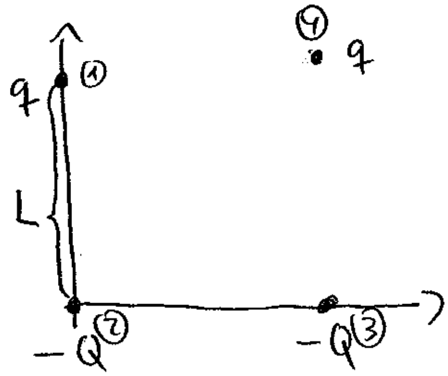
Problems chapter 1

1. $F_r = \dfrac{q^2}{4\pi\varepsilon_0 r^2} \Rightarrow q = \sqrt{4\pi\varepsilon_0 F}\cdot r = 2\cdot 10^{-6}\,C$

2. $\vec{F} = \dfrac{q_0 q}{4\pi\varepsilon_0 (a^2+H^2)^{3/2}}\Big[\underbrace{a\vec{i}_x + H\vec{i}_y}_{\text{left charge}}\ \underbrace{-a\vec{i}_x + H\vec{i}_y}_{\text{right charge}}\Big]$

$$= \frac{q_0 q}{4\pi\varepsilon_0}\,\frac{2H}{(a^2+H^2)^{3/2}}\,\vec{i}_y$$

3.



$$\vec{F}_{14} = \frac{q^2}{4\pi\varepsilon_0 L^2}\,\vec{i}_x$$

$$\vec{F}_{24} = -\frac{qQ}{4\pi\varepsilon_0 (2L^2)}\,\frac{1}{\sqrt{2}}\left(\vec{i}_x + \vec{i}_y\right)$$

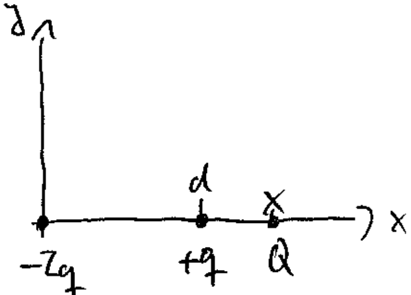
$$\vec{F}_{34} = -\frac{qQ}{4\pi\varepsilon_0 L^2}\,\vec{i}_y$$

$$\vec{F}_{tot} = \vec{F}_{14} + \vec{F}_{24} + \vec{F}_{34} = \frac{q}{4\pi\varepsilon_0 L^2}\left[\left(q - \frac{Q}{2\sqrt{2}}\right)\vec{i}_x - Q\left(1 + \frac{1}{2\sqrt{2}}\right)\vec{i}_y\right]$$

$$F_x = 9\cdot 10^9\,\frac{1.5\cdot 10^{-7}}{0.01}\left(1.5\cdot 10^{-7} - \frac{3\cdot 10^{-7}}{2\sqrt{2}}\right) \simeq 5.93\cdot 10^{-3}\,N$$

$$F_y = -9\cdot 10^9 \, \frac{3\cdot 1.5\cdot 10^{-14}}{0.01}\left(1+\frac{1}{2\sqrt{2}}\right) \approx 54.82\cdot 10^{-3}\,C$$

4.



(a) look for solution for charge Q on x-axis

Suppose Q is positive

Then: $\vec{F} = F_x \vec{i}_x$ with

$$F_x = \frac{1}{4\pi\varepsilon_0}\left[-\frac{2qQ}{x^2} + \frac{\operatorname{sign}(x-d)\,qQ}{(x-d)^2}\right]$$

Case 1: $x > d$:

$$-\frac{2}{x^2} + \frac{1}{(x-d)^2} = 0$$

$$(x-d)^2 = \frac{x^2}{2}$$

$$\frac{x^2}{2} - 2dx + d^2 = 0$$

$$x^2 - 4dx + 2d^2 = 0$$

$$x_{1/2} = 2d \pm \sqrt{4d^2 - 2d^2}$$

$$= (2 \pm \sqrt{2})\,d$$

Since we assumed $x > d$, we must have the upper sign

Then $x = (2+\sqrt{2})\,d \Rightarrow x \approx 3.41\,d$

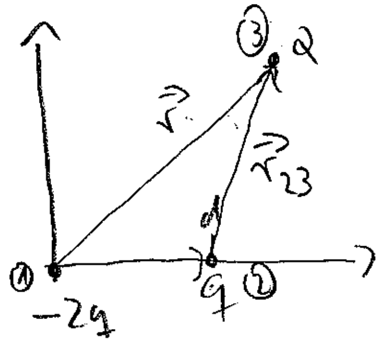
case 2: $x < d$

Everything remains the same, but we have

$$-\frac{2}{x^2}-\frac{1}{(x-d)^2}=0$$

which never can be fulfilled since the LHS is always < 0 for all $x \in \mathbb{R}$.

(b) how about placing Q somewhere away from the x-axis? We can choose the y axis such that Q is in the xy plane. Then we have



$$\vec{r} := x\vec{i}_x + y\vec{i}_y$$

$$F_{13,x} = -\frac{2qQ}{4\pi\varepsilon_0}\frac{x}{(x^2+y^2)^{3/2}}$$

$$F_{13,y} = -\frac{2qQ}{4\pi\varepsilon_0}\frac{y}{(x^2+y^2)^{3/2}}$$

$$\vec{r}_{23} = \vec{r} - d\vec{i}_x = (x-d)\,\vec{i}_x + y\,\vec{i}_y$$

$$F_{23,x} = \frac{qQ}{4\pi\varepsilon_0}\frac{x-d}{[(x-d)^2+y^2]^{3/2}}$$

$$F_{23,y} = \frac{qQ}{4\pi\varepsilon_0}\frac{y}{[(x-d)^2+y^2]^{3/2}}$$

Thus we must have

$$F_{13,y} + F_{23,y} = 0$$

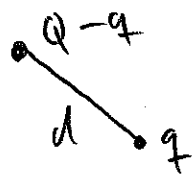
$$\Rightarrow \frac{2y}{r^3} = \frac{y}{r_{23}^3}$$

If $y \neq 0$, then we must have

$$r = r_{23}$$

or $x^2 + y^2 = (x-d)^2 + y^2 \Rightarrow x^2 = (x-d)^2$ which cannot be fulfilled. Thus the only possible solution is the one where q is located also on the y axis!

(5)



$$F = \frac{1}{4\pi\varepsilon_0} \frac{(Q-q)\,q}{d^2}$$

Find q such that F becomes maximal. Thus we have to maximize

$$f(q) = (Q-q)q = Qq - q^2$$

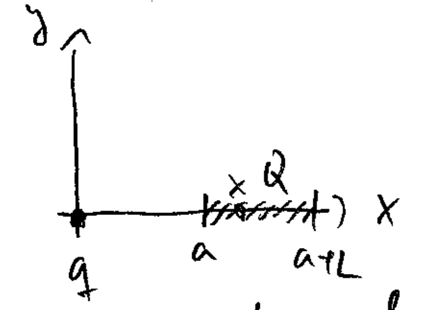
Derivative must be 0:

$$f'(q) = Q - 2q \stackrel{!}{=} 0 \Rightarrow q = \frac{Q}{2}$$

Check whether we have maximum

$$f''(q) = -2 < 0 \Rightarrow \text{maximum}$$

(6)



Superposition for each charge element $dQ = \frac{Q}{L}dx$

$$\vec{F} = F_x \vec{i}_x$$

$$F_x = -\int_a^{a+L} \frac{1}{4\pi\varepsilon_0}\, q \cdot \frac{Q}{L}dx\, \frac{1}{x^2}$$

$$= -\frac{qQ}{4\pi\varepsilon_0 L}\int_a^{a+L}\frac{dx}{x^2} = -\frac{qQ}{4\pi\varepsilon_0 L}\left(-\frac{1}{x}\right)_a^{a+L}$$

$$= -\frac{qQ}{4\pi\varepsilon_0 L}\left(\frac{1}{a} - \frac{1}{a+L}\right)$$

$$= -\frac{qQ}{4\pi\varepsilon_0 L}\,\frac{a+L-a}{a(a+L)}$$

$$F_x = -\frac{qQ}{4\pi\varepsilon_0\, a(a+L)}$$

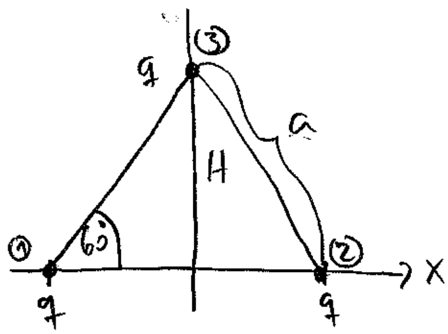
Exercises Chapter 1

(1) $-Q = 10 \cdot 10^{23} \cdot 1.6 \cdot 10^{-19}\,C = 1.6 \cdot 10^{5}\,C$

$$F = \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{1m^2} = 9 \cdot 10^9 \cdot (1.6 \cdot 10^5)^2\,N = 2.304 \cdot 10^{20}\,N$$

$(\approx 5.17 \cdot 10^{19}\,lbs)$

(2)



$$\vec{F}_{13} = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{a^2} \left(\cos 60^\circ \vec{i}_x + \sin 60^\circ \vec{i}_y \right)$$

$$\vec{F}_{23} = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{a^2} \left(-\cos 60^\circ \vec{i}_x + \sin 60^\circ \vec{i}_y \right)$$

$$\vec{F}_{tot} = \vec{F}_{13} + \vec{F}_{23} = \frac{1}{4\pi\varepsilon_0} \frac{2q^2}{a^2} \sin 30^\circ \vec{i}_y$$

$$\sin 60^\circ = \frac{H}{a} = \frac{\sqrt{a^2 - \frac{a^2}{4}}}{a} = \frac{\sqrt{3}}{2}$$

$$F_y = \frac{1}{4\pi\varepsilon_0} \frac{\sqrt{3}\,q^2}{a^2} = 9 \cdot 10^9 \cdot \frac{\sqrt{3}\,(6 \cdot 10^{-5})^2}{0.12^2}\,N$$

$$\approx 3.9 \cdot 10^3\,N$$

③

$$\vec{F}_{34} = \frac{1}{4\pi\varepsilon_0} \frac{q_3 q_4}{b^2} \vec{i}_y$$

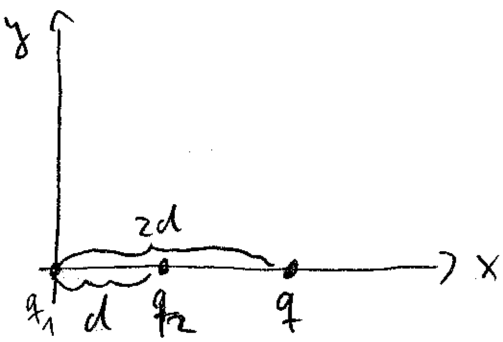
$$\vec{F}_{14} = 1.97 \cdot 10^{-4} N \vec{i}_x$$

$$\vec{F}_{24} = 1.41 \cdot 10^{-4} N \vec{i}_x + 1.06 \cdot 10^{-4} N \vec{i}_y$$

$$\vec{F}_{34} = 3.5 \cdot 10^{-4} N \vec{i}_y$$

$$\vec{F}_{kr} = 3.38 \cdot 10^{-4} N \vec{i}_x + 4.56 \cdot 10^{-4} N \vec{i}_y$$

(5)



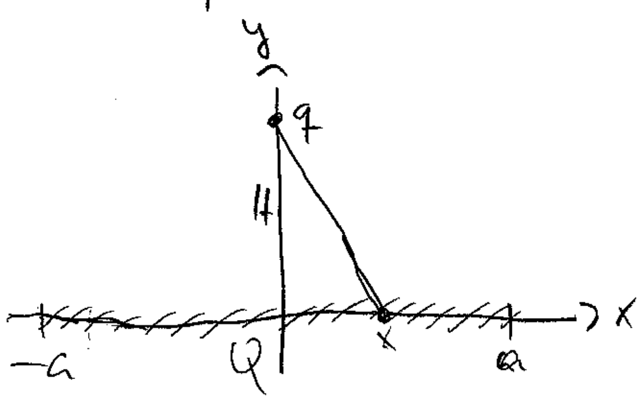
$$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0} \frac{q q_2}{d^2} (+\vec{i}_x)$$

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0} \frac{q q_1}{4d^2} (+\vec{i}_x)$$

$$\vec{F}_{kr} = \vec{F}_1 + \vec{F}_2 = 0$$

$$\Rightarrow \quad q_2 + \frac{q_1}{4} = 0 \Rightarrow q_2 = -\frac{q_1}{4}$$

(6)



$$dQ = \frac{Q}{2a}\, dx$$

$$\vec{F} = \int_{-a}^{a} dx \, \frac{qQ}{2a} \frac{1}{4\pi\varepsilon_0} \frac{1}{(x^2+H^2)^{3/2}} \begin{pmatrix} -x \\ H \end{pmatrix}$$

$$= \frac{qQH}{8\pi\varepsilon_0 a} \int_{-a}^{a} dx \begin{pmatrix} 0 \\ 1 \end{pmatrix} \frac{1}{(x^2+H^2)^{3/2}}$$

$$F_y = \frac{qQH}{4\pi\varepsilon_0 a} \int_{0}^{a} dx \, \frac{1}{(x^2+H^2)^{3/2}}$$

$$= \frac{qQH}{4\pi\varepsilon_0 a} \left. \frac{x}{H^2\sqrt{x^2+H^2}} \right|_0^a$$

$$= \frac{qQ}{4\pi\varepsilon_0 a H} \frac{a}{\sqrt{a^2+H^2}}$$

$$F_y = \frac{qQ}{4\pi\varepsilon_0 H\sqrt{H^2+a^2}}$$