Hadrons in hot and dense matter III

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Outline

Transport theory and hydrodynamics

Phase-space distribution

- classical many-body system of relativistic particles
- all particles are on their mass shell: $E = E_p := \sqrt{\vec{p}^2 + m^2}$
- Boltzmann equation [dvv80, CK02, Hee15]:
  dynamical equation for phase-space distribution function $f(t, \vec{x}, \vec{p})$
- relativistic covariance of phase-space distribution
  - $f(t, \vec{x}, \vec{p})$ defined as Lorentz scalar quantity
  - particle number N: $\mathrm{d}N = \mathrm{d}^3\vec{x}\, \mathrm{d}^3\vec{p}\, f(t, \vec{x}, \vec{p})$
  - particle-number four-vector current $(N^\mu) = (n, \vec{N})$

$$N^\mu = \int_{\mathbb{R}^3} \mathrm{d}^3\vec{p}\, \frac{p^\mu}{E_p} f(t, \vec{x}, \vec{p})$$

  - flow-velocity of fluid cell ("Eckart frame")

$$\vec{v}_{\text{Eck}}(x) = \frac{\vec{N}(x)}{N^0(x)}, \quad u^\mu_{\text{Eck}} = \frac{N^\mu}{\sqrt{N_\mu N^\mu}} = \frac{N^\mu}{n_0}$$

  - n_0: particle density in local fluid (Eckart) restframe

Relativistic Boltzmann equation

- particles moving along trajectories $(\vec{x}(t), \vec{p}(t))$
- for infinitesimal time step $\mathrm{d}t$

$$\mathrm{d}N(t+\mathrm{d}t) = f(t+\mathrm{d}t, \vec{x}+\mathrm{d}t\,\vec{v}, \vec{p}+\mathrm{d}t\,\vec{F})\mathrm{d}^6\xi(t+\mathrm{d}t), \quad \mathrm{d}^6\xi = \mathrm{d}^3\vec{x}\,\mathrm{d}^3\vec{p}$$

- Jacobian for phase-space volume

$$\mathrm{d}^6\xi(t+\mathrm{d}t) = \mathrm{d}^6\xi(t)\det\left(\frac{\partial(\vec{x}+\mathrm{d}t\,\vec{v}, \vec{p}+\mathrm{d}t\,\vec{F})}{\partial(\vec{x},\vec{p})}\right) = \mathrm{d}^6\xi(t)\left(1+\mathrm{d}t\,\vec{\nabla}_p \cdot \vec{F}\right) + \mathcal{O}(\mathrm{d}t^2)$$

- total change of $\mathrm{d}N$

$$\mathrm{d}N(t+\mathrm{d}t) - \mathrm{d}N(t) = \mathrm{d}^6\xi(t)\mathrm{d}t\left[\frac{\partial f}{\partial t} + \frac{\vec{p}}{E}\cdot\frac{\partial \vec{f}}{\partial \vec{x}} + \frac{\partial(\vec{F}f)}{\partial \vec{p}}\right]$$

Relativistic Boltzmann equation

- covariance: $\mathrm{d}\tau = \mathrm{d}t \sqrt{1-\vec{v}^2}$ proper time, $\vec{v} = \vec{p}/E_p$, $\sqrt{1-\vec{v}^2} = m/E_p$

$$\mathrm{d}t \left[\frac{\partial f}{\partial t} + \frac{\vec{p}}{E} \cdot \frac{\partial \vec{f}}{\partial \vec{x}} \right] = \mathrm{d}\tau \frac{p^\mu}{m} \frac{\partial f}{\partial x^\mu} \Rightarrow \text{covariant!}$$

- covariant equation of motion for point particle

$$\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = K^\mu, \quad p_\mu p^\mu = m^2 = \text{const} \Rightarrow$$

$$K^0 = \frac{\vec{p}}{E_p} \cdot \vec{K} \Rightarrow \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \vec{F} = \vec{K} \frac{m}{E_p}$$

$$\frac{E_p}{m} \vec{\nabla}_p (\vec{F} f) = \frac{\partial}{\partial p^\mu} (K^\mu f) \Rightarrow \text{covariant!}$$

$$\mathrm{d}N(t+\mathrm{d}t) - \mathrm{d}N(t) = \mathrm{d}t \left[\frac{\partial f}{\partial t} + \frac{\vec{p}}{E} \cdot \frac{\partial \vec{f}}{\partial \vec{x}} + \frac{\partial (\vec{F} f)}{\partial \vec{p}} \right] = \mathrm{d}\tau \left[\frac{p^\mu}{m} \frac{\partial f}{\partial x^\mu} + \frac{\partial (K^\mu f)}{\partial p^\mu} \right]$$

Relativistic Boltzmann equation

- change of particle number due to collisions
  - short-range interactions: collisions at one point (local) in space
  - invariant cross section

$$\mathrm{d}N_{\text{coll}}(p_1', p_2' \leftarrow p_1, p_2) = \mathrm{d}^4 x \frac{\mathrm{d}^3 \vec{p}_1}{E_1} \frac{\mathrm{d}^3 \vec{p}_2}{E_2} \frac{\mathrm{d}^3 \vec{p}\,'_1}{E_1'} \frac{\mathrm{d}^3 \vec{p}\,'_2}{E_2'} f_1 f_2 W(p_1', p_2' \leftarrow p_1, p_2),$$

$$\mathrm{d}\sigma = \frac{W(p_1', p_2' \leftarrow p_1, p_2) \mathrm{d}^4 x \frac{\mathrm{d}^3 \vec{p}_1}{E_1} \frac{\mathrm{d}^3 \vec{p}_2}{m} \frac{\mathrm{d}^3 \vec{p}\,'_1}{E_1'} \frac{\mathrm{d}^3 \vec{p}\,'_2}{E_2'} f_1 f_2}{\mathrm{d}^4 x \mathrm{d}^3 \vec{p}_1 v_{\text{rel}} f_1 \mathrm{d}^3 \vec{p}_2 f_2},$$

$$\mathrm{d}\sigma = \frac{\mathrm{d}^3 \vec{p}\,'_1}{E_1'} \frac{\mathrm{d}^3 \vec{p}\,'_2}{E_2'} \frac{W(p_1', p_2' \leftarrow p_1, p_2)}{I}, \quad I = \sqrt{(p_1 \cdot p_2)^2 - m^4}$$

  - important: v_{rel} is velocity of particle 1 in rest frame of particle 2
  - from relativistic covariance (or unitarity of S-matrix!) $\Rightarrow$ detailed-balance relation

$$W(p_1', p_2' \leftarrow p_1, p_2) = W(p_1, p_2 \leftarrow p_1', p_2')$$

- Boltzmann equation (manifestly covariant form)

$$p^\mu \frac{\partial f}{\partial x^\mu} + m \frac{\partial (K^\mu f)}{\partial p^\mu} = \frac{1}{2} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 \vec{p}_2}{E_2} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 \vec{p}\,'_1}{E_1'} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 \vec{p}\,'_2}{E_2'} W(p_1', p_2' \leftarrow p, p_2)(f_1' f_2' - f f_2$$

- collision integral: "gain minus loss"

Entropy

- input from quantum mechancis: particle in a cubic box (periodic boundary cond.)

$$\vec{p} = \frac{2\pi}{L} \vec{n}, \quad \vec{n} \in \mathbb{Z}^3$$

- $\Delta^6 \xi_j = L^3 \Delta^3 \vec{p}$ ("microscopically large, macroscopically small")
- contains G_j single-particle states (g: degeneracy due to spin, isospin, ...)

$$G_j = g \frac{\Delta^6 \xi_j}{(2\pi)^3}$$

- statistical weight for N_j particles in $\Delta^6 \xi_j$:
- factor $1/N_j!$: indistinguishability of particles

$$\Delta\Gamma_j = \frac{1}{N_j!} G_j^{N_j}$$

- entropy a la Boltzmann

$$S = \sum_j \ln \Delta\Gamma_j \simeq \sum_j [N_j \ln G_j - N_j(\ln N_j - 1)]$$
$$= -\int \mathrm{d}^3\vec{x}\, \mathrm{d}^3\vec{p}\, f(x,p)\{\ln[(2\pi)^3 f(x,p)/g] - 1\}$$

The Boltzmann H theorem

- H = greek Eta: Boltzmann's notation for entropy
- covariant description of entropy: entropy four-flow

$$S^{\mu}(x) = -\int_{\mathbb{R}^3} \frac{\mathrm{d}^3 \vec{p}}{E} p^{\mu} f(x,p) \{\ln[(2\pi)^3 f(x,p)/g] - 1\}$$

- Boltzmann equation + symmetries of $W(p_1' p_2' \leftarrow p_1 p_2)$

$$\frac{\partial S^{\mu}}{\partial x^{\mu}} := \zeta = +\frac{1}{4} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 \vec{p}}{E} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 \vec{p}_2}{E_2} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 \vec{p}\,'_1}{E_1'} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 \vec{p}\,'_2}{E_2'} f f_2 \times \left[\frac{f_1' f_2'}{f f_2} - \ln\left(\frac{f_1' f_2'}{f f_2} \right) - 1 \right] W(p_1' p_2' \leftarrow p, p_1) \geq 0$$

- (on average) entropy can never decrease with time!
- equilibrium $\Leftrightarrow$ S maximal!
- bracket must vanish $\Rightarrow$ Maxwell-Boltzmann distribution

$$f_{\text{eq}}(x,p) = \frac{g}{(2\pi)^3} \exp\left[-\beta(x) \Big(u(x) \cdot p - \mu(x) \Big) \right], \quad p^0 = E = \sqrt{m^2 + \vec{p}^2}$$

- $\beta = 1/T$: inverse temperature, u: fluid four-velocity, μ: chemical potential
- temperature, chemical potential are Lorentz scalars!

Hydrodynamics

- in the limit of very small mean-free path: system in local thermal equilibrium
- switch to ideal hydrodynamics description
- forget about “particles” ⇒ fluid description
- equations of motion for $\vec{v}(t,\vec{x})$: conservation laws

$$\partial_\mu N^\mu = 0, \quad \partial_\mu T^{\mu\nu} = 0$$

- N^μ: net-baryon number, $T^{\mu\nu}$: energy-momentum tensor
- ideal hydrodynamics

$$N^\mu = n u^\mu, \quad T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu - P \eta^{\mu\nu}$$
$$\partial_\mu N^\mu = 0, \quad \partial_\mu T^{\mu\nu} = 0$$

- n: proper net-baryon density, ϵ: proper energy density, P: pressure
- 5 equations of motion, 6 unknowns: $\vec{v}$, n, ϵ, P
- need also equation of state $\epsilon = \epsilon(P)$
- hadron-resonance gas EoS (low energies)
  lQCD based cross-over phase transition (high energies)

Transport simulations (UrQMD and GiBUU)

The GiBUU Model

GiBUU
The Giessen Boltzmann-Uehling-Uhlenbeck Project

- Boltzmann-Uehling-Uhlenbeck (BUU) framework for hadronic transport
- reaction types: pA, πA, γA, eA, νA, AA
- open-source modular Fortran 95/2003 code
- version control via Subversion
- publicly available realeases: https://gibuu.hepforge.org
- Review on hadronic transport (GiBUU): [BGG+12]
- all calculations for dileptons: J. Weil

The Boltzmann-Uehling-Uhlenbeck Equation

- time evolution of phase-space distribution functions

$$[\partial_t + (\vec{\nabla}_p H_i) \cdot \vec{\nabla}_x - (\vec{\nabla}_x H_i) \cdot \vec{\nabla}_p] f_i(t, \vec{x}, \vec{p}) = I_{\text{coll}}[f_1, \ldots, f_i, \ldots, f_j]$$

- use Monte-Carlo simulation for test particles
- transition probability W in collision term used to define stochastic process ("random numbers" on the computer)
- Hamiltonian H_i
  - selfconsistent hadronic mean fields, Coulomb potential, "off-shell potential"
- collision term I_{coll}
  - two- and three-body decays/collisions
  - multiple coupled-channel problem
  - resonances described with relativistic Breit-Wigner distribution

$$\mathscr{A}(x,p) = -\frac{1}{\pi} \frac{\operatorname{Im}\Pi}{(p^2 - M^2 - \operatorname{Re}\Pi)^2 + (\operatorname{Im}\Pi)^2}; \quad \operatorname{Im}\Pi = -\sqrt{p^2}\Gamma$$

  - off-shell propagation: test particles with off-shell potential

Ultra-relativistic Molecular Dynamics (UrQMD)

- transport model for hadrons
  - all hadrons (resonances) with masses up to 2.2 GeV included
  - cross sections adapted to experimental data
  - no explicit medium modifications of hadrons implemented
- quantum molecular dynamics
  - hadrons represented by quantum-mechanical Gaussian wave packets

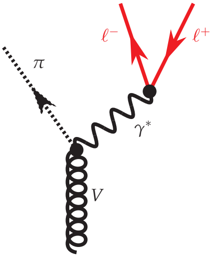
$$\psi_i(t,\vec{x}) = \left(\frac{2}{\pi L}\right)^{1/4} \exp\left\{-\frac{2}{L}[\vec{x}-q_i(t)]^2 + \mathrm{i}\vec{p}_i(t)\cdot\vec{x}\right\}$$

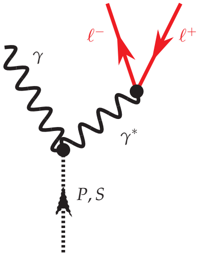
  - N-body state = product state (no Bose/Fermi symmetrization!)
  - classical equations of motion from Lagrangian

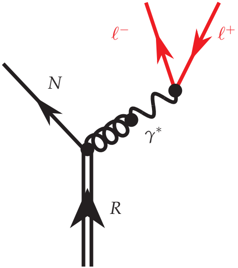
$$L = \sum_i \left[-\dot{q}_i \cdot \vec{p}_i + \langle T_i \rangle + \frac{1}{2}\sum_{ij} \left\langle V_{ij}^{(2)} \right\rangle - \frac{3}{2Lm} \right]$$

  - interaction potentials: similar resonance model as in GiBUU
- all calculations for dileptons: S. Endres

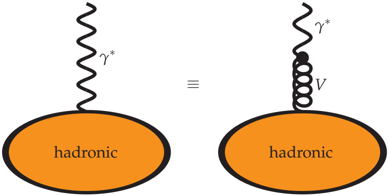
Dalitz decays







- Dalitz decay:
  1 particle → 3 particles
- $V: \omega \rightarrow \pi + \gamma^* \rightarrow \pi + \ell^+ + \ell^-$
- $P, S: \pi, \eta \rightarrow \gamma + \gamma^* \rightarrow \gamma + \ell^+ + \ell^-$
- R: Baryon resonances
  $\Delta, N^* \rightarrow N + V \rightarrow N + \gamma^* \rightarrow N + \ell^+ + \ell^-$
- vector-meson dominance

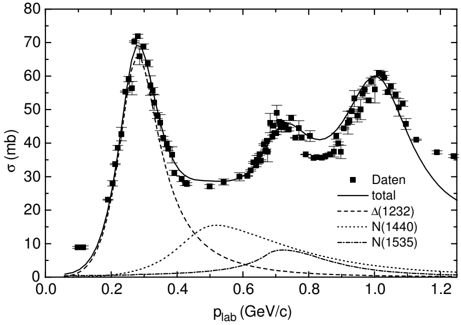


Resonance Model

- reactions dominated by resonance scattering: $a\,b \rightarrow R \rightarrow c\,d$
- Breit-Wigner cross-section formula

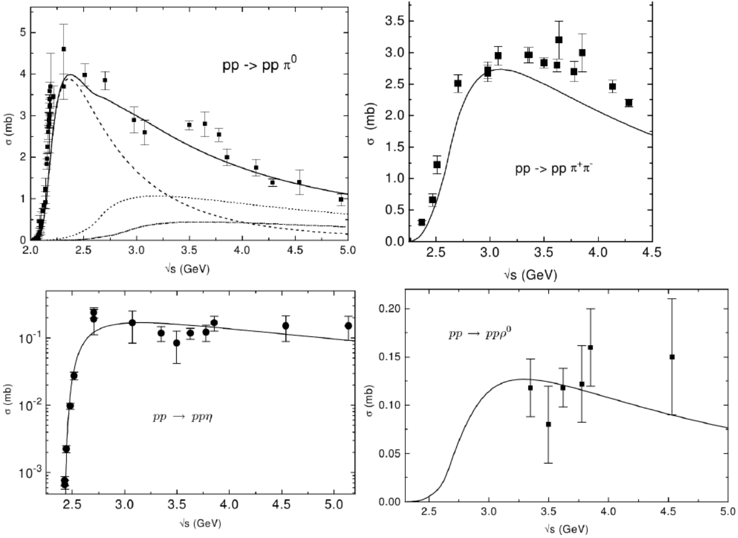
$$\sigma_{ab \rightarrow R \rightarrow cd} = \frac{2s_R + 1}{(2s_a + 1)(2s_b + 1)} \frac{4\pi}{p_{\text{lab}}^2} \frac{s\Gamma_{ab\rightarrow R}\Gamma_{R\rightarrow cd}}{(s - m_R^2)^2 + s\Gamma_{\text{tot}}^2}$$

- applicable for low-energy nuclear reactions $E_{\text{kin}} \lesssim 1.1$ GeV
- example: $\sigma_{\pi^- p \rightarrow \pi^- p}$ [Teis (PhD thesis 1996), data: Baldini et al, Landolt-Börnstein **12** (1987)]



GiBUU: Resonance Model

- further cross sections



GiBUU: Extension to HADES energies

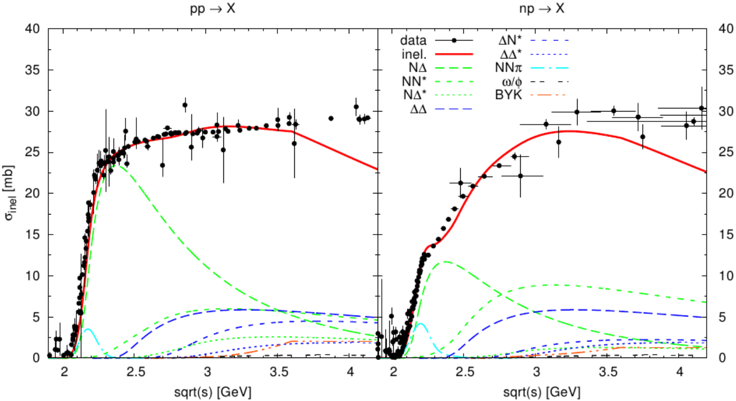
- [WHM12, WM13]
- keep same resonances (parameters from Manley analysis)

	rating	M_0 [MeV]	Γ_0 [MeV]	$\lvert\mathcal{M}^2\rvert/16\pi$ [mb GeV^2] NR	ΔR	branching ratio in % πN	ηN	$\pi\Delta$	ρN	σN	$\pi N^*(1440)$	$\sigma\Delta$
$P_{11}(1440)$	****	1462	391	70	—	69	—	22_P	—	9	—	—
$S_{11}(1535)$	***	1534	151	8	60	51	43	—	2_S+1_D	1	2	—
$S_{11}(1650)$	****	1659	173	4	12	89	3	2_D	3_D	2	1	—
$D_{13}(1520)$	****	1524	124	4	12	59	—	5_S+15_D	21_S	—	—	—
$D_{15}(1675)$	****	1676	159	17	—	47	—	53_D	—	—	—	—
$P_{13}(1720)$	*	1717	383	4	12	13	—	—	87_P	—	—	—
$F_{15}(1680)$	****	1684	139	4	12	70	—	10_P+1_F	5_P+2_F	12	—	—
$P_{33}(1232)$	****	1232	118	OBE	210	100	—	—	—	—	—	—
$S_{31}(1620)$	**	1672	154	7	21	9	—	62_D	25_S+4_D	—	—	—
$D_{33}(1700)$	*	1762	599	7	21	14	—	74_S+4_D	8_S	—	—	—
$P_{31}(1910)$	****	1882	239	14	—	23	—	—	—	—	67	10_P
$P_{33}(1600)$	***	1706	430	14	—	12	—	68_P	—	—	20	—
$F_{35}(1905)$	***	1881	327	7	21	12	—	1_P	87_P	—	—	—
$F_{37}(1950)$	****	1945	300	14	—	38	—	18_F	—	—	—	44_F

- production channels in Teis: $NN \to N\Delta$, $NN \to NN^*, N\Delta^*$, $NN \to \Delta\Delta$
- extension to $NN \to \Delta N^*, \Delta\Delta^*$, $NN \to NN\pi$,
  $NN \to NN\rho, NN\omega, NN\pi\omega, NN\phi$,
  $NN \to BYK$ $(B = N, \Delta,\ Y = \Lambda, \Sigma)$

GiBUU Extension to HADES energies

- good description of total pp, pn (inelastic) cross section

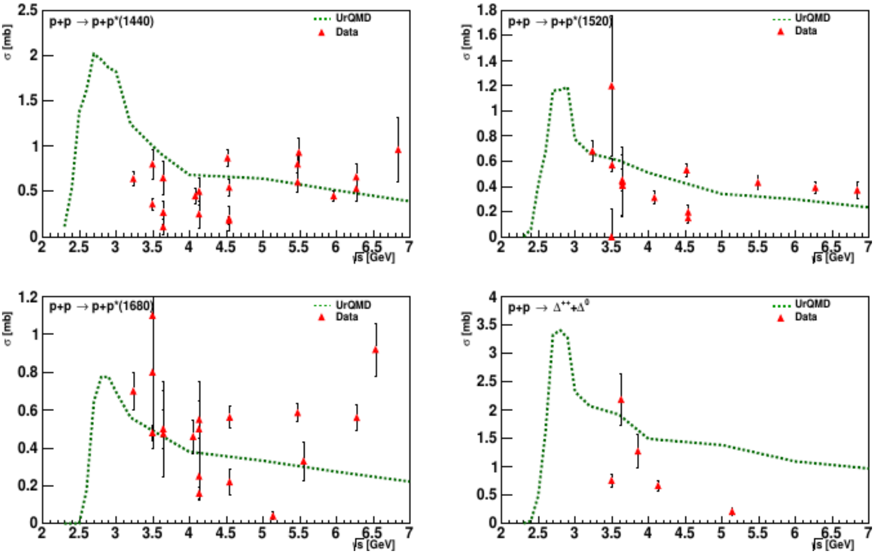


- dilepton sources
  - Dalitz decays: $\pi^0, \eta \rightarrow \gamma \ell^+ \ell^-$; $\omega \rightarrow \pi^0 \ell^+ \ell^-$, $\Delta \rightarrow N \ell^+ \ell^-$
  - $\rho, \omega, \phi \rightarrow \ell^+ \ell^-$: invariant mass $\ell^+ \ell^-$ spectra $\Rightarrow$ spectral properties of vector mesons
  - for details, see [WHM12]

UrQMD: Baryon resonances

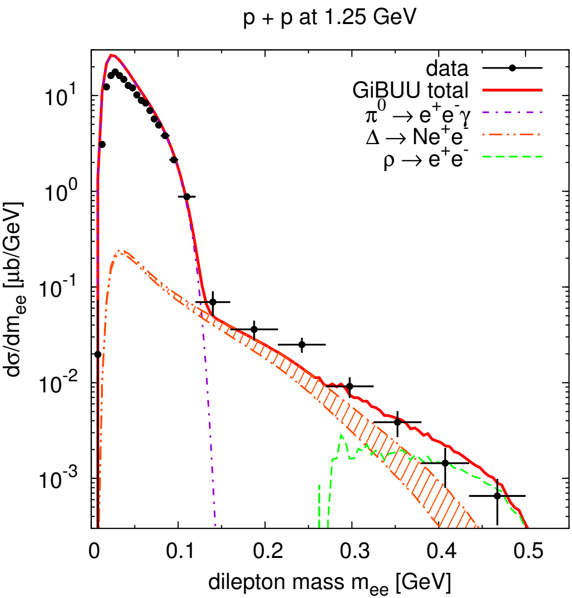
Resonance	Mass	Width	$N\pi$	$N\eta$	$N\omega$	$N\varrho$	$N\pi\pi$	$\Delta_{1232}\pi$	$N^*_{1440}\pi$	ΛK	ΣK	$f_0 N$	$a_0 N$
N^*_{1440}	1.440	350	0.65				0.10	0.25					
N^*_{1520}	1.515	120	0.60			0.15	0.05	0.20					
N^*_{1535}	1.550	140	0.60	0.30			0.05		0.05				
N^*_{1650}	1.645	160	0.60	0.06		0.06	0.04	0.10	0.05	0.07	0.02		
N^*_{1675}	1.675	140	0.40					0.55	0.05				
N^*_{1680}	1.680	140	0.60			0.10	0.10	0.15	0.05				
N^*_{1700}	1.730	150	0.05			0.20	0.30	0.40	0.05				
N^*_{1710}	1.710	500	0.16	0.15		0.05	0.21	0.20	0.10	0.10	0.03		
N^*_{1720}	1.720	550	0.10			0.73	0.05			0.10	0.02		
N^*_{1900}	1.850	350	0.30	0.14	0.39	0.15				0.02			
N^*_{1990}	1.950	500	0.12			0.43	0.19	0.14	0.05	0.03		0.04	
N^*_{2080}	2.000	550	0.42	0.04	0.15	0.12	0.05	0.10		0.12			
N^*_{2190}	2.150	470	0.29			0.24	0.10	0.15	0.05	0.12			
N^*_{2220}	2.220	550	0.29		0.05	0.22	0.17	0.20		0.12			
N^*_{2250}	2.250	470	0.18			0.25	0.20	0.20	0.05	0.12			
Δ_{1232}	1.232	115	1.00										
Δ^*_{1600}	1.700	350	0.10					0.65	0.25				
Δ^*_{1620}	1.675	160	0.15			0.05		0.65	0.15				
Δ^*_{1700}	1.750	350	0.20			0.25		0.55					
Δ^*_{1900}	1.840	260	0.25			0.25		0.25	0.25				
Δ^*_{1905}	1.880	350	0.18			0.80		0.02					
Δ^*_{1910}	1.900	250	0.30			0.10		0.35	0.25				
Δ^*_{1920}	1.920	200	0.27					0.40	0.30	0.03			
Δ^*_{1930}	1.970	350	0.15			0.22		0.20	0.28	0.15			
Δ^*_{1950}	1.990	350	0.38			0.08		0.20	0.18	0.12			0.04

UrQMD: Baryon resonances

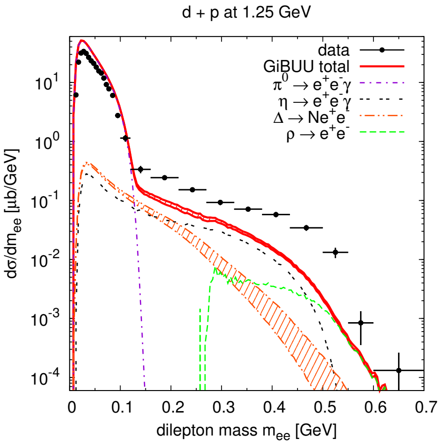


Dileptons in pp, pA, and AA collisions at SIS energies

GiBUU: p p at HADES ($E_{\mathrm{kin}} = 1.25$ GeV)

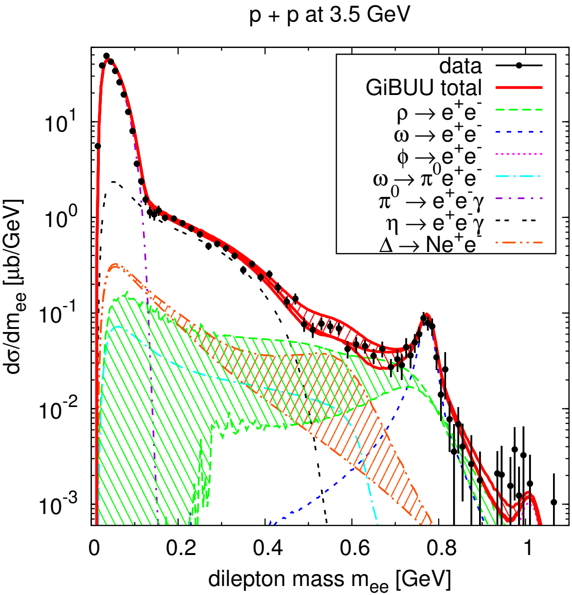


d p at HADES ($E_{kin} = 1.25$ GeV)



- triggered on forward protons → quasifree np scattering
- model uncertainties:
  - ρ production through $D_{13}(1525)$ (isospin symmetric?)
  - $S_{11}(1535)$ [enhanced in np; (from η production)]
  - d-wave function treatable as quasiclassical "distribution"?
  - bremsstrahlung contributions

GiBUU: p p at HADES ($E_{\text{kin}} = 3.5$ GeV)



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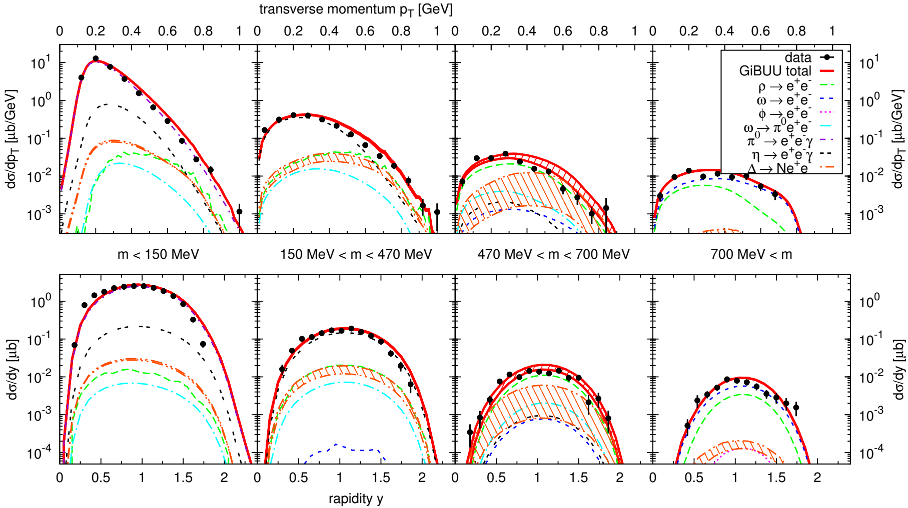
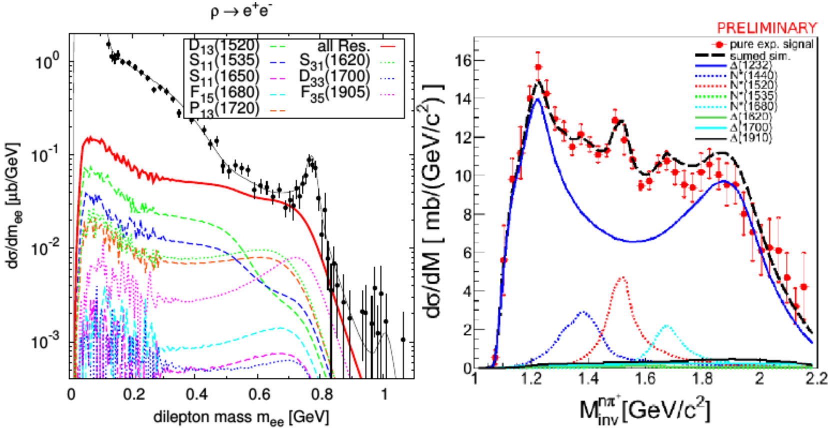


GiBUU: “ρ meson” in pp

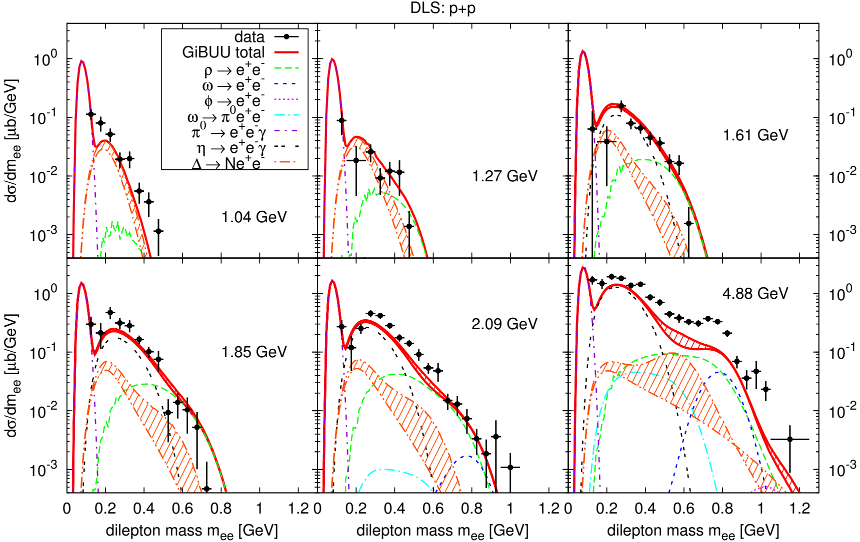
- production through hadron resonances
  $NN \rightarrow NR \rightarrow NN\rho$, $NN \rightarrow N\Delta \rightarrow NN\pi\rho$



- “ρ”-line shape “modified” already in elementary hadronic reactions
- due to production mechanism via resonances

GiBUU: Comparison to old DLS data (pp)

- HADES data consistent with DLS data
- checked by comparing HADES data within DLS acceptance



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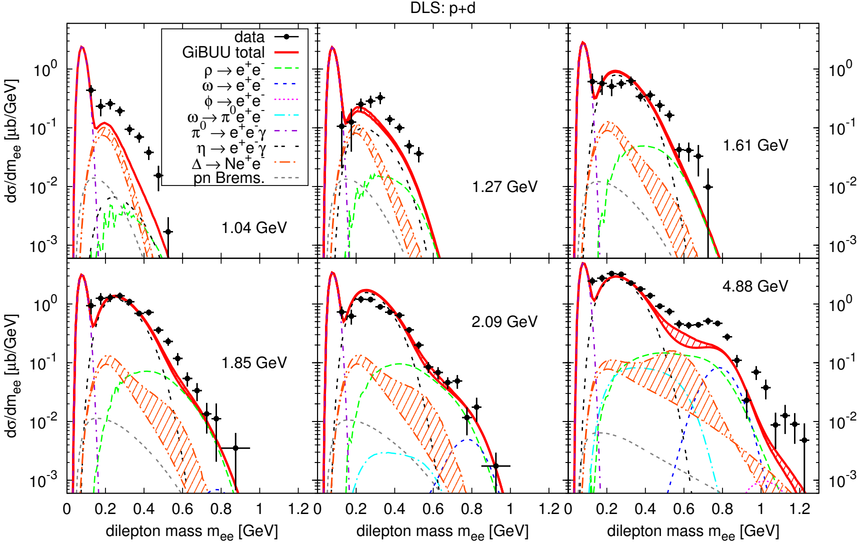
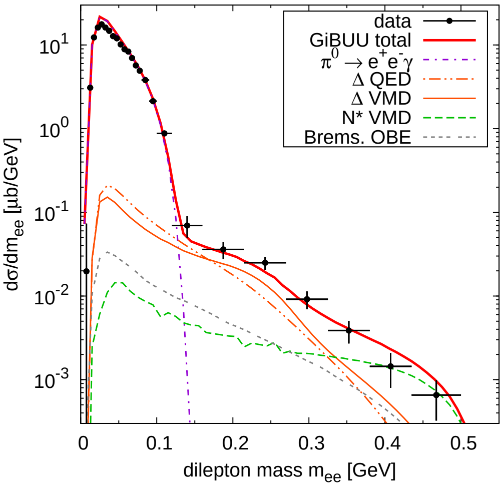


GiBUU: Newest development: Δ(1232) in VMD model

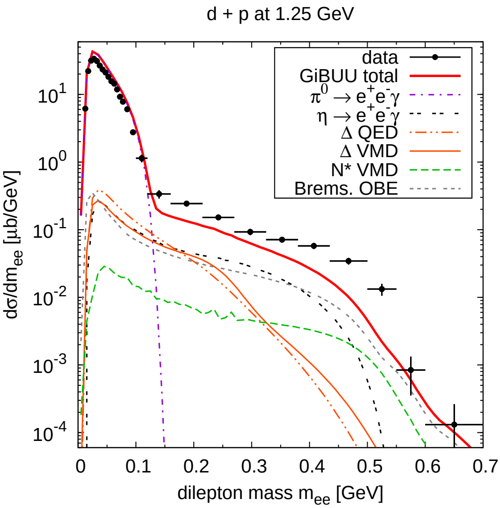
- so far: Δ-Dalitz decay treated separately from other resonances
- now: treating Δ as all other resonances via VMD model

p + p at 1.25 GeV



GiBUU: Newest development: Δ(1232) in VMD model

- so far: Δ-Dalitz decay treated separately from other resonances
- now: treating Δ as all other resonances via VMD model



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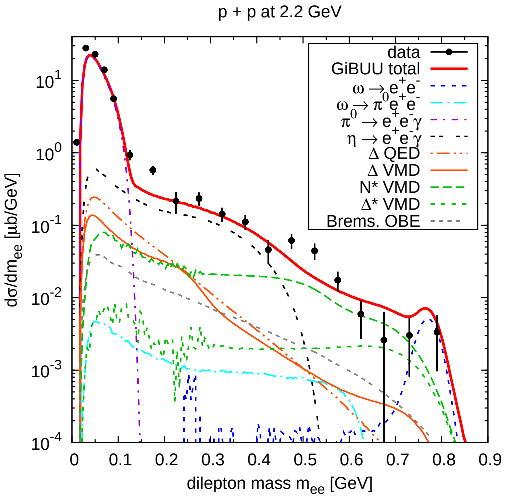

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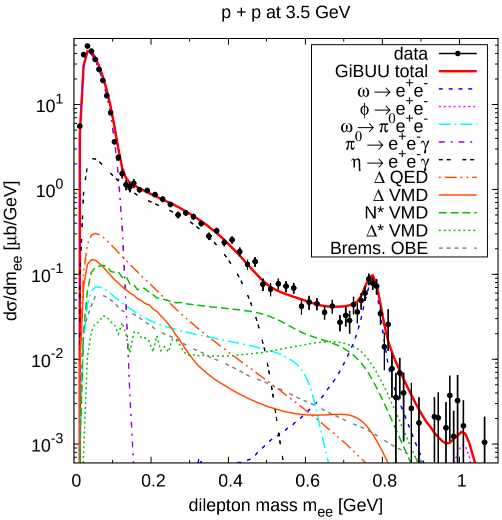
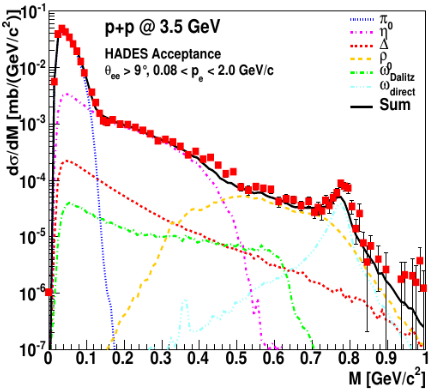
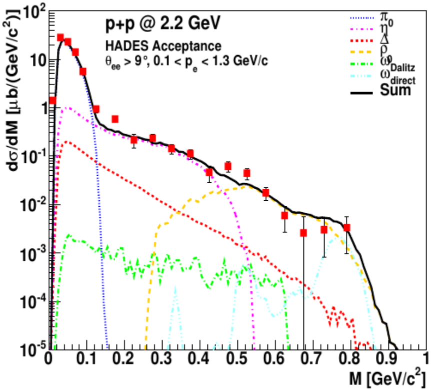


UrQMD: p p at HADES ($E_{\text{kin}} = 2.2$ GeV and 3.5 GeV





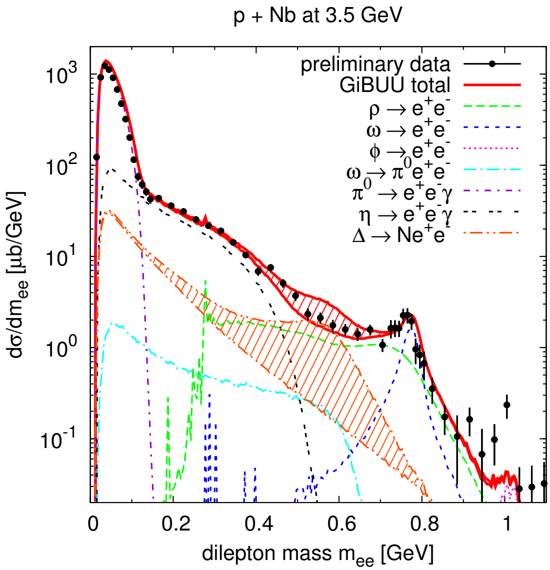
GiBUU: p Nb at HADES (3.5 GeV)

GiBUU:

- medium effects built in transport model
  - binding effects, Fermi smearing, Pauli blocking
  - final-state interactions
  - production from secondary collisions
- sensitivity to additional in-medium modifications of vector mesons?

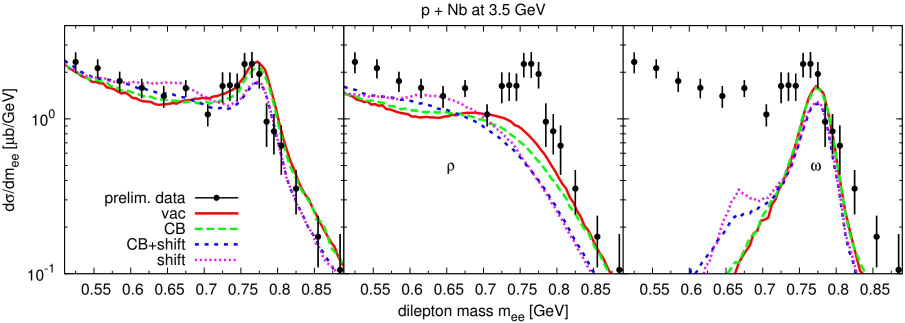
GiBUU: p Nb at HADES (3.5 GeV)

- with vacuum spectral functions:



GiBUU: p Nb at HADES (3.5 GeV)

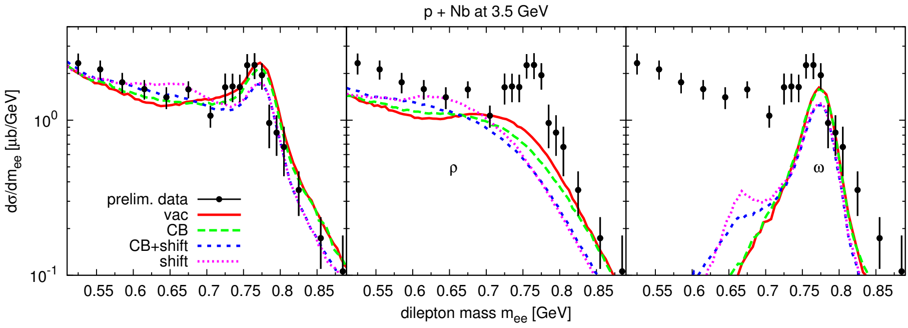
- with medium modified spectral functions:



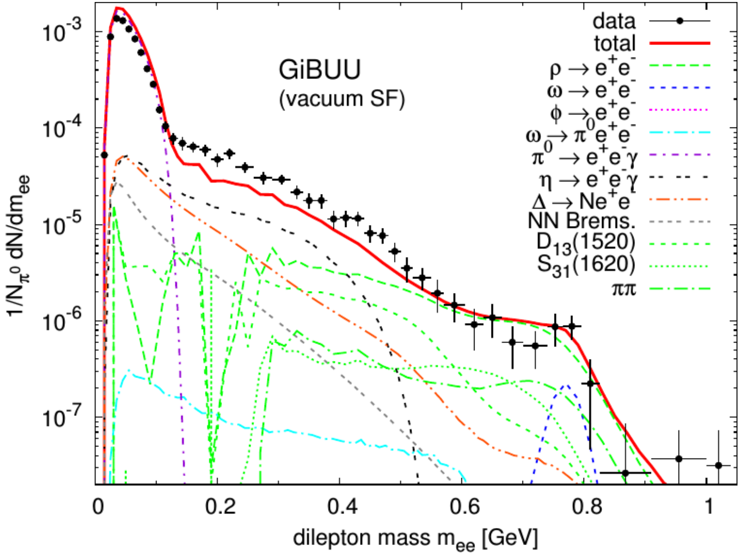
- no definite hint for medium modifications in p Nb

GiBUU: p Nb at HADES (3.5 GeV)

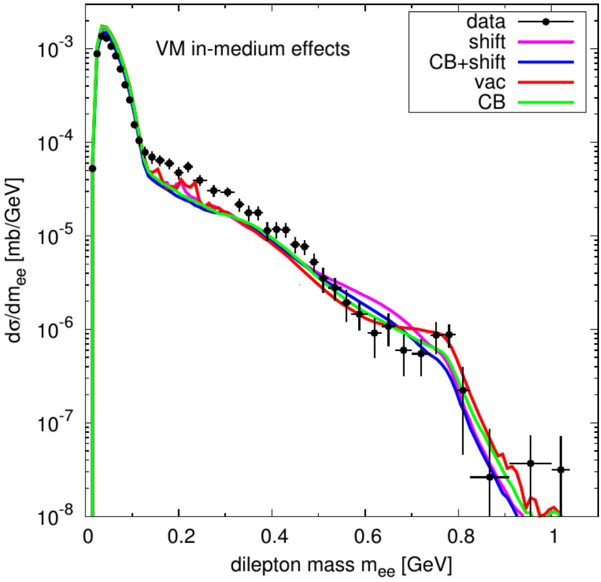
- medium effects built in transport model
  - binding effects, Fermi smearing, Pauli blocking
  - final-state interactions
  - production from secondary collisions
- sensitivity on medium effects of vector-meson spectral functions?



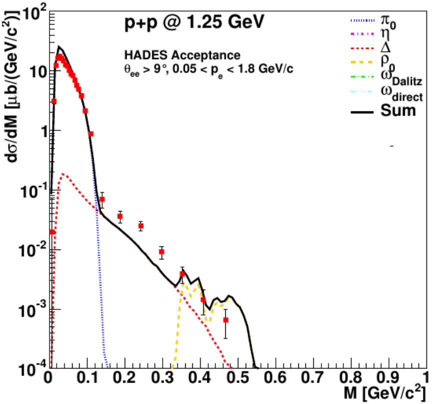
GiBUU: Ar KCl at HADES

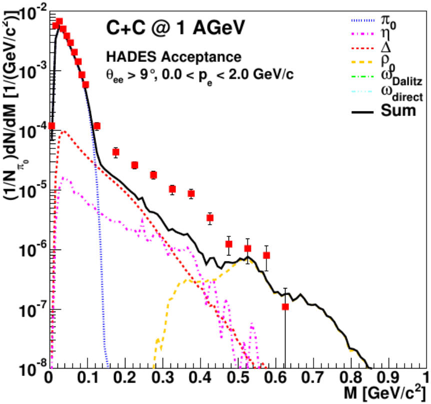


GiBUU: Ar KCl at HADES

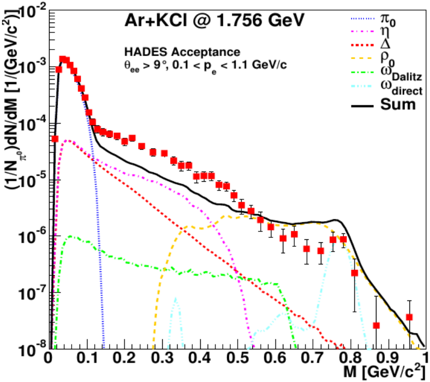


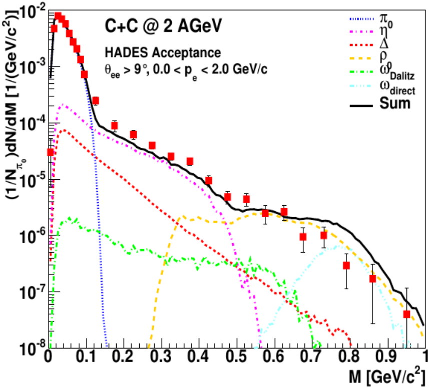
UrQMD: pp and CC at HADES (lowest energies)





UrQMD: Ar KCl and CC at HADES





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Quiz

Quiz

1. What's the difference between the simulation algorithms used in GiBUU (test-particle Monte Carlo simulation) and in UrQMD (quantum molecular dynamics simulation)?
2. Which is the most important empirical input we need for transport models in low-energy heavy-ion collisions?
3. Why are the ρ-meson properties in the particle-data booklet defined solely through reactions like $e^+ + e^- \rightarrow \pi + \pi$ and not with $p + p \rightarrow$ hadrons?
4. what's the fundamental difficulty in making use of (quantitative) many-body-QFT calculations of medium-modified spectral functions?
5. how can one solve this approximately and what are the caveats?