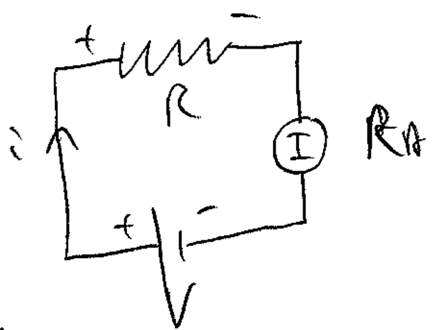
Problems Chapt. VIII

(1) Let R_A denote the resistance of the Amp meter. Then according to Kirchhoff's 2nd law



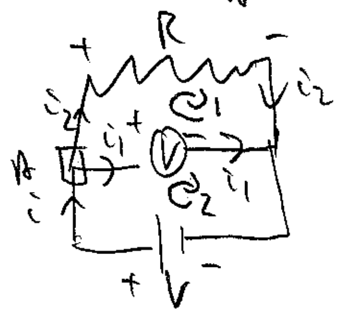
we have

$$V - iR - iR_A = 0 \Rightarrow i = \frac{V}{R+R_A}$$

Only for $R_A \ll R$ we have $i = \frac{V}{R}$. Thus an ideal amp meter has $R_A = 0$ inner resistance. Then the power dissipated in the Amp meter would be

$$P_A = i\, U_A = R_A i^2 = 0$$

(2)



R_V : resistance of volt meter

A : $i_1 + i_2 - i = 0$

1 : $-R i_2 + R_V i_1 = 0$

2 : $V - R_V i_1 = 0 \Rightarrow R_V i_1 = V$

$V = R i_2$

$i = V\left(\frac{1}{R} + \frac{1}{R_V}\right)$

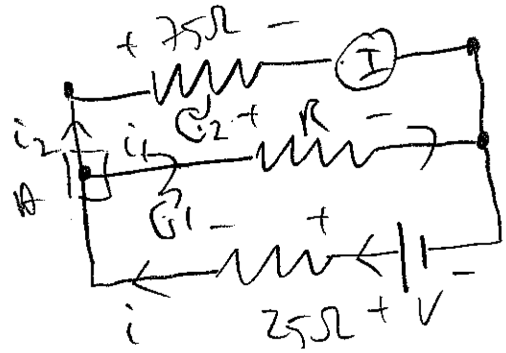
which goes to $\frac{V}{R}$ for $R_V \to \infty \Rightarrow$ Ideal volt meter has infinite resistance. Since then $i_1 = \frac{V}{R_V} \to 0$, no power is dissipated on the voltmeter in this case.

(3) If both switches are open, we have

$$i = \frac{V}{(50+25+75)\Omega} = \frac{V}{150\Omega} = 20\,mA$$

That's the current through the 75Ω resistor.

If both switches are closed the effective circuit is



$$A: \quad i = i_1 + i_2$$

$$1: \quad V - 25\Omega\, i - R i_1 = 0$$

$$2: \quad +R i_1 - 75\Omega\, i_2 = 0$$

$$\Rightarrow \quad i_2 = \frac{R}{75\Omega} i_1$$

$$V = 25\Omega\, i + R i_1 = 25\Omega\left(i_1 + \frac{R}{75\Omega} i_1\right) + R i_1$$

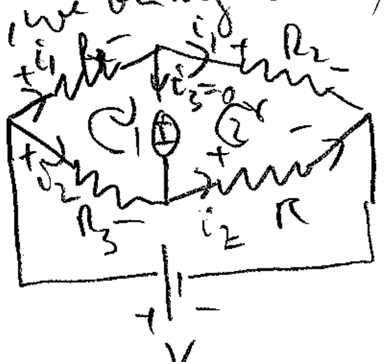
$$= \left(25\Omega + \frac{4}{3}R\right) i_1$$

$$V = \left(25\Omega + \frac{4R}{3}\right) \cdot \frac{75\Omega}{R} i_2$$

$$V = \left(\frac{1875\,\Omega^2}{R} + 100\,\Omega \right) i_2$$

$$R = \frac{1875\,\Omega^2\, i_2}{V - 100\,\Omega\, i_2} = 37.5\,\Omega$$

(4) No current through the amp meter makes it easy to solve, we only need,



We need only

$$1:\ R_3 i_2 - R_1 i_1 = 0$$

$$2:\ R i_2 - R_2 i_1 = 0$$

$$\Rightarrow \quad i_2 = \frac{R_1}{R_3} i_1$$

$$\Rightarrow \quad R = R_2 \frac{i_1}{i_2} = \frac{R_2 i_1}{\frac{R_1}{R_3} i_1} = \frac{R_2 R_3}{R_1}$$

(5) We need the current through the 25 Ω resistance only. Thus we first sort together the parallelly hooked resistors to 1 resistor:

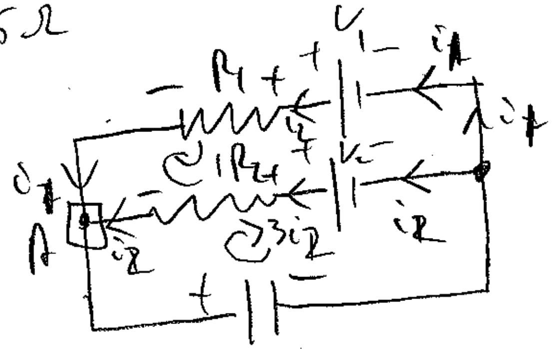
$$\frac{1}{R} = \left(\frac{1}{3} + \frac{1}{2} + \frac{1}{6} + \frac{1}{3} \right) \frac{1}{100\,\Omega} = \frac{4}{300\,\Omega}$$

$\Rightarrow R = 75\Omega$

Thus $i = \dfrac{V}{100\Omega} = 100\,mA = 0.1\,A$

$P_{25\Omega} = 25\Omega\, i^2 = 0.25W$

(6)



We need only V_C, A and ① first:

A: $-i_1 - i_2 = 0 \Rightarrow i_2 = -i_1$

$V_2 - R_1 i_1 + R_2 i_2 - V_1 = 0$

$V_1 - V_2 = R_1 i_1 - R_2 i_2 = (R_1 + R_2) i_1$

$\Rightarrow i_1 = \dfrac{V_1 - V_2}{R_1 + R_2}$

$i_2 = \dfrac{V_2 - V_1}{R_1 + R_2}$

To get V_C we use ③

$V_C + R_2 i_2 - V_2 = 0$

$\Rightarrow V_C = V_2 - R_2 i_2 = V_2 - \dfrac{V_2 - V_1}{R_1 + R_2} R_2$

$V_C = \dfrac{R_1 V_2 + R_2 V_1}{R_1 + R_2}$

$$Q = CV_c = C\,\frac{R_1 V_1 + R_2 V_1}{R_1 + R_2}$$

This shows that you can always introduce more currents than necessary. After using Kirchhoff's Laws, you cancel the superfluous currents anyway!

(7) is not a good idea. It's more safe to use the Gauss elimination algorithm. Note that there are two typos on page 164. It must be:

$$i_1 = \frac{(R_2 + R_3 + R_4)V_1 - (R_2 + R_3)V_2}{\det a}$$

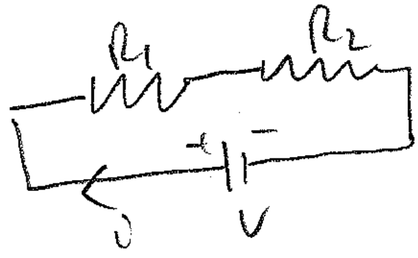
$$V = \frac{[R_1 R_2 + R_4(R_1 + R_2 + R_3)]V_2 - R_3 R_4 V_1}{\det a}$$

$\det a = R_1(R_2 + R_3 + R_4) + R_4(R_2 + R_3)$ (correct in book)

i_2 and i_3 are correct either

Exercises – Chpt. 8

(1)



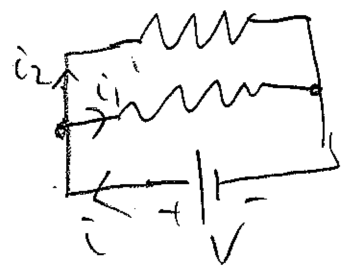
(a) $(R_1+R_2)i = V \Rightarrow i = \dfrac{V}{R_1+R_2}$

$$P_1 = R_1 i^2 = \frac{R_1}{(R_1+R_2)^2} V^2$$

$$P_2 = R_2 i^2 = \frac{R_2}{(R_1+R_2)^2} V^2$$

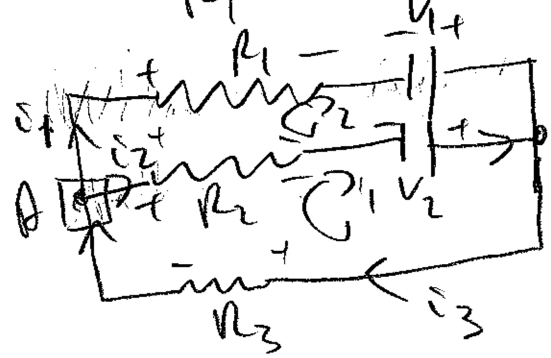
$$\Rightarrow \frac{P_1}{P_2} = \frac{R_1}{R_2}$$

(b)



$$P_1 = \frac{V^2}{R_1} \; ; \; P_2 = \frac{V^2}{R_2} \Rightarrow \frac{P_1}{P_2} = \frac{R_2}{R_1}$$

(2)



2: $V_1 - V_2 + R_2 i_2 - R_1 i_1 = 0$

A: $i_1 + i_2 = i_3$

1: $V_2 - R_3 i_3 - R_2 i_2 = 0$

②

$$R_3 i_3 + R_2 i_2 = V_2$$

$$R_3 (i_1 + i_2) + R_2 i_2 = V_2$$

$$R_3 i_1 + (R_2 + R_3) i_2 = V_2$$

$$R_1 i_1 - \quad R_2 i_2 = V_1 - V_2$$

$$R_3 i_1 + (R_2 + R_3) i_2 = V_2$$

$$\left[- R_1 (R_2 + R_3) - R_3 R_2\right] i_2 = - R_1 V_2 + R_3 (V_1 - V_2)$$

$$i_2 = \frac{-R_3 V_1 + (R_1 + R_3) V_2}{R_1 R_2 + R_1 R_3 + R_3 R_2}$$

$$i_1 = \frac{1}{R_1} \left(V_1 - V_2 + R_2 \frac{-R_3 V_1 + (R_1 + R_3) V_2}{R_1 R_2 + R_1 R_3 + R_2 R_3} \right)$$

$$= \frac{1}{R_1} \left(\frac{(V_1 - V_2)(R_1 R_2 + R_1 R_3 + R_2 R_3) - R_2 R_3 V_1 + R_2 (R_1 + R_3) V_2}{D} \right.$$

$$= \frac{1}{R_1} \left(\frac{V_1 (R_1 R_2 + R_1 R_3) - R_1 R_3 V_2}{D} \right)$$

$$= \frac{V_1 (R_2 + R_3) - R_3 V_2}{D}$$

$$i_3 = i_1 + i_2 = \frac{R_2 V_1 + R_1 V_2}{R_1(R_2+R_3) + R_2 R_3}$$

(3) We can substitute the resistance by R' with

$$\frac{1}{R'} = \frac{1}{2R} + \frac{1}{2R} = \frac{1}{R} \Rightarrow R' = R$$

$$i = \frac{V}{R} = 4.5\,A$$

(4) We immediately see that $i_3 = 0$

Then the voltage on R_2

$$V_2 = \frac{R_2}{R_2+R_3} V = \frac{12}{18} 9V = 6V$$

and $Q_1 = Q_2$ with

$$\frac{Q_1}{C_1} + \frac{Q_2}{C_2} = V_2$$

$$Q_1 = \frac{V_2}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{V_2 C_2 C_1}{C_1 + C_2}$$

$$= 6V \frac{40}{13} \mu F = \frac{240}{13} \cdot 10^{-6} C$$

$$= 1.846 \cdot 10^{-5} C$$

5 a)

$$Q_1 = Q_2 = \frac{V C_1 C_2}{C_1 + C_2} = 6V \ \frac{4}{5}\mu F = \frac{24}{5} 10^{-6} C$$

$$= 4.8 \cdot 10^{-6} C$$

(b) $$Q_1 = C_1 V_1 = C_1 \frac{R_1}{R_1 + R_2} V = 1\mu F \cdot \frac{20}{60} \cdot 6V$$

$$= 2 \cdot 10^{-6} C$$

$$Q_2 = C_2 V_2 = C_2 \frac{R_2}{R_1 + R_2} V = 4\mu F \cdot \frac{40}{60} \cdot 6V$$

$$= 16 \cdot 10^{-6} C$$