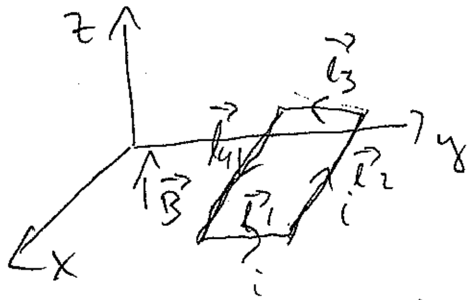
Problems - Chapter 9

(1) $\vec{B}$ is uniform



(a) $\vec{B} = B\,\vec{i}_z$; $\vec{l}_1 = l\,\vec{i}_y$; $\vec{l}_2 = -w\,\vec{i}_x$; $\vec{l}_3 = -l\,\vec{i}_y$; $\vec{l} = w\,\vec{i}_x$

$$\Rightarrow \vec{F}_1 = i\,\vec{l}_1 \times \vec{B} = i\,l\,B\,\vec{i}_x$$

$$\vec{F}_2 = i\,\vec{l}_2 \times \vec{B} = -i\,w\,B\,\vec{i}_x \times \vec{i}_z = i\,w\,B\,\vec{i}_y$$

$$\vec{F}_3 = -\vec{F}_1 = -i\,l\,B\,\vec{i}_x$$

$$\vec{F}_4 = -\vec{F}_2 = -i\,w\,B\,\vec{i}_y$$

(b) $\vec{l}_1{}' = l(\cos\vartheta\,\vec{i}_y + \sin\vartheta\,\vec{i}_z) \Rightarrow \vec{F}_1{}' = i\,\vec{l}_1 \times \vec{B} = i\,l\,B\cos\vartheta\,\vec{i}_x$

$\vec{l}_2{}' = -w\,\vec{i}_x \Rightarrow \vec{F}_2{}' = \vec{F}_2 = -i\,w\,B\,\vec{i}_y$

$\vec{l}_3{}' = -\vec{l}_1{}' \Rightarrow \vec{F}_3{}' = -\vec{F}_1{}' = -i\,l\,B\cos\vartheta\,\vec{i}_x$

$\vec{l}_4{}' = -\vec{l}_2{}' \Rightarrow \vec{F}_4{}' = \vec{F}_4 = -i\,w\,B\,\vec{i}_y$

(2) $\vec{C} = \vec{l}_1 \times \vec{F}_2{}' = \vec{l}_1 \times (\vec{l}_2{}' \times \vec{B}) = -\vec{l}_1 \times i\,w\,B\,\vec{i}_y$

$$= -i\,w\,B\,l\sin\vartheta\,\vec{i}_x$$

(3) According to the lecture:

$$R = \frac{M v}{|q| B} \quad (\text{Radius of trajectory} \perp \vec{B})$$

but we have

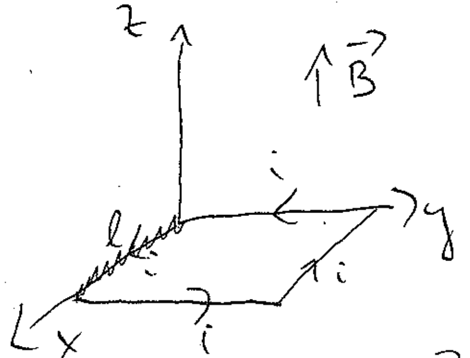
$$\frac{M}{2} v^2 = |q|\,V_0 \Rightarrow v = \sqrt{\frac{2|q|\,V_0}{M}}$$

$$\Rightarrow r = \sqrt{\frac{2 m V_0}{|q|}}\,\frac{1}{B}$$

or $$m = \frac{B^2 r^2 |q|}{2 V_0}$$

where $|q| = n e$ ($e = 1.6 \cdot 10^{-19}$ C, elementary charge)

(4)



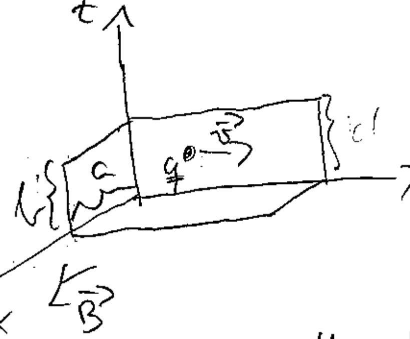
$$\vec{F} = i\,\vec{l} \times \vec{B} = i\,l\,B\,\vec{i}_x \times \vec{i}_z = -i\,l\,B\,\vec{i}_y$$

$$m\,\ddot{y} = -i\,l\,B$$

$$\Rightarrow \dot{y} = -\frac{i\,l}{m} B t$$

That's the solution given in the book. We just used another labelling of the axes.

(5)



$$\vec{B} = B\,\vec{i}_x \;;\; \vec{v} = v\,\vec{i}_y$$

$$\vec{F}_{mag} = q\,\vec{v} \times \vec{B} = q\,v\,B\,(-\vec{i}_z)$$

if $B > 0$

The charges are forced to the lower ($q>0$) or upper boundary ($q<0$) of the conductor. In the steady state the force $\vec{F}_{mag}$ is compensated by the force of the electric field, built up by this separation

Then we have

$$\vec{F}_{el} = q\vec{E} = qE_z\vec{i}_z \stackrel{!}{=} -\vec{F}_{mag}$$

$$\Rightarrow \quad qvB = -qE_z$$

$$\Rightarrow E_z = -vB$$

The voltage difference between the lower and the upper plate is

$$V_H = -\int_0^d dz\, E_z = -vBb \qquad \text{(Hall voltage)}$$

Now, if the current is given by

$$\vec{j} = qn\vec{v} = j\vec{i}_y = \frac{i}{ab}\vec{i}_y$$

we have

$$\vec{v} = \frac{i}{qnab}\vec{i}_y \Rightarrow \vec{v} = \frac{i}{qnab}\vec{i}_y$$

Thus if the current is running in positive $\vec{i}_y$ direction, we have

$$V_H = -\frac{i}{qnab}Bb = -\frac{iB}{qna}$$

That means that the voltage is negative for $q>0$ and positive for $q<0$. In this way one has found out that indeed the negative electrons are responsible for the current in a wire.

(6) was already done in lecture

Exercises Chapter 9

(1) $\vec{F}_{mag} = q\,\vec{v} \times \vec{B}$

$= -1.6 \cdot 10^{-19} C \, (6 \cdot 10^{5} \vec{i}_x + 4 \cdot 10^{5} \vec{i}_y) \frac{m}{s} \times 0.1 \vec{i}_x \frac{Wb}{m^2}$

$= +6.4 \cdot 10^{-15} N \, \vec{i}_z$

(2) $c \cong 3 \cdot 10^{8} \frac{m}{s} \Rightarrow 0.01\, c = 3 \cdot 10^{6} \frac{m}{s}$

$R = \frac{m v}{e B} = \frac{1}{1.76 \cdot 10^{11}} \cdot \frac{3 \cdot 10^{6}}{6 \cdot 10^{-5}} m = 28.4 \, cm$

(3)(a) $|\vec{F}_G| = m g$

Current must go to the right such that F_{mag} points up. Then

$|\vec{F}| = i l B \overset{!}{=} m g \Rightarrow i = \frac{m g}{l B}$

(b) The current is a flow of charged particles and thus they feel the force $q\,\vec{v} \times \vec{B}$.

(4) $\vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B})$

We need a B field such that

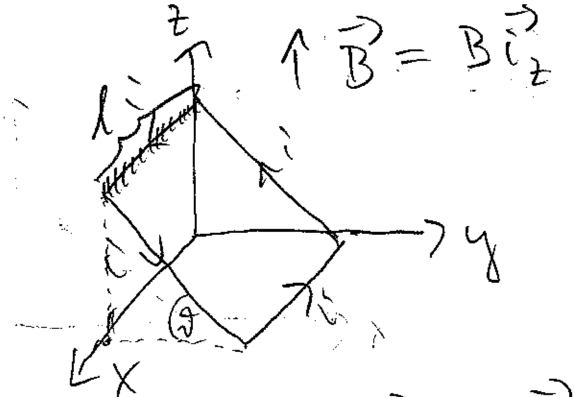
$\vec{E} + \vec{v} \times \vec{B} = 0$

Since $\vec{E} = E\,\vec{i}_y$ and $\vec{v} = v\,\vec{i}_x$ we must have $\vec{B} = B\,\vec{i}_z$

$\Rightarrow E - vB = 0 \Rightarrow B = \frac{E}{v}$

Since $E > 0$ and $v > 0$ we have $B > 0$.

(5)



$$\vec{l} = +l\vec{i}_x \Rightarrow \vec{F}_M = i\vec{l} \times \vec{B} = -ilB\vec{i}_y$$

$$\vec{F}_g = -mg\,\vec{i}_z$$

$$\vec{F}_{M+g} = -ilB\vec{i}_y - mg\,\vec{i}_z$$

Now we need the component in direction of the tracks.
The direction is given by the unit vector

$$\vec{n}_{\parallel} = \cos\vartheta\,\vec{i}_y - \sin\vartheta\,\vec{i}_z$$

The friction force is $\mu(\vec{F}_{M+g}\cdot\vec{n}_\perp)\,\vec{n}_\parallel$

$$\vec{n}_\perp = +\vec{i}_x \times \vec{n}_\parallel = +\cos\vartheta\,\vec{i}_z + \sin\vartheta\,\vec{i}_y$$

$$F_{fr\parallel} = \pm(-ilB\sin\vartheta - mg\cos\vartheta)\mu \quad \begin{cases}\text{for sliding down}\\ \text{for sliding up}\end{cases}$$

$$\vec{F}_{total}\,\vec{n}_\parallel = -ilB\cos\vartheta + mg\sin\vartheta \pm (-ilB\sin\vartheta - mg\cos\vartheta)\mu$$

$$= -ilB(\cos\vartheta \pm \mu\sin\vartheta) + mg(\sin\vartheta \mp \mu\cos\vartheta)$$

$$\stackrel{!}{=} 0 \Rightarrow B = \frac{mg(\sin\vartheta \mp \mu\cos\vartheta)}{il(\cos\vartheta \pm \mu\sin\vartheta)} \quad \begin{cases}\text{for sliding down}\\ \text{for sliding up}\end{cases}$$