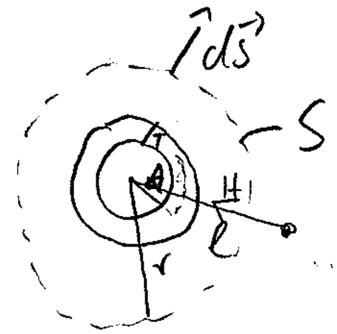
(1) We use Gauss's law with spheres as Gaussian surfaces



From the spherical symmetry we know that

$$\vec{E} = E(r)\,\vec{i}_r \quad \text{(spherical coordinates)}$$

Since $d\vec{S} = r^2 \sin\vartheta \, d\vartheta \, d\varphi$

We have

$$\oint_S d\vec{S}\cdot\vec{E} = \int_0^{\pi} d\vartheta \int_0^{2\pi} d\varphi \; r^2 \sin\vartheta \, E(r)$$

$$= 4\pi r^2 E(r) \overset{!}{=} \frac{Q_{inside}}{\varepsilon_0}$$

Now

$$Q_{inside} = \begin{cases} 0 & \text{for } r < A \\ \dfrac{Q}{\frac{4}{3}\pi\left[(A+T)^3 - A^3\right]}\,\dfrac{4}{3}\pi\,(r^3 - A^3) & \text{for } A < r < A+T \\ Q & \text{for } r > A+T \end{cases}$$

$$= \begin{cases} 0 & \text{for } r < A \\ Q\,\dfrac{r^3 - A^3}{\left[(A+T)^3 - A^3\right]} & \text{for } A < r < A+t \\ Q & \text{for } r > A+t \end{cases}$$

So we have

$$E(r) = \begin{cases} 0 & \text{for } r < A \\ Q\dfrac{r^3 - A^3}{[(A+T)^3 - A^3]}\dfrac{1}{4\pi\varepsilon_0 r^2} & \text{for } A \le r \le A+T \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & \text{for } r > A+T \end{cases}$$

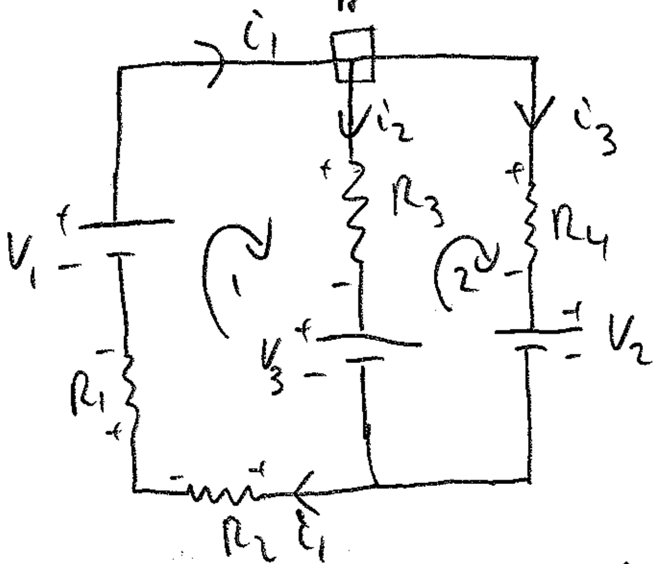
$$\Delta V = V(0) - V(H) = -\int_{\ell} d\vec{r}\,\vec{E}(\vec{r})$$

$$= \int_A^{A+T} dr\, Q\,\frac{r^3 - A^3}{[(A+T)^3 - A^3]}\,\frac{1}{4\pi\varepsilon_0 r^2} + \int_{A+T}^{H} dr\,\frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\Rightarrow \Delta V = \frac{Q}{4\pi\varepsilon_0[(A+T)^3 - A^3]}\left\{\frac{1}{2}\left[(A+T)^2 - A^2\right] - A^3\left(\frac{1}{A} - \frac{1}{A+T}\right)\right\}$$

$$+ \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{A+T} - \frac{1}{H}\right]$$

(2)



(a) Using Kirchhoff II for loop 1 gives

$$V_1 - R_3 i_2 - V_3 - R_2 i_1 - R_1 i_1 = 0$$

given that $i_2 = 0$, we find

$$i_1 = \frac{V_1 - V_3}{R_1 + R_2}$$

(b) Kirchhoff II for loop 2

$$V_3 + R_3 i_2 - R_4 i_3 - V_2 = 0$$

Kirchhoff I for node A:

$$i_1 = i_2 + i_3$$

With $i_2 = 0$ and $i_1 = i_3 = \dfrac{V_1 - V_3}{R_1 + R_2}$

$$V_3 = V_2 + R_4 i_3 = V_2 + \frac{R_4}{R_1 + R_2}(V_1 - V_3)$$

$$\Rightarrow V_3 \left(1 + \frac{R_4}{R_1 + R_2}\right) = V_2 + \frac{R_4}{R_1 + R_2} V_1$$

$$\Rightarrow V_3 \left(\frac{R_1 + R_2 + R_4}{R_1 + R_2}\right) = V_2 + \frac{R_4}{R_1 + R_2} V_1$$

$$\Rightarrow V_3 = \frac{(R_1 + R_2) V_2 + R_4 V_1}{R_1 + R_2 + R_4}$$

(c) Then still no current flows through R_3, i.e. all results above apply, and we must have

$$Q = C V_3 = C \frac{(R_1 + R_2) V_2 + R_4 V_1}{R_1 + R_2 + R_4}$$

(3) 1st configuration: 2nd configuration:

$$R_1 = \frac{\rho L}{W H} \qquad R_2 = \frac{\rho H}{W L}$$

(a) $i_2 = \frac{V}{R_2} = 4 i_1 = \frac{4V}{R_1}$

$$\Rightarrow \frac{R_1}{R_2} = 4 \Rightarrow 4 = \frac{\cancel{\rho} L}{\cancel{W} H} \frac{\cancel{W} L}{\cancel{\rho} H} = \left(\frac{L}{H}\right)^2$$

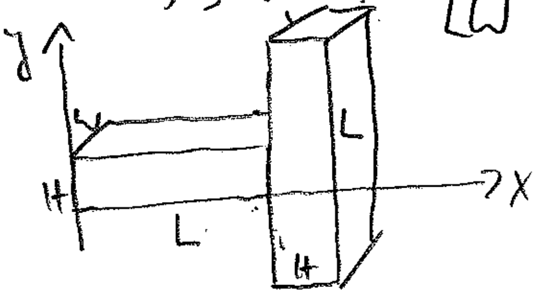
$$\Rightarrow L = 2H$$

(b) Series of resistors:

$$R = R_1 + R_2 = \frac{\rho}{W}\left(\frac{L}{H} + \frac{H}{L}\right) = \frac{5}{2}\frac{\rho}{W}$$

$$i = \frac{V}{R} = \frac{2W}{5\rho} V$$

$$\vec{j} = \frac{2WV}{5\rho}\vec{i}_x \cdot \begin{cases} \frac{1}{HW} & \text{for } 0 \le x < L \\ \frac{1}{LW} & \text{for } L < x \le L+H \end{cases}$$



$$\vec{E} = \rho \vec{j} = \frac{2V\vec{i}_x}{5} \cdot \begin{cases} \frac{1}{H} & \text{for } 0 \le x < L \\ \frac{1}{L} & \text{for } L < x \le L+H \end{cases}$$

(c) $$\frac{\sigma}{\varepsilon_0} = \vec{i}_x \cdot \left[\vec{E}(L+\varepsilon) - \vec{E}(L-\varepsilon)\right]$$

$$= \frac{2V}{5}\left(\frac{1}{L} - \frac{1}{H}\right)$$

$$Q = \sigma H W = \frac{2V\varepsilon_0 W}{5}\left(\frac{H}{L} - 1\right) = -\frac{\varepsilon_0 W V}{5}$$

(4) (a) charges spread over inner cylinder (outer radius)

$$\sigma_1 = \frac{Q}{2\pi L(A+T)} \quad \text{at } r = A+T$$

Induces charges on inner edge of outer cylinder

$$\sigma_2 = -\frac{Q}{2\pi L B} \quad \text{at } r = B$$

and

$$\sigma_3 = + \frac{Q}{2\pi L(B+T)} \quad \text{at } r = B+T$$

(b) $$\vec{E} = \begin{cases} \dfrac{Q}{2\pi\varepsilon_0 L r}\,\vec{i}_r & \text{for } A+T < r < B \text{ and } r > B+T \\ 0 & \text{everywhere else} \end{cases}$$

$$\Rightarrow \Delta V = \int_{A+T}^{B} dr\, \frac{Q}{2\pi\varepsilon_0 L r} = \frac{Q}{2\pi\varepsilon_0 L} \ln\left(\frac{B}{A+T}\right)$$

(c) $$C = \frac{Q}{\Delta V} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{B}{A+T}\right)}$$