Name: .... Section: ....

Physics 208 Quiz 7

April 04, 2007; due April 11

Problem 1 (20 Points)

Take the magnetic field of an infinitely long current-carrying wire, which reads in our standard cylinder coordinates

$$\vec{B} = \frac{\mu_0 i}{2\pi\rho}\vec{i}_\varphi(\varphi). \tag{1}$$

(a) Express the field in terms of Cartesian coordinates.

(b) Express the field in terms of spherical coordinates.

(c) Show that the $\vec{B}$ field fulfills the condition

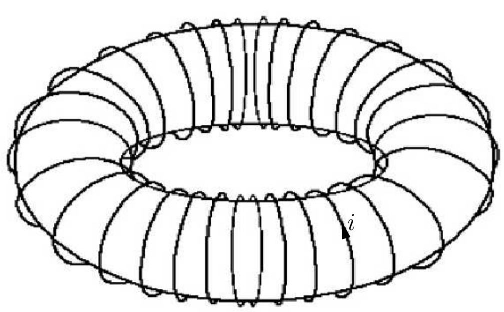
$$\oint_{S_R} \mathrm{d}\vec{S}\cdot\vec{B} = 0, \tag{2}$$

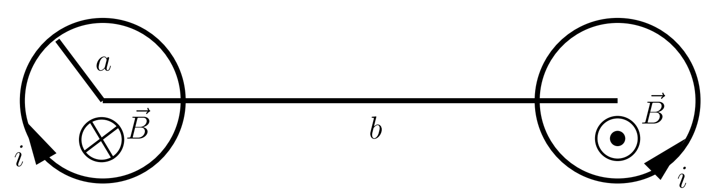
where S_R is the sphere (surface) with radius R around the origin of the coordinate system.

Note: As explained in the lecture, Eq. (2) expresses the fact that there are no magnetic monopoles found in nature. This is one of the fundamental laws of electromagnetism which is always valid, not only for time-independent fields. It is one of Maxwell's equations (in integral form)!

Problem 2 (40 Points)

A wire is wrapped N times around a torus ("donut") as shown in the figures, and a current is made running through the coil as indicated.





Use Ampère's Circuital Law to calculate the magnetic field, $\vec{B}$, everywhere!

See problem 3 on next page!

Problem 3 (40 Points)

Use the Biot-Savart Law to calculate the magnetic field, $\vec{B}(z)$, along the axis of a tightly wound cylindrical coil of radius, R, and finite length, L, carrying a current, i. Discuss the limiting cases $L \gg R$ and $L \ll R$.

For extra credit: Can you find the magnetic dipole moment by taking $z \gg L$?

Hints: If A is the cross-sectional area of the wire, the current-density vector per unit cylinder length (in standard-cylinder coordinates) is given by

$$\vec{j}\,' = \frac{Ni}{LA}\vec{i}_\varphi, \tag{3}$$

where N is the total number of loops. In the Biot-Savart Law, at each point along the cylinder axis you have to integrate over the volume of the wire looping around the cylinder there, but the volume element simplifies to $\mathrm{d}^3\vec{r}\,' = AR\mathrm{d}\varphi'$. Finally you have to integrate along the cylinder axis to sum up all loops. Thus, here the Biot-Savart Law takes the form

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int_{-L/2}^{L/2}\mathrm{d}z'\int_0^{2\pi}\mathrm{d}\varphi' AR\frac{\vec{j}\,'(\vec{r}\,')\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}, \tag{4}$$

where I have written $\vec{r}\,'$ in terms of cylinder coordinates ρ', φ', z'.

You can use the integrals

$$\int \mathrm{d}\varphi'\,\vec{i}_\rho(\varphi') = 0, \quad \int \mathrm{d}x\frac{1}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}. \tag{5}$$

Note also that you can solve this problem only along the symmetry axis of the coil, i.e., for $\vec{r} = z\vec{i}_z$. So do not struggle to calculate the field elsewhere!