphi^2 \\ & + (\partial_\mu\eta^*)(\partial_\mu\eta) - \xi m^2\eta^*\eta\end{aligned}$$

Example: vector-meson dominance model

- so far: free ρ meson and free ghosts
- ghosts only relevant for **ideal gas thermodynamics**
 - V^μ : four bosonic field degrees
(3 transverse with mass m , 1 longitudinal with mass $\sqrt{\xi}m$)
 - φ : 1 bosonic Stückelberg ghost with mass $\sqrt{\xi}m$
 - η^*, η : 2 pseudofermionic Faddeev Popov fields with mass $\sqrt{\xi}m$
 - **in partition sum**: 3 bosons with mass m + 2 bosons with mass $\sqrt{\xi}m$ – 2 FP ghosts with mass $\sqrt{\xi}m \Rightarrow$ effectively three bosons with mass m
 - partition sum independent of gauge parameter, ξ !
 - $\xi \rightarrow \infty$: “unitary gauge” \rightarrow only three bosonic ρ -degrees of freedom!
- Coupling to pions: **obey gauge invariance!** (like scalar QED)

$$\mathcal{L}_\pi = (D_\mu \pi)^* (D^\mu \pi) - m_\pi^2 |\pi|^2 - \frac{\lambda}{8} |\pi|^4$$

- $D_\mu = \partial_\mu + igV_\mu$; g : $\rho\pi\pi$ coupling

Example: vector-meson dominance model

- add photons: $D_\mu = \partial_\mu + igV_\mu + ieA_\mu$
- Lagrangian for photons: usual gauge fixed QED
- additional direct $\rho\gamma$ mixing [KLZ67]

$$\mathcal{L}_{\rho\gamma} = -\frac{e}{2g_{\rho\gamma}} V_{\mu\nu} A^{\mu\nu}$$

- classical field equations: \Rightarrow **electromagnetic current**

$$j_{\text{em}}^\nu = \partial_\mu A^{\mu\nu} = ie \left(1 - \frac{g}{g_{\rho\gamma}} \right) \pi \overleftrightarrow{D}^\nu \pi^* + \frac{e}{g_{\rho\gamma}} m^2 V^\mu + \frac{e^2}{g_{\rho\gamma}^2} \partial_\mu A^{\mu\nu}$$

- for $g_{\rho\gamma} = g$: $j_{\text{em}}^\nu = \frac{e}{g} m^2 V^\nu + \mathcal{O}(e^2)$: \Rightarrow **“vector-meson dominance”**

Example: vector-meson dominance model

- calculate ρ selfenergy

$$i\Sigma_{\rho}^{\mu\nu}(p) = \mu \text{ [diagram 1]} + \mu \text{ [diagram 2]}$$

Diagram 1: A loop diagram for the ρ meson self-energy. It consists of two wavy lines representing ρ mesons. The left wavy line has momentum p and the right wavy line has momentum p . A dashed line with an arrow represents a lepton l circulating in a loop. The incoming lepton line has momentum l and the outgoing lepton line has momentum $l+p$.

Diagram 2: A tadpole diagram for the ρ meson self-energy. It consists of a single wavy line representing a ρ meson with momentum ν . A dashed line with an arrow represents a lepton l forming a loop that connects to the wavy line at a single vertex.

- transversality from gauge invariance:

$$\Sigma_{\rho}^{\mu\nu}(q) = \left(q^2 g^{\mu\nu} - q^{\mu} q^{\nu} \right) \tilde{\Sigma}(q^2)$$

- electromagnetic form factor of pions

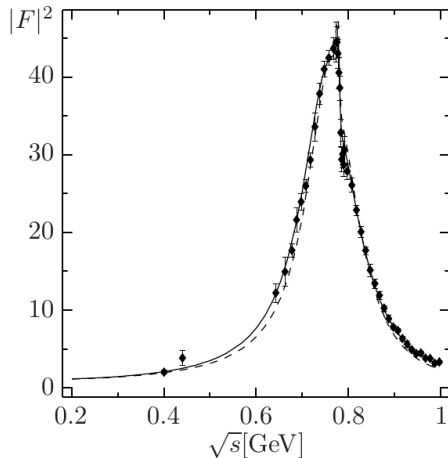
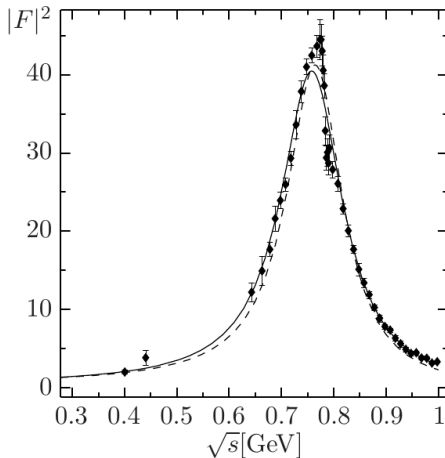
$$F(k^2) = \frac{\text{[diagram 1]}}{\text{[diagram 2]}}$$

Diagram 1 (Numerator): A diagram for the electromagnetic form factor of a pion. On the left, a red line represents an incoming π^- and another red line represents an outgoing π^+ . On the right, two black lines with arrows represent outgoing electrons (e). A green wavy line (representing a ρ meson) connects the pion vertex to the electron vertex.

Diagram 2 (Denominator): A diagram for the pion propagator. On the left, a red line represents an incoming π^- and another red line represents an outgoing π^+ . On the right, two black lines with arrows represent outgoing electrons (e). A black wavy line (representing a photon) connects the pion vertex to the electron vertex.

Example: vector-meson dominance model

- fit to observables: **em. form factor of π**

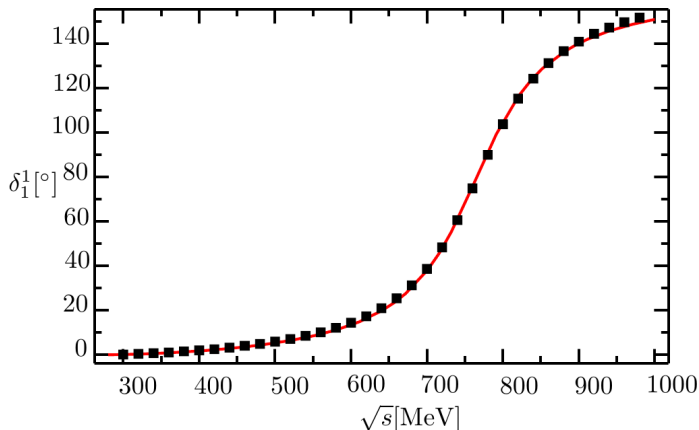


- best fit: $g = 5.683$, $g_{\rho\gamma} = 5.171$, $m_\rho = 765 \text{ MeV}/c^2$
strict VMD: $g = g_{\rho\gamma} = 5.38$, $m_\rho = 770 \text{ MeV}/c^2$
data: [B⁺85]

Example: vector-meson dominance model

- $\pi\pi \rightarrow \pi\pi$ phase shift in $I = 1$ channel

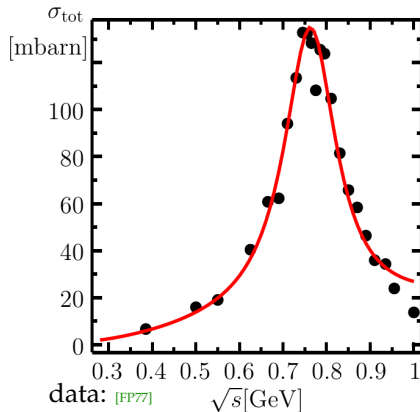
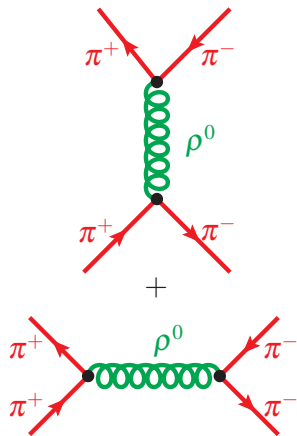
$$\delta_1^1 = \arccos \frac{\text{Re } G_\rho}{|G_\rho|}$$



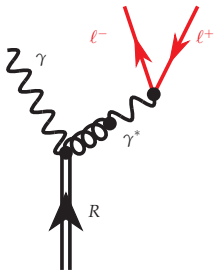
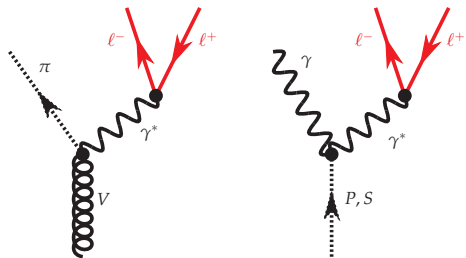
data: [FP77]

Example: vector-meson dominance model

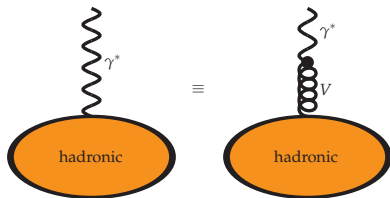
- $\pi\pi \rightarrow \pi\pi$ total cross section

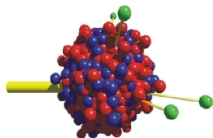


Dalitz decays



- **Dalitz decay:**
1 particle \rightarrow 3 particles
- $V: \omega \rightarrow \pi + \gamma^* \rightarrow \pi + e^+ + e^-$
- $P, S:$
 $\pi, \eta \rightarrow \gamma + \gamma^* \rightarrow \gamma + e^+ + e^-$
- R : Baryon resonances
 $\Delta, N^* \rightarrow N + V \rightarrow N + \gamma^* \rightarrow N + e^+ + e^-$
- vector-meson dominance





GiBUU

The Giessen Boltzmann-Uehling-Uhlenbeck Project

- Boltzmann-Uehling-Uhlenbeck (BUU) framework for hadronic transport
- reaction types: pA , πA , γA , eA , νA , AA
- open-source modular Fortran 95/2003 code
- version control via Subversion
- publicly available releases: <http://gibuu.physik.uni-giessen.de>
- Review on hadronic transport (GiBUU): [BCG⁺12]

The Boltzmann-Uehling-Uhlenbeck Equation

- time evolution of **phase-space distribution functions**

$$[\partial_t + (\vec{\nabla}_p H_i) \cdot \vec{\nabla}_x - (\vec{\nabla}_x H_i) \cdot \vec{\nabla}_p] f_i(t, \vec{x}, \vec{p}) = I_{\text{coll}}[f_1, \dots, f_i, \dots, f_j]$$

- Hamiltonian H_i
 - selfconsistent hadronic mean fields, Coulomb potential, “off-shell potential”
- collision term I_{coll}
 - two- and three-body decays/collisions
 - multiple coupled-channel problem
 - resonances described with relativistic Breit-Wigner distribution

$$\mathcal{A}(x, p) = -\frac{1}{\pi} \frac{\text{Im } \Pi}{(p^2 - M^2 - \text{Re } \Pi)^2 + (\text{Im } \Pi)^2}; \quad \text{Im } \Pi = -\sqrt{p^2} \Gamma$$

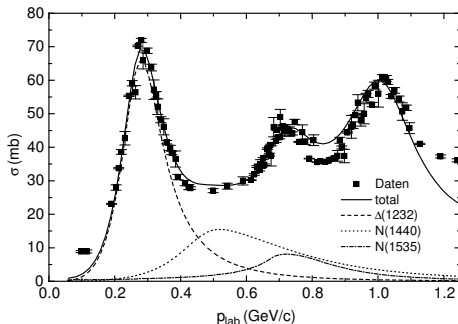
- off-shell propagation: test particles with **off-shell potential**

Resonance Model

- reactions dominated by resonance scattering: $ab \rightarrow R \rightarrow cd$
- Breit-Wigner cross-section formula

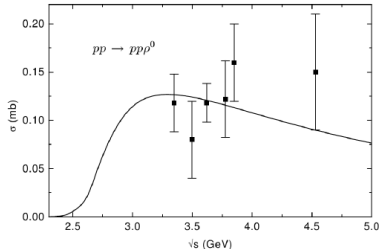
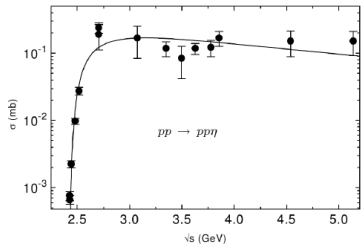
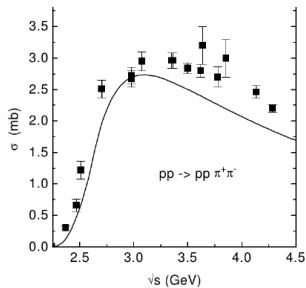
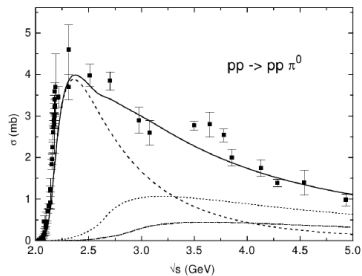
$$\sigma_{ab \rightarrow R \rightarrow cd} = \frac{2s_R + 1}{(2s_a + 1)(2s_b + 1)} \frac{4\pi}{p_{\text{lab}}^2} \frac{s\Gamma_{ab \rightarrow R}\Gamma_{R \rightarrow cd}}{(s - m_R^2)^2 + s\Gamma_{\text{tot}}^2}$$

- applicable for low-energy nuclear reactions $E_{\text{kin}} \lesssim 1.1 \text{ GeV}$
- example: $\sigma_{\pi^- p \rightarrow \pi^- p}$ [Teis (PhD thesis 1996), data: Baldini et al, Landolt-Börnstein 12 (1987)]



Resonance Model

- further cross sections



Extension to HADES energies

• [WHM12]

• keep same resonances (parameters from Manley analysis)

| | rating | M_0 [MeV] | Γ_0 [MeV] | $ \mathcal{M}^2 /16\pi$ [mb GeV ²] | | branching ratio in % | | | | | | |
|------------------------|--------|----------------|---------------------|--|------------|----------------------|----------|----------------------------------|----------------------------------|------------|-----------------|-----------------|
| | | | | NR | ΔR | πN | ηN | $\pi \Delta$ | ρN | σN | $\pi N^*(1440)$ | $\sigma \Delta$ |
| P ₁₁ (1440) | **** | 1462 | 391 | 70 | — | 69 | — | 22 _P | — | 9 | — | — |
| S ₁₁ (1535) | *** | 1534 | 151 | 8 | 60 | 51 | 43 | — | 2 _S + 1 _D | 1 | 2 | — |
| S ₁₁ (1650) | **** | 1659 | 173 | 4 | 12 | 89 | 3 | 2 _D | 3 _D | 2 | 1 | — |
| D ₁₃ (1520) | **** | 1524 | 124 | 4 | 12 | 59 | — | 5 _S + 15 _D | 21 _S | — | — | — |
| D ₁₅ (1675) | **** | 1676 | 159 | 17 | — | 47 | — | 53 _D | — | — | — | — |
| P ₁₃ (1720) | * | 1717 | 383 | 4 | 12 | 13 | — | — | 87 _P | — | — | — |
| F ₁₅ (1680) | **** | 1684 | 139 | 4 | 12 | 70 | — | 10 _P + 1 _F | 5 _P + 2 _F | 12 | — | — |
| P ₃₃ (1232) | **** | 1232 | 118 | OBE | 210 | 100 | — | — | — | — | — | — |
| S ₃₁ (1620) | ** | 1672 | 154 | 7 | 21 | 9 | — | 62 _D | 25 _S + 4 _D | — | — | — |
| D ₃₃ (1700) | * | 1762 | 599 | 7 | 21 | 14 | — | 74 _S + 4 _D | 8 _S | — | — | — |
| P ₃₁ (1910) | **** | 1882 | 239 | 14 | — | 23 | — | — | — | — | 67 | 10 _P |
| P ₃₃ (1600) | *** | 1706 | 430 | 14 | — | 12 | — | 68 _P | — | — | 20 | — |
| F ₃₅ (1905) | ** | 1881 | 327 | 7 | 21 | 12 | — | 1 _P | 87 _P | — | — | — |
| F ₃₇ (1950) | **** | 1945 | 300 | 14 | — | 38 | — | 18 _F | — | — | — | 44 _F |

• production channels in Teis: $NN \rightarrow N\Delta, NN \rightarrow NN^*, N\Delta^*, NN \rightarrow \Delta\Delta$

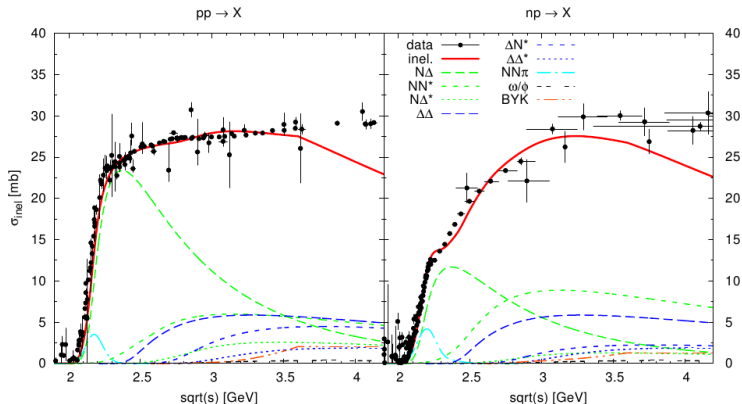
• extension to $NN \rightarrow \Delta N^*, \Delta\Delta^*, NN \rightarrow NN\pi,$

$NN \rightarrow NN\rho, NN\omega, NN\pi\omega, NN\phi,$

$NN \rightarrow BYK$ ($B = N, \Delta, Y = \Lambda, \Sigma$)

Extension to HADES energies

- good description of total pp, pn (inelastic) cross section

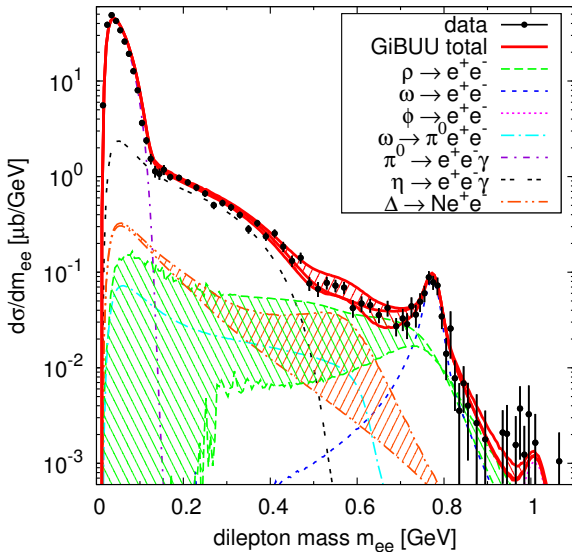


- dilepton sources

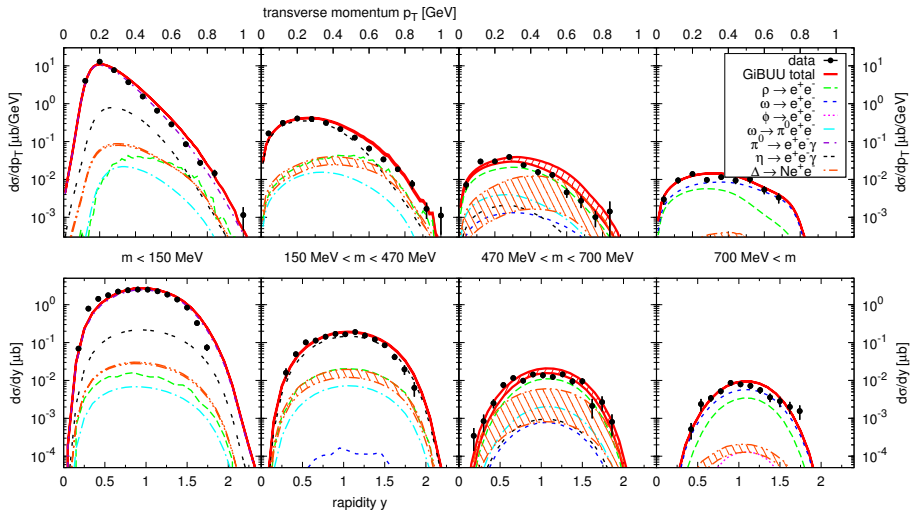
- Dalitz decays: $\pi^0, \eta \rightarrow \gamma l^+ l^-$; $\omega \rightarrow \pi^0 l^+ l^-$, $\Delta \rightarrow N l^+ l^-$
- $\rho, \omega, \phi \rightarrow l^+ l^-$: invariant mass $l^+ l^-$ spectra \Rightarrow spectral properties of vector mesons
- for details, see [WHM12]

p p at HADES ($E_{\text{kin}} = 3.5 \text{ GeV}$)

p + p at 3.5 GeV

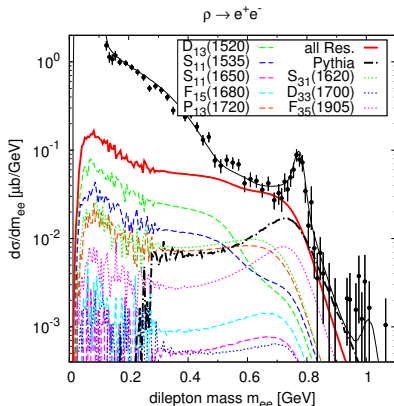
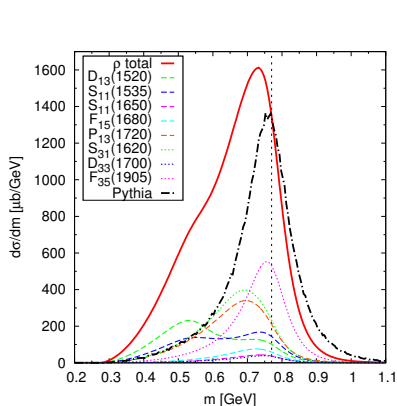


p p at HADES ($E_{\text{kin}} = 3.5 \text{ GeV}$)



“ ρ meson” in pp

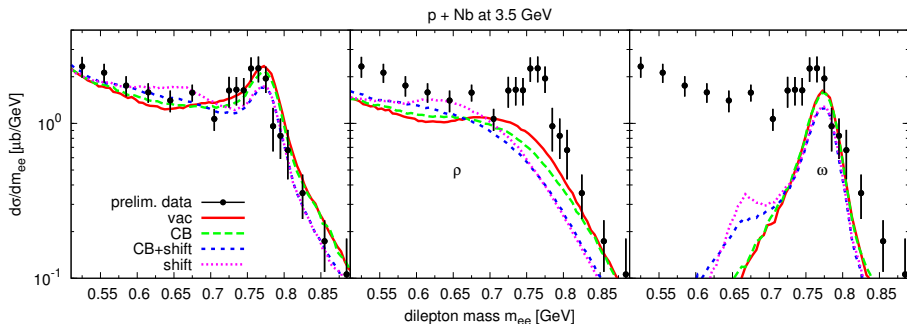
- production through hadron resonances
 $NN \rightarrow NR \rightarrow NN\rho, NN \rightarrow N\Delta \rightarrow NN\pi\rho$



- “ ρ ”-line shape “modified” already in elementary hadronic reactions
- due to production mechanism via resonances

p Nb at HADES (3.5 GeV)

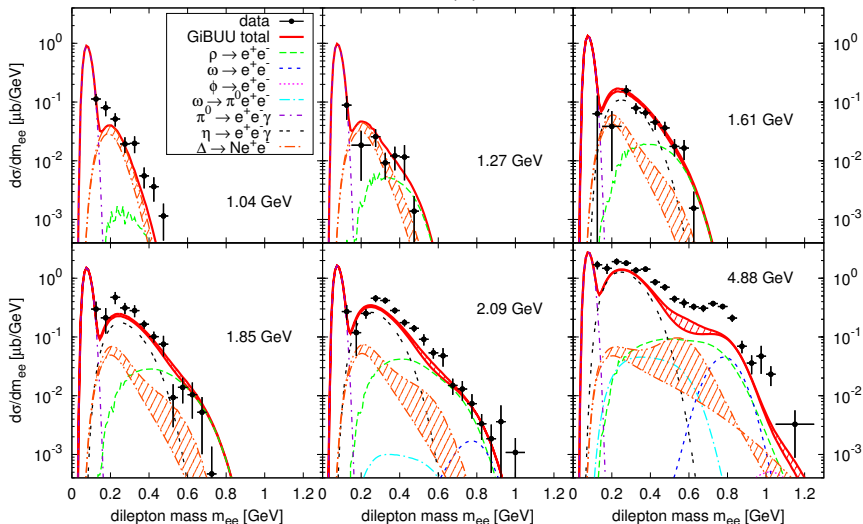
- medium effects built in transport model
 - binding effects, Fermi smearing, Pauli blocking
 - final-state interactions
 - production from secondary collisions
- sensitivity on medium effects of vector-meson spectral functions?



Comparison to old DLS data (pp)

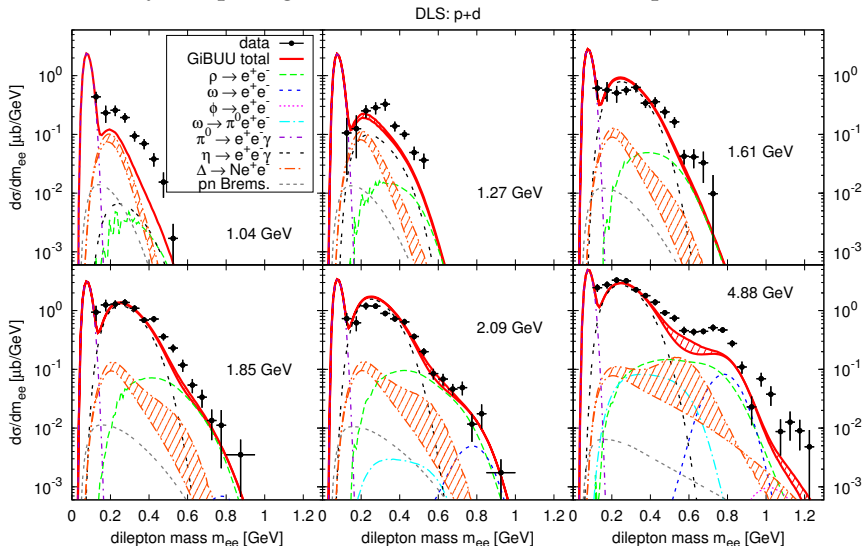
- HADES data consistent with DLS data
- checked by comparing HADES data within DLS acceptance

DLS: p+p



Comparison to old DLS data (pd)

- HADES data consistent with DLS data
- checked by comparing HADES data within DLS acceptance



Conclusions and Outlook

- dilepton spectra \Leftrightarrow in-medium em. current correlator
- effective hadronic models for dilepton sources
 - vector-meson dominance model (VMD)
 - low-mass region $0 \leq M \lesssim 1 \text{ GeV}$: $j_{\text{em}}^{(\text{had})\mu} \propto V^\mu$ ($V \in \{\rho, \omega, \phi\}$)
 - direct relation between dilepton signal and VM spectral functions
 - interactions with mesons and baryons
 - models constrained by phenomenology in pp, pn, pA
 - medium modifications predicted by finite-temperature QFT
- Elementary reactions at SIS energies
 - GiBUU for pp, pn with resonance model for all HADES energies
 - pn still a problem?
 - p Nb, AA work in progress

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