Gauss's Law
Some Figures for Lecture PHYS-208

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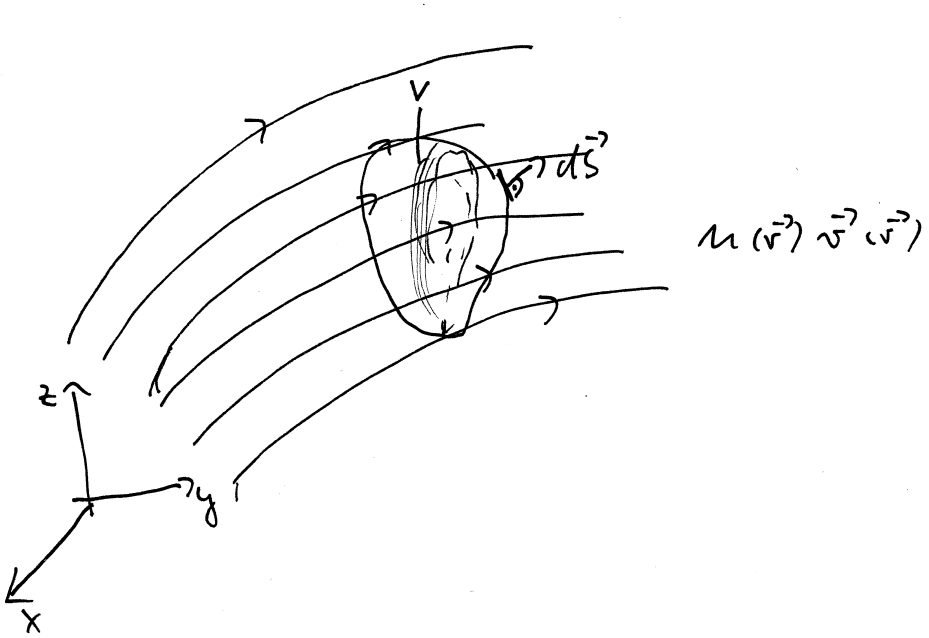
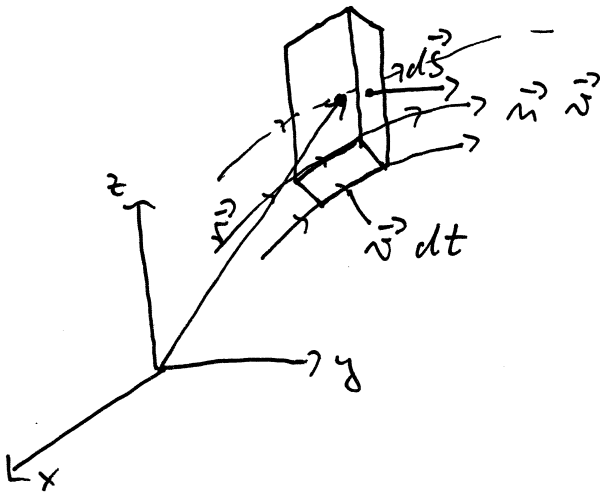
Fluid Flow



Flow of a fluid with particle density, n and velocity field $\vec{v}$ through a volume, V.

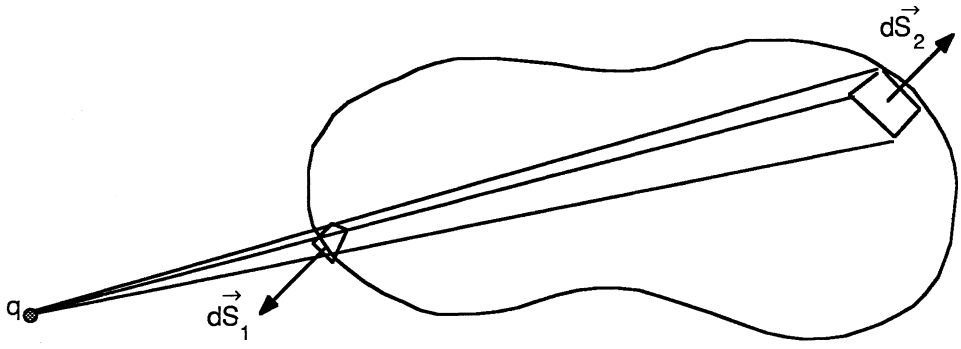
Particle Flux through Surface Element



- In time interval $\mathrm{d}t$ all particles inside the volume element $\mathrm{d}V = \vec{v}(\vec{r})\ \mathrm{d}\vec{S}\ \mathrm{d}t$ flow through surface.
- number of those particles: $\mathrm{d}N = n(\vec{r})\mathrm{d}V$
- total number of particles per time, flowing out of V:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \text{flux} = \int_{\partial V} \mathrm{d}\vec{S}\ n(\vec{r})\vec{v}(\vec{r}), \quad \partial V\text{: boundary of } V$$

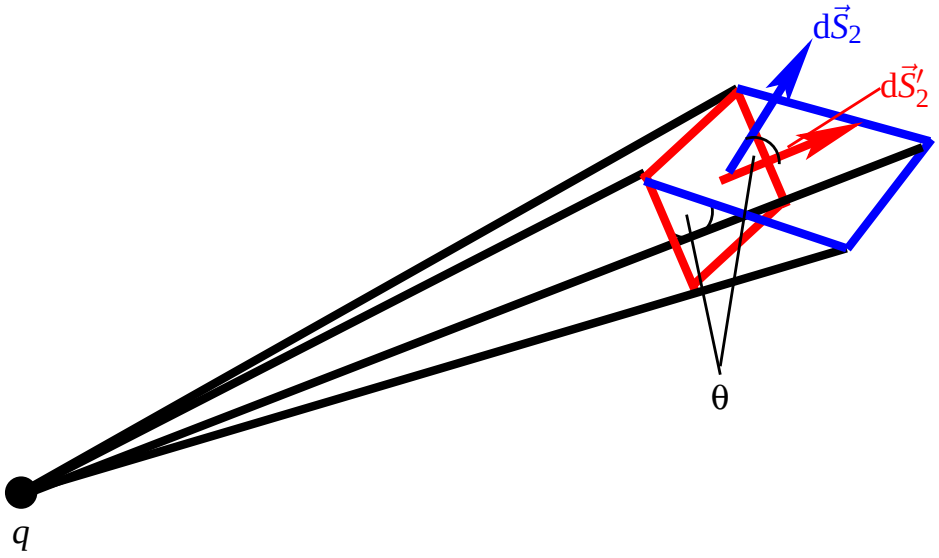
Gauss's Law (Proof for point particle outside V I)



- split ∂V in rectangular surface elements
- draw cone through surface elements $\mathrm{d}\vec{S}_1$ and $\mathrm{d}\vec{S}_2$

Figure from: W. H. Bassichis, DON'T PANIC, Vol. II, 5th Edition, OR Publishing (2005)

Gauss's Law (Proof for point particle outside V II)



- Surface element from ∂V (blue) with its cone (black) compared to intersection of cone with sphere (red)

$$|\mathrm{d}\vec{S}'_2| = \vec{i}_r \mathrm{d}\vec{S}'_2 = |\mathrm{d}\vec{S}_2| \cos\theta = \vec{i}_r \mathrm{d}\vec{S}_2$$