Electromagnetic Probes in Heavy-Ion Collisions I

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March 31-April 04, 2014

Outline

1. Plan of the lectures and motivation
2. Electromagnetic Probes: Phenomenology
3. The standard model in a nutshell
   - Particles and forces
   - Quantum Electrodynamics (QED)
   - Quantum Chromodynamics (QCD) and chiral symmetry
   - Quantum flavordynamics (QFD)
4. Strongly interacting matter: QCD/hadronic models at finite T, μ
5. References
6. Quiz

Plan of the Lectures

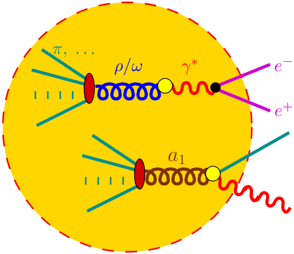
- Lecture I: Fundamentals
  - symmetries and conservation laws in (quantum) field theory
  - the Standard Model in a nutshell
  - QCD, chiral symmetry, and the relation with electromagnetic probes
- Lecure II: theory descriptions of heavy-ion collisions and em. probes
  - transport and hydrodynamics
  - collective flow
  - radiation of electromagnetic probes from a thermal transparent medium (McLerran-Toimela formula)
  - effective hadronic models for vector mesons

Plan of the Lectures

- Lecture III: Dileptons in heavy-ion collisions (SIS@GSI)
  - hadronic models for transport models: baryon resonances
  - Gießen Boltzmann-Uehling-Uhlenbeck (GiBUU)
  - Ultrarelativistic Quantum Molecular Dynamics (UrQMD)
  - medium modifications:
    "transport-hydro hybrid" and "coarse-graining" approach
- Lecture IV: Electromagnetic probes in heavy-ion collisions (SPS@CERN, RHIC@BNL, LHC@CERN)
  - hard-thermal-loop approved dilepton rates (emission from QGP)
  - hadronic many-body theories (emission from hadron gas)
  - dileptons at SPS and RHIC
  - photons at RHIC and LHC ("the photon-v_2 puzzle")

Why Electromagnetic Probes?

- $\gamma, \ell^{\pm}$: only e. m. interactions
- reflect whole "history" of collision
- chance to see chiral symm. rest. directly?



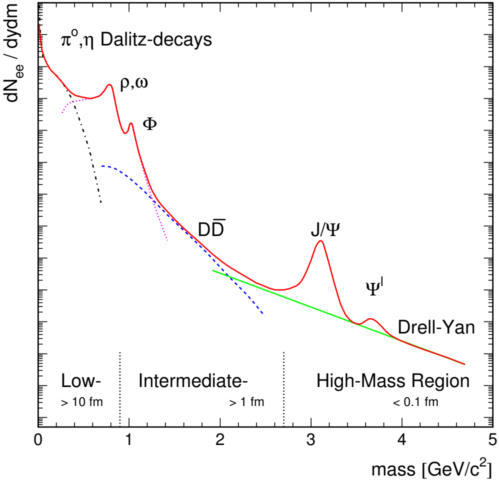
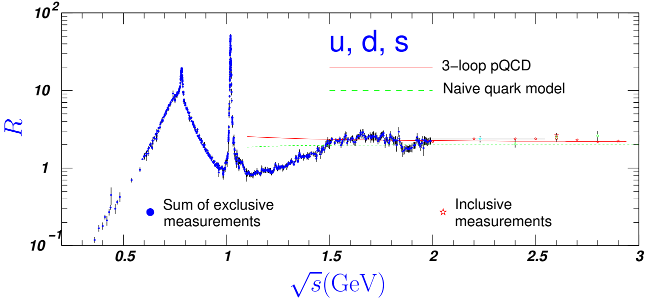


Fig. by A. Drees (from [RW00])

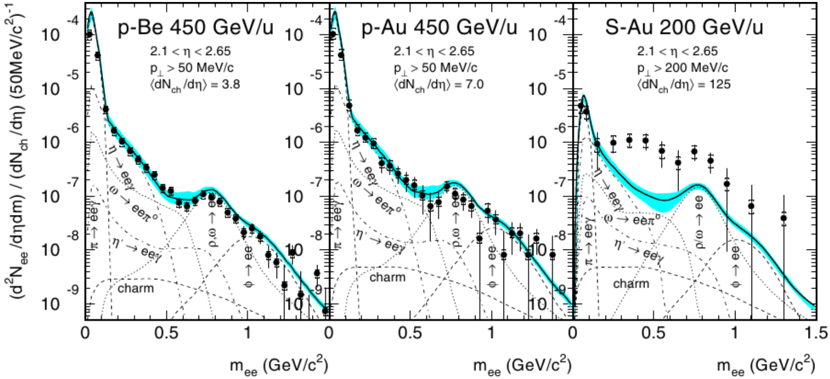
Vacuum Baseline: $e^+e^- \to$ hadrons



$$R := \frac{\sigma_{\mathrm{e^+e^-\to hadrons}}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$$

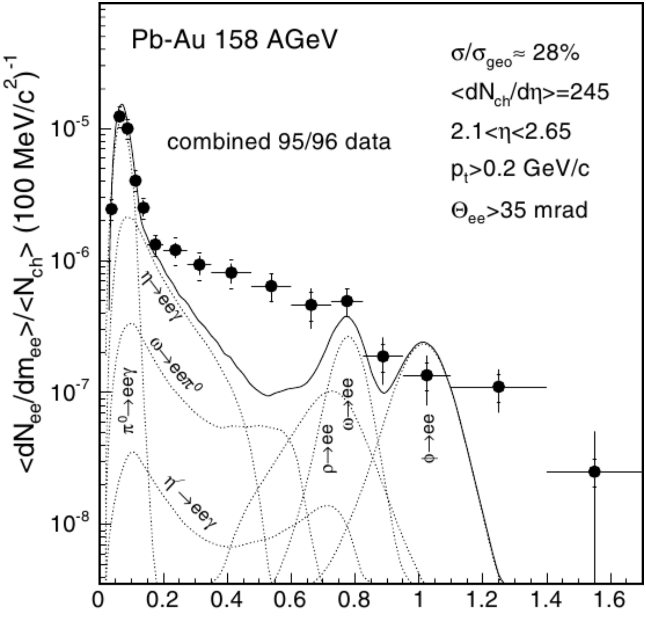
- probes all hadrons with quantum numbers of γ^*
- $R_{\mathrm{QM}} = N_c \sum_{f=u,d,s} Q_f^2 = 3 \times [(2/3)^2 + (-1/3)^2 + (-1/3)^2] = 2$
- Our aim pp $\to \ell^+\ell^-$, pA $\to \ell^+\ell^-$, AA $\to \ell^+\ell^-$ ($\ell = \mathrm{e}, \mu$)

The CERES findings: Dilepton enhancement

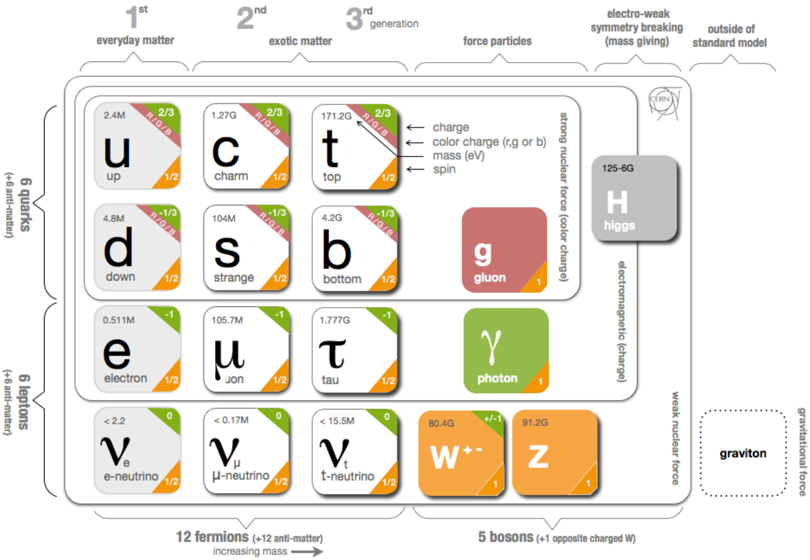


- pp (pBe): "elementary reactions"; baseline (mandatory to understand first!)
- pA: "cold nuclear matter effects"; next step (important as baseline for other observables like "J/ψ suppression")
- AA: "medium effects"; hope to learn something about in-medium properties of vector mesons, fundamental QCD properties

The CERES findings: Dilepton enhancement



The standard model in a nutshell: particles and forces



[graphics from http://www.isgtw.org/spotlight/go-particle-quest-first-cern-hackfest]

Quantum Electrodynamics (QED)

Literature: [Nac90, DGH92, B+12], conventions as in [Nac90]

- electrons+positrons: massive spin-1/2 Dirac field $\psi \in \mathbb{C}^4$
- describes electron (charge $q_\mathrm{e} = -1$) and antielectron (=positron)
- photon: massless vector field A_μ
- antisymmmetric field-strength tensor $\rightarrow (\vec{E}, \vec{B})$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \begin{pmatrix} 0 & E^1 & E^2 & E^3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{pmatrix}$$

- Lagrangian ($e > 0$: em. coupling constant $e^2/(4\pi) = \alpha_\mathrm{em} \simeq 1/137$

$$\mathscr{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\psi}[\mathrm{i}(\not{\partial} + \mathrm{i} q_\mathrm{e} e \not{A})]\psi, \quad q_e = -1$$

- Dirac matrices: $\gamma^\mu \in \mathbb{C}^{4\times 4}$, $\mu \in \{0,1,2,3\}$, $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$, $\overline{\psi} = \psi^\dagger \gamma^0$
- "Feynman slash" $\not{A} = A_\mu \gamma^\mu$, $\not{\partial} = \gamma^\mu \partial_\mu = \gamma^\mu \frac{\partial}{\partial x^\mu}$

Symmetries of QED

- as a classical field theory: Least-action principle ⇒ equations of motion
- action (Lorentz invariant!)

$$S[A,\psi] = \int \mathrm{d}^4 x \mathscr{L}$$

- symmetries lead to conservation laws (Noether's Theorem)
- space-time symmetries
  - time translations: energy conservation
  - space translations: momentum conservation
  - rotations: angular-momentum conservation
- intrinsic symmetry: invariant under change of phase factor
  $\psi \rightarrow \exp(-\mathrm{i} q_{\mathrm{e}} e \alpha)\psi$, $\alpha \in \mathbb{R} \Rightarrow$ electric-charge conservation

$$j_{\mathrm{em}}^{(\mathrm{e})\mu} = q_{\mathrm{e}} e \overline{\psi} \gamma^{\mu} \psi, \quad \partial_{\mu} j_{\mathrm{em}}^{(\mathrm{e})\mu} = 0$$

- here even local gauge symmetry:

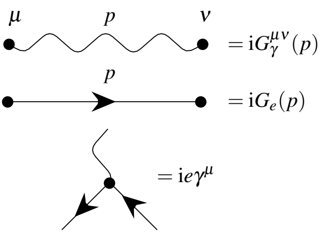
$$\psi \rightarrow \exp[-\mathrm{i} q_{\mathrm{e}} e \chi(x)]\psi, \quad A_{\mu} \rightarrow A_{\mu} + q_{\mathrm{e}} \partial_{\mu} \chi$$

- local symmetry ⇔ gauge boson

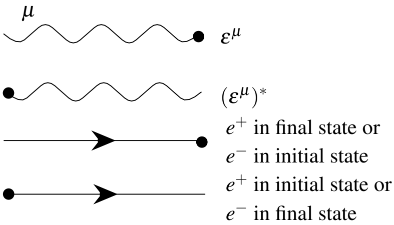
Quantization

- fields ⇒ operators
- physical quantities S-matrix elements: $|T_{fi}|^2$ transition probabilities for scattering from asymptotic free initial to asymptotic free final state
- local, microcausal quantum field theory with stable ground state
  - spin-statistics relation:
    half-integer spin ⇔ fermions, integer spin ⇔ bosons
- can only evaluate in perturbation theory ⇒ Feynman rules

Internal lines: Propagators



External lines: Initial and final states



- $G_\gamma^{\mu\nu} = -\eta_{\mu\nu}/(p^2 + i0^+)$, $G_e = (\not{p} - m)/(p^2 - m^2 + i0^+)$

Quantum Chromodynamics: QCD

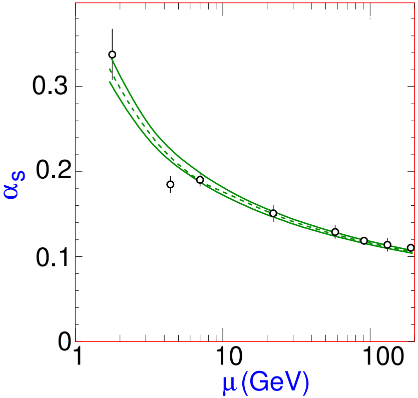
- Theory for strong interactions: QCD

$$\mathscr{L}_{\text{QCD}} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \overline{\psi}(\mathrm{i}\not{D} - \hat{M})\psi$$

- non-Abelian gauge group $\mathrm{SU}(3)_{\text{color}}$
  - each quark: color triplet
  - covariant derivative: $\mathrm{D}_\mu = \partial_\mu + \mathrm{i} g \hat{T}_a A^a$ ($a \in \{1, \ldots, 8\}$)
  - field-strength tensor $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^a{}_{bc} A^b_\mu A^c_\nu$
  - group structure constants: $[\hat{T}^a, \hat{T}^b] = \mathrm{i} f^a{}_{bc} \hat{T}^b \hat{T}^c$, $\hat{T}^a = (\hat{T}^a)^\dagger \in \mathbb{C}^{3\times 3}$
- Particle content:
  - ψ: Quarks with flavor $(u, d; c, s; t, b)$ (mass eigenstates!)
  - $\hat{M} = \mathrm{diag}(m_u, m_d, m_s, \ldots)$ = current quark masses
  - A^a_μ: gluons, gauge bosons of $\mathrm{SU}(3)_{\text{color}}$
- Symmetries
  - fundamental building block: local $\mathrm{SU}(3)_{\text{color}}$ symmetry
  - in light-quark sector: approximate chiral symmetry ($\hat{M} \to 0$)
  - dilation symmetry (scale invariance for $\hat{M} \to 0$)

Features of QCD

- asymptotically free: at large momentum transfers $\alpha_s = 4\pi g_s^2 \to 0$
- running from renormalization group (due to self-interactions of gluons!): Nobel prize 2004 for Gross, Wilczek, Politzer



- quarks and gluons confined in hadrons
- theoretically not fully understood (nonperturbative phenomenon!)
- need of effective hadronic models at low energies: (Chiral) symmetry!

Chiral Symmetry of (massless) QCD

- Consider only light u, d quarks
- iso-spin 1/2 doublet: $\psi = \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$
- NB: ψ has three “indices”: Dirac spinor, color, flavor iso-spin!
- γ matrices: $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}\mathbb{1}$, $\gamma_5 := \mathrm{i}\gamma_0\gamma_1\gamma_2\gamma_3$, $\gamma_5\gamma_\mu = -\gamma_\mu\gamma_5$, $\gamma_5^\dagger = \gamma_5$, $\gamma_5^2 = \mathbb{1}$
- Diracology of left and right-handed components

$$\psi_L = \frac{\mathbb{1} - \gamma_5}{2}\psi = P_L\psi, \quad \psi_R = \frac{\mathbb{1} + \gamma_5}{2}\psi = P_R\psi,$$

$$P_{L/R}^2 = P_{L/R}, \quad P_R P_L = P_L P_R = 0, \quad P_{L/R}\gamma_5 = \gamma_5 P_{L/R} = \mp P_{L/R}$$

$$P_{L/R}\gamma_\mu = \gamma_\mu P_{R/L}, \quad \overline{P_L\psi} = \overline{\psi}P_R, \quad \overline{P_R\psi} = \overline{\psi}P_L$$

$$\overline{\psi}\gamma_\mu\psi = \overline{\psi_L}\gamma_\mu\psi_L + \overline{\psi_R}\gamma_\mu\psi_R, \quad \overline{\psi}\psi = \overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L$$

- $\overline{\psi} := \psi^\dagger\gamma_0$, $\overline{\gamma_5\psi} = \psi^\dagger\gamma_5^\dagger\gamma_0 = -\overline{\psi}\gamma_5$
- in the massless limit ($m_u = m_d = 0$)

$$\mathscr{L}_{u,d} = \overline{\psi}\mathrm{i}\not{D}\psi = \overline{\psi_L}\mathrm{i}\not{D}\psi_L + \overline{\psi_R}\mathrm{i}\not{D}\psi_R$$

Chiral Symmetry

- in the massless limit ($m_u = m_d = 0$)
- a lot of global chiral symmetries:
  - change of independent phases for left and right components:

$$\psi_L(x) \rightarrow \exp(-\mathrm{i}\phi_L)\psi_L(x), \quad \psi_R(x) \rightarrow \exp(-\mathrm{i}\phi_R)\psi_R(x)$$

  - symmetry group $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$
  - independent "iso-spin rotations"

$$\psi_L(x) \rightarrow \exp(-\mathrm{i}\vec{\alpha}_L \cdot \vec{T})\psi_L(x), \quad \psi_R(x) \rightarrow \exp(-\mathrm{i}\vec{\alpha}_R \cdot \vec{T})\psi_R(x)$$

  - $\vec{T} = \vec{\tau}/2$, $\vec{\tau}$: Pauli matrices; symmetry group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$
- alternative notation scalar-pseudoscalar phases/iso-spin rotations

$$\psi \rightarrow \exp(-\mathrm{i}\phi_s)\psi, \quad \psi \rightarrow \exp(-\mathrm{i}\gamma_5\phi_a)\psi$$
$$\psi \rightarrow \exp(-\mathrm{i}\vec{\alpha}_V \cdot \vec{T})\psi, \quad \psi \rightarrow \exp(-\mathrm{i}\gamma_5\vec{\alpha}_A \cdot \vec{T})\psi$$

- $\mathrm{U}(1)_s$ and $\mathrm{SU}(2)_V$ are subgroups that are symmetries even if $m_u = m_d \neq 0 \Rightarrow$ Heisenberg's iso-spin symmetry!

Currents: relation to mesons

- based on [Koc97, Sch03, Din11]
- Noether: each global symmetry leads to a conserved quantity
- from chiral symmetries

$$j_s^\mu = \overline{\psi}\gamma^\mu \psi, \quad j_a^\mu = \overline{\psi}\gamma^\mu \gamma_5 \psi$$
$$\vec{j}_V^\mu = \overline{\psi}\gamma^\mu \vec{T}\psi, \quad \vec{j}_A^\mu = \overline{\psi}\gamma^\mu \gamma_5 \vec{T}\psi$$

- Link to mesons: Build Lorentz-invariant objects with corresponding quantum numbers
  - σ: $\overline{\psi}\psi$ (scalar and iso-scalar)
  - π's: $\mathrm{i}\overline{\psi}\vec{T}\gamma_5\psi$ (pseudoscalar and iso-vector)
  - ρ's: $\overline{\psi}\gamma_\mu\vec{T}\psi$ (vector and iso-vector)
  - a_1's: $\overline{\psi}\gamma_\mu\gamma_5\vec{T}\psi$ (axialvector and iso-axialvector)
- in nature: σ and π's; ρ's and a_1's do not have same mass!
- reason: QCD ground state not symmetric under pseudoscalar and pseudovector trafos since $\langle \text{vac} | \overline{\psi}\psi | \text{vac} \rangle \neq 0$

Spontaneous symmetry breaking

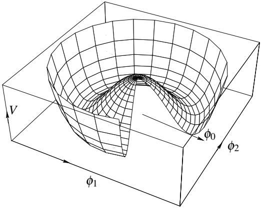
- spontaneously broken symmetry: ground state not symmetric
- vacuum necessarily degenerate
- vacuum invariant under scalar and vector transformations: $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$ broken to $\mathrm{U}(1)_s$; $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ broken to $\mathrm{SU}(2)_V$
- for each broken symmetry massless scalar Goldstone boson
- there are three pions which are very light compared to other hadrons (finite masses due to explicit breaking through m_u, m_d!)
- but no pseudoscalar isoscalar light particle! ($m_\eta \simeq 548$ MeV)
- reason: $\mathrm{U}(1)_a$ anomaly
    - axialscalar symmetry does not survive quantization!
    - good for explanation of correct decay rate for $\pi_0 \rightarrow \gamma\gamma$
    - axialscalar current not conserved $\partial_\mu j_a^\mu = 3/8 \alpha_s \varepsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a$
- explicit breaking due to quark masses
    - can be treated perturbatively $\Rightarrow$ chiral perturbation theory
    - axial-vector current only approximately conserved $\Rightarrow$ PCAC
    - a lot of low-energy properties of hadrons derivable

The minimal linear σ model

- chiral symmetry realized by SO(4): meson fields $\phi \in \mathbb{R}^4$
- describes σ and pions $(\pi^{\pm}, \pi^0)$

$$\mathscr{L}_{\chi\text{limit}} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{\lambda}{4}(\phi^2 - f_\pi^2)^2$$

- spontaneous symmetry breaking: mexican-hat potential



- doesn't cost energy to excite field in direction of the rim
  $\Rightarrow$ massless Nambu-Goldstone bosons (pions)
- vacuum expectation value $\langle \phi^0 \rangle = f_\pi \neq 0$
- symmetry spontaneously broken from SO(4) to SO(3)_V
- particle spectrum: 4 field-degrees of freedom $\Rightarrow$ vacuum inv. 3-dim SO(3)
  $\Rightarrow$ 3 massless pions $\Rightarrow$ $4 - 3 = 1$ massive σ

Explicit symmetry breaking

- explicit χ-symmetry breaking due to m_{quark}: $m_\pi \simeq 140\,\text{MeV}$
- Gell-Mann-Oakes-Renner relation: $m_\pi^2 f_\pi^2 = -m \langle \overline{q} q \rangle$
- vector (isospin) symmetry only fulfilled for $m_u = m_d$
- in reality: $m_u \simeq$ 1.7-3.3 MeV, $m_d \simeq$ 4.1-3.3MeV
- isospin symmetry as strongly broken as χ symmetry!

Quantum flavordynamics: QFD

- unified description of weak and electromagnetic interaction
- based on local chiral gauge symmetry $\mathrm{SU}(2)_{\text{wiso}} \times U(1)_{\text{hyper}}$
- left-handed fermions: $\mathrm{SU}(2)_{\text{wiso}}$ doublets
- right-handed fermions: $\mathrm{SU}(2)_{\text{wiso}}$ singlets
- spontaneously broken to $\mathrm{U}(1)_{\text{em}}$
- $\mathrm{SU}(2)_{\text{wiso}}$ scalar-boson doublet (4 real fields)
- Higgs mechanism: local symmetry $\Rightarrow$ Goldstone bosons eaten by gauge bosons
- gauge bosons become massive without violating gauge invariance!
- 4-dim gauge group spont broken to 1-dim gauge group
- 3 Goldstone bosons eaten up $\Rightarrow$ 3 massive gauge bosons $W^{\pm}$, Z and $4-3=1$ massless photon
- 1 massive scalar boson left as observable particle $\Rightarrow$ Higgs boson!
- flavors grouped into 3 families $\Psi_i = (\nu_i, \ell_i^-, u_i, d_i')$
- flavor eigenstates $\neq$ mass eigenstates
- Cabbibbo-Kobayashi-Maskawa quark-mixing matrix: $d_i' = \sum_j V_{ij} d_j$ ($\hat{V}$ unitary)

Lagrangian of QFD

- quantum numbers of leptons and quarks
  - $\vec{t}$: su(2) matrices for weak isospin
    $t \in \{0, 1/2, 1, \ldots\}$ isospin representation eigenvalues of $\vec{t}^2$: $t(t+1)$
    eigenvalues of t_3: $\{-t, -t+1, \ldots, t-1, t\}$
  - Y: weak isospin, $Q = Y + t_3$ electric charge

Particles			t	t^3	Y	Q
(Higgs)	ϕ		$1/2$	$-1/2$	$1/2$	0
ν_{eL}	$\nu_{\mu L}$	$\nu_{\tau L}$	$1/2$	$1/2$	$-1/2$	0
e_L	μ_L	τ_L	$1/2$	$-1/2$	$-1/2$	-1
e_R	μ_R	τ_R	0	0	-1	-1
u_L	c_L	t_L	$1/2$	$1/2$	$1/6$	$2/3$
d'_L	s'_L	b'_L	$1/2$	$-1/2$	$1/6$	$-1/3$
u_R	c_R	t_R	0	0	$2/3$	$2/3$
d_R	s_R	b_R	0	0	$-1/3$	$-1/3$

- Lagrangian must be invariant under local $\mathrm{SU}(2)_{\text{wiso}} \times U(1)_{\text{hyper}}$
- local symmetry chiral
  $\Rightarrow$ no “naive mass terms” for quarks, leptons, and gauge bosons allowed!
- all masses must come from spontaneous symmetry breaking!

Lagrangian of QFD

- gauge bosons acting in wiso-hypercharge space
- $\mathrm{D}_\mu = \partial_\mu + \mathrm{i} g W_\mu^a \hat{t}_a + \mathrm{i} g' B_\mu \hat{Y}$
- $W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g \varepsilon^{abc} W_\mu^b W_\mu^c$
- $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

$$\mathscr{L} = -\frac{1}{4} W_{\mu\nu}^a W_{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \mathscr{L}_{\text{Yuk}} + (\mathrm{D}_\mu \phi)^\dagger (\mathrm{D}^\nu \phi) - V(\phi)$$
$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

- $\mu^2 < 0 \Rightarrow$ mexican-hat potential $\Rightarrow \langle \phi \rangle = h_0/\sqrt{2} \in \mathbb{R}$
- symmetry local: can gauge "phase" away

$$\phi(x) = \exp[-\mathrm{i} g \vec{\alpha}(x) \cdot \hat{\vec{t}}] \begin{pmatrix} [h_0 + h(x)]/\sqrt{2} \\ 0 \end{pmatrix}, \quad h \in \mathbb{R}$$

- in this "unitary gauge" Goldstone modes eaten completely by gauge bosons $\Rightarrow$ 3 massive, 1 massless gauge boson
- 1 physical Higgs boson left

Lagrangian of QFD

- after symmetry breaking: diagonalize gauge-boson fields $\Rightarrow$ mass eigenstates

$$W^{\pm} = \frac{1}{\sqrt{2}}(W^1 \pm \mathrm{i} W^2), \quad \begin{pmatrix} W^{3\mu} \\ B^{\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta_{\mathrm{W}} & \sin\theta_{\mathrm{W}} \\ -\sin\theta_{\mathrm{W}} & \cos\theta_{\mathrm{W}} \end{pmatrix} \begin{pmatrix} Z \\ A^{\mu} \end{pmatrix}$$

- Weinberg angle: $\cos\theta_{\mathrm{W}} = g/G$, $\sin\theta_{\mathrm{W}} = g'/G$, $G = \sqrt{g^2 + g'^2}$
- gauge- and Higgs-boson Lagrangian

$$\begin{aligned}\mathscr{L}_{\text{gauge+Higgs}} = & \frac{1}{2}(\partial_{\mu} h)(\partial^{\mu} h) - \frac{G^2}{8}(h_0 + h)^2 \left[(W_{\mu}^1 W^{1\mu} + W_{\mu}^2 W^{2\mu}) \cos^2\theta_w + Z_{\mu} Z^{\mu}\right] \\ & - \frac{m_h^2}{2} h^2 \left(1 + \frac{m_h^2}{h_0} h + \frac{m_h^2}{4 h_0^2} h^2\right) \\ & - \frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}\end{aligned}$$

- physical parameters:

$$G, \quad \theta_w, \quad h_0, \quad m_h^2 = 2\mu^2/\lambda \;\Rightarrow\; m_W^2 = \frac{G^2}{4} h_0^2 \cos^2\theta_w, \quad m_Z^2 = \frac{G^2}{4} h_0^2$$

Leptons and quarks

- kinetic matter Lagrangian + gauge interactions (no explicit mass term!)

$$\mathscr{L}_{\text{matter-gauge bosons}} = \overline{\Psi} \not{D} \Psi$$

- covariant derivatives different for left- and right-handed part

$$\mathrm{D}_{\mathrm{L}\mu} \Psi_{i,\mathrm{L}} = (\partial_\mu + \mathrm{i} g \vec{W}_\mu \cdot \hat{\vec{T}}_{\mathrm{L}} + \mathrm{i} g' B_\mu \hat{Y}_{\mathrm{L}}) \Psi_{i,\mathrm{L}},$$
$$\mathrm{D}_{\mathrm{R}\mu} \Psi_{i,\mathrm{R}} = (\partial_\mu + \mathrm{i} g' B_\mu \hat{Y}_R) \Psi_{i,\mathrm{R}}, \quad \hat{T}_{\mathrm{R}} \equiv 0$$

Leptons and quarks

- Yukawa couplings (assume massless neutrinos!)

$$\mathscr{L}_{\text{leptons}}^{\text{Yuk}} = -\overline{\Psi}_{i,\text{R}}^{\text{lept}} \hat{C}_{\text{lept}} \phi^{\dagger} \Psi_{i,\text{L}}^{\text{lept}} + \text{h.c.}$$

$$\mathscr{L}_{\text{Yuk}}^{\text{quarks}(1)} = -\overline{\Psi}_{i,\text{R}}^{D} \hat{C}_{\text{quarks}} \phi^{\dagger} \Psi_{i,\text{L}}^{UD} + \text{h.c.}$$

$$\mathscr{L}_{\text{Yuk}}^{\text{quarks}(2)} = -\overline{\Psi}_{i,\text{R}}^{U} \hat{C}'_{\text{quarks}} \phi^{T} \hat{\varepsilon} \Psi_{i,\text{L}}^{UD} + \text{h.c.}, \quad \hat{\varepsilon} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- can redefine the basis of family members with equal quantum numbers

$$\hat{C}_{\text{lept}} \to \hat{U}_1^{\dagger} \hat{C}_{\text{lept}} \hat{V}_1, \quad \hat{C}'_{\text{quarks}} \to \hat{U}_2^{\dagger} \hat{C}'_{\text{quarks}} \hat{V}_2, \quad \hat{C}_{\text{quarks}} \to \hat{U}_3^{\dagger} \hat{C}_{\text{quarks}} \hat{V}_2,$$

$\hat{U}_j, \hat{V}_k \in \text{U}(3)$

- standard choice

$$\hat{C}_{\text{lept}} = \text{diag}(c_e, c_\mu, c_\tau) \quad \text{with} \quad c_e, c_\mu, c_\tau \in \mathbb{R}_{>0},$$

$$\hat{C}'_{\text{quarks}} = \text{diag}(c_u, c_c, c_t) \quad \text{with} \quad c_u, c_c, c_t \in \mathbb{R}_{>0},$$

$$\hat{C}_{\text{quarks}} = \hat{V} \text{diag}(c_d, c_s, c_b) \hat{V}^{\dagger} \quad \text{with} \quad c_d, c_s, c_b \in \mathbb{R}_{>0}, \quad \hat{V} \in \text{U}(3)$$

- $\hat{V}$: Cabibbo-Kobayashi-Maskawa matrix (3 mixing angles + 1 CP-viol. phase)

Leptons and quarks

- matter Lagrangian in terms of physical fields

$$\mathscr{L}_{\text{matter-gauge bosons}} = \overline{\Psi} \mathrm{i} \not{\partial} \Psi - e \left\{ A_\mu J^\mu_{\text{em}} + \frac{1}{\sin\theta_{\text{W}} \cos\theta_{\text{W}}} Z_\mu J^\mu_{\text{NC}} + \frac{1}{\sqrt{2}\sin\theta_{\text{W}}} (W^+_\mu J^\mu_{\text{CC}} + W^-_\mu J^\dagger_{\text{CC}}) \right\}$$

- with the currents

$$J^\mu_{\text{em}} = \overline{\Psi} \gamma^\mu (\hat{T}_3 + \hat{Y}) \Psi,$$

$$J^\mu_{\text{NC}} = \overline{\Psi} \gamma^\mu \left[\hat{T}_3 - \sin^2\theta_{\text{W}} (\hat{T}_3 + \hat{Y}) \right] \Psi,$$

$$J^\mu_{\text{CC}} = \overline{\Psi} \gamma^\mu (\hat{T}_1 + \mathrm{i} \hat{T}_2) \Psi,$$

Leptons and quarks

- fields for particles of definite mass
  - massive leptons and quarks: ψ_j, $\psi_{j,L} = (1-\gamma_5)\psi_j/2$, $\psi_{j,R} = (1+\gamma_5)\psi_j/2$ $(j \in \{e,\mu,\tau,u,d,c,s,t,b\})$
  - quarks: mass (unprimed) vs. flavor eigenstates (primed) $\psi'_{l'} = V_{l'l}\psi_l$ $l',l \in \{d,s,b\}$, $\hat{V} \in \mathrm{U}(3)$: CKM mixing matrix
  - neutrinos (treated as massless): only left-handed part $\nu_{k,\mathrm{L}}$ $(k \in \{e,\mu,\tau\})$
- Yukawa terms

$$\mathscr{L}_{\text{Yuk}} = -(\overline{\psi}_e, \overline{\psi}_\mu, \overline{\psi}_\tau)\text{diag}(m_e, m_\mu, m_\tau)(\psi_e, \psi_\mu, \psi_\tau) - \overline{\psi}_q \text{diag}(m_u, m_d, \ldots, m_b)\psi_q$$

- masses: $m_j = c_j h_0/\sqrt{2}$
  - NB: most of the mass of matter surrounding us is not from Higgs mechanism!
  - “elementary” (“current”) light-quark masses: $m_u \simeq$ 1.7-3.3 MeV, $m_d \simeq$ 4.1-3.3MeV
  - proton: bound state of *uud* but mass $m_p \simeq 938$ MeV
  - most of the proton mass dynamically generated by strong interaction!

Quarks and leptons

- currents in terms of mass eigenstates ($\psi_{R/L} = (1 \pm \gamma_5)/2$)

$$J^{\mu}_{\text{CC}} = (\overline{\nu}_{e,\text{L}}, \overline{\nu}_{\tau,\text{L}}, \overline{\nu}_{\tau,\text{L}})\gamma^{\mu} \begin{pmatrix} \psi_{e,\text{L}} \\ \psi_{\nu,\text{L}} \\ \psi_{\tau,\text{L}} \end{pmatrix} + (\overline{\psi}_{u,\text{L}}, \overline{\psi}_{c,\text{L}}, \overline{\psi}_{t,\text{L}})\gamma^{\mu} \hat{V} \begin{pmatrix} \psi_{d,\text{L}} \\ \psi_{s,\text{L}} \\ \psi_{b,\text{L}} \end{pmatrix}$$

$$\begin{aligned} J_{\text{NC}} = {} & (\overline{\nu}_e, \overline{\nu}_\tau, \overline{\nu}_\tau)\gamma^{\mu} \frac{1}{2} \frac{1-\gamma_5}{2} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \\ & + (\overline{\psi}_e, \overline{\psi}_\nu, \overline{\psi}_\tau)\gamma^{\mu} \left(-\frac{1}{2} \frac{1-\gamma_5}{2} + \sin^2\theta_{\text{W}} \right) \begin{pmatrix} \psi_{\text{e}} \\ \psi_\mu \\ \psi_\tau \end{pmatrix} \\ & + (\overline{\psi}_u, \overline{\psi}_c, \overline{\psi}_t) \left(\frac{1}{2} \frac{1-\gamma_5}{2} - \frac{2}{3}\sin^2\theta_{\text{W}} \right) \begin{pmatrix} \psi_u \\ \psi_c \\ \psi_t \end{pmatrix} \\ & + (\overline{\psi}_d, \overline{\psi}_s, \overline{\psi}_b) \left(-\frac{1}{2} \frac{1-\gamma_5}{2} + \frac{1}{3}\sin^2\theta_{\text{W}} \right) \begin{pmatrix} \psi_d \\ \psi_s \\ \psi_b \end{pmatrix} \end{aligned}$$

- no flavor-changing NC $\Leftrightarrow$ Glashow-Iliopoulous-Maiani (GIM) mechanism

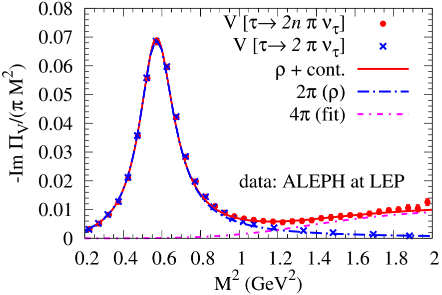
Quarks and leptons

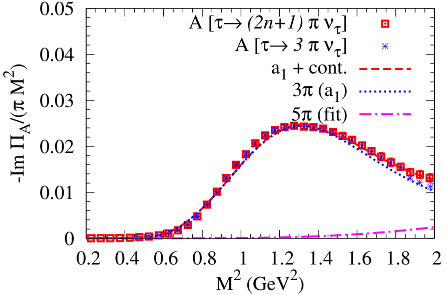
- currents in terms of mass eigenstates ($\psi_{R/L} = (1 \pm \gamma_5)/2$)
- electromagnetic current

$$J^{\mu}_{\text{em}} = -(\overline{\psi}_e, \overline{\psi}_\nu, \overline{\psi}_\tau)\gamma^{\mu} \begin{pmatrix} \psi_{\text{e}} \\ \psi_{\mu} \\ \psi_{\tau} \end{pmatrix}$$
$$+ \frac{2}{3}(\overline{\psi}_u, \overline{\psi}_c, \overline{\psi}_t)\gamma^{\mu} \begin{pmatrix} \psi_u \\ \psi_c \\ \psi_t \end{pmatrix}$$
$$- \frac{1}{3}(\overline{\psi}_d, \overline{\psi}_s, \overline{\psi}_b)\gamma^{\mu} \begin{pmatrix} \psi_d \\ \psi_s \\ \psi_b \end{pmatrix}.$$

Most accurate experiment related to χSB

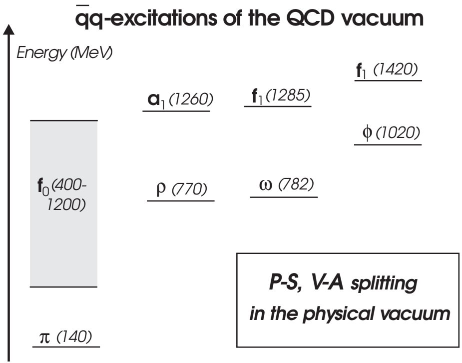
- weak decay $\tau \rightarrow \nu + n \cdot \pi$
- weak interactions: charged currents $\propto j_V^\mu - j_A^\mu$
- n even: must go through vector current
  n odd: must go through axialvector current





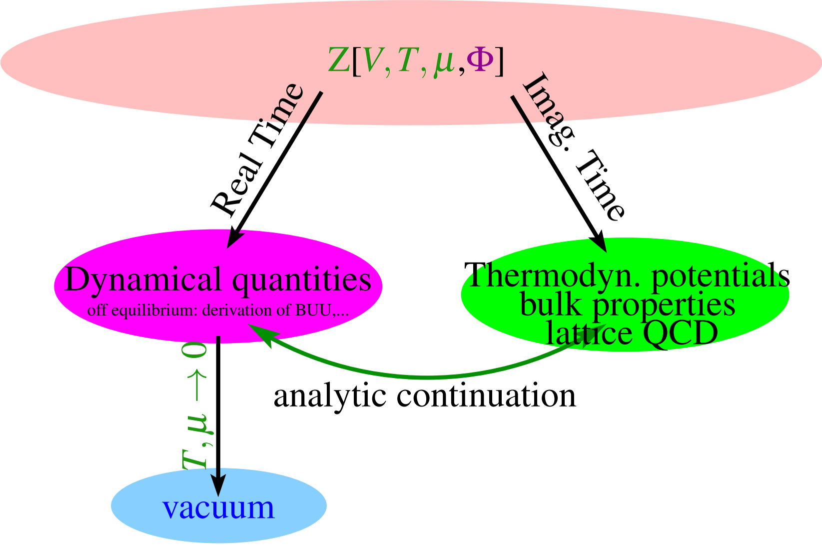
Phenomenology from Chiral Symmetry

- Use (approximate) chiral symmetry to build effective models
- Ward identities
  - PCAC: $\left\langle 0 \left| \partial^\mu j^k_{A\mu} \right| \pi^j(\vec{k}) \right\rangle = \mathrm{i} F_\pi^2 m_\pi^2 \delta^{kj}$
  - $m_\pi^2 F_\pi^2 = -(m_u + m_d) \langle 0 | \overline{u} u | 0 \rangle$
    (Gell-Mann-Oakes-Renner relation)
- Spontaneous breaking causes splitting of chiral partners:



Finite Temperature/Density: Idealized theory picture

- partition sum: $Z(V,T,\mu_q,\Phi) = \mathrm{Tr}\{\exp[-(\mathrm{H}[\Phi] - \mu_q \mathrm{N})/T]\}$



[CSHY85, Lv87, LeB96, KG06]

Finite Temperature

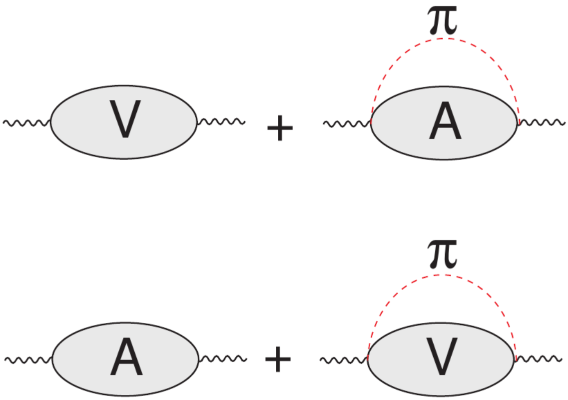
- Asymptotic freedom
  - quark condensate melts at high enough temperatures/densities
- all bulk properties from partition sum:

$$Z(V,T,\mu_q) = \mathrm{Tr}\{\exp[-(\mathrm{H}-\mu_q \mathrm{N})/T]\}$$

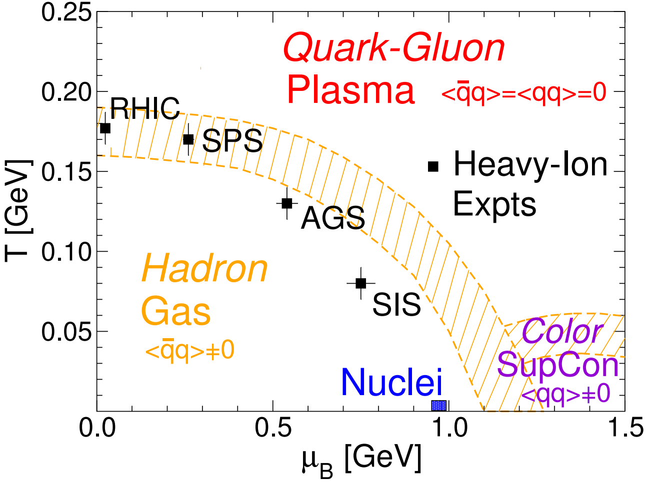
- Free energy: $\Omega = -\frac{T}{V} \ln Z = -P$
- Quark condensate: $\left\langle \overline{\psi}_q \psi_q \right\rangle_{T,\mu_q} = \frac{V}{T} \frac{\partial P}{\partial m_q}$
- Lattice QCD (at $\mu_q = 0$)
  - chiral symmetry $\Leftrightarrow \langle \overline{\psi} \psi \rangle$
  - deconfinement transition $\Leftrightarrow$ Polyakov Loop $\mathrm{tr} \left\langle P \exp(\mathrm{i} \int_0^\beta \mathrm{d}\tau A^0) \right\rangle$
  - Chiral symmetry restoration and deconfinement transition at same T_c

Vector-Axialvector Mixing in the Medium

- in the medium: vector-axialvector currents mix
- due to thermal pions
- possible mechanism for χSR!
- in low-density/temperature approximation: model independent
- see [DEI90a, DEI90b, UBW02, SYZ96, SYZ97]



The QCD Phase Diagram



What can we learn from em. probes in heavy-ion collisions?

- only penetrating probe
  - leptons and photons leave hot and dense fireball unaffected
  - they are produced during the entire fireball evolution
  - dileptons provide information on in-medium spectral properties of hadrons
- theoretical challenge
  - need an understanding of QCD medium at all stages of its evolution
    $\Rightarrow$ transport models, hydrodynamics
  - need to identify all sources of dileptons and photons
  - perturbative QCD not applicable
    $\Rightarrow$ non-perturbative QCD, effective hadronic models
  - evaluate dilepton and photon rates $\Rightarrow$ QFT at finite T and μ_{B}

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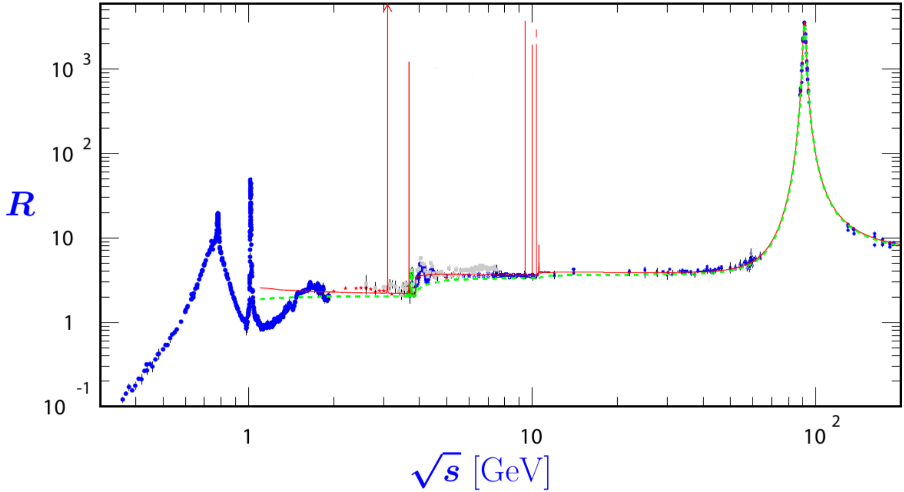
Quiz

1. Why do we want to measure dileptons in HICs?
2. What are the peaks in the following figure of $R_{e^+e^- \to \mathrm{hadrons}}$?
3. Can you explain the horizontal lines (values: 2, 3.333, 3.667)?



Quiz

4. What are the “fundamental” and “accidental” symmetries of QCD?
5. What’s chiral symmetry?
6. Why is it (intuitively) only true for massless quarks?
7. What’s the main consequence of spontaneous symmetry breaking?
8. Why can one measure the vector and axial-vector current-current correlators from $\tau \rightarrow$ even/odd number of pions $+ \nu$?