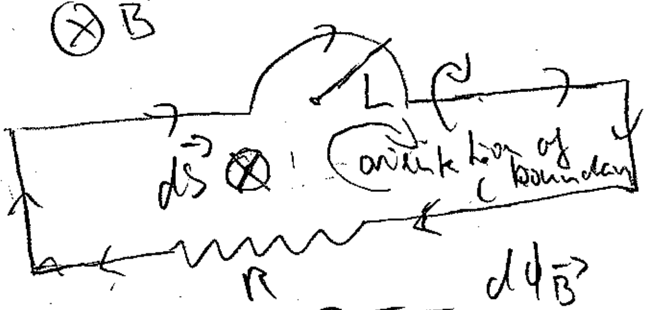
Problems Chpt. 11

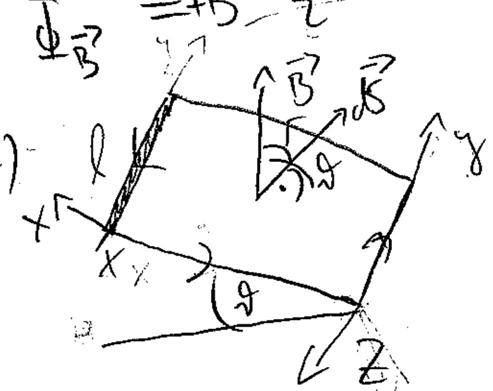
(1) $\otimes \vec B$



$$\oint \vec E\, d\vec r = iR = -\frac{d\Phi_{\vec B}}{dt}$$

$$\Phi_{\vec B} = +B\frac{\pi L^2}{2}\cos(\omega t) + \text{const} \Rightarrow i = -\frac{1}{R}\frac{d\Phi_{\vec B}}{dt} = +\frac{\omega}{R}\frac{\pi L^2}{2} B \sin(\omega t)$$

(2)



$$\Phi_{\vec B} = B l x \cos\vartheta \Rightarrow$$

$$-B l \dot x \cos\vartheta = iR$$

$$\vec B = -\vec i_z B\cos\vartheta + \vec i_x B\sin\vartheta$$

$$\vec F_M = i\vec l\times\vec B = -\frac{B l \dot x\cos\vartheta}{R}(-l\vec i_y)\times B(-\vec i_z\cos\vartheta + \vec i_x\sin\vartheta)$$

$$= \frac{B^2 l^2 \dot x\cos\vartheta}{R}(\vec i_x\cos\vartheta - \vec i_z\sin\vartheta)$$

$$\vec F_G = -m\vec g = mg(\vec i_z\cos\vartheta - \vec i_x\sin\vartheta)$$

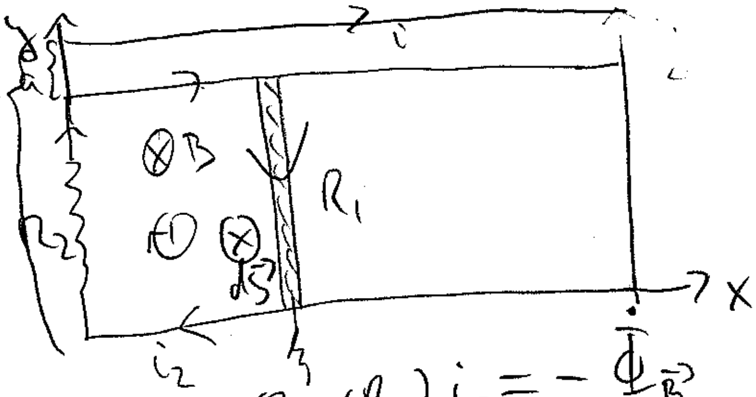
The force in z direction is counteracted by the slide. The terminal velocity is given by $F_x = 0$

$$\Rightarrow \frac{B^2 l^2 \cos^2\vartheta\, \dot x}{R} = mg\sin\vartheta$$

$$\Rightarrow \boxed{\dot x = \frac{R m g \sin\vartheta}{B^2 l^2 \cos^2\vartheta}}$$

See also alternative solution at the end

(3)



$$\oint \vec{E}\, d\vec{r} = (R_1+R_2)\, i_2 = -\dot{\Phi}_{\vec{B}}$$

$$\Phi_{\vec{B}} = \int_0^{b-a} dy \int_0^{\xi} dx \frac{\mu_0 i}{2\pi (b-y)}$$

$$= \frac{\mu_0 \xi i}{2\pi}\left[-\ln(b-y)\right]_0^{b-a}$$

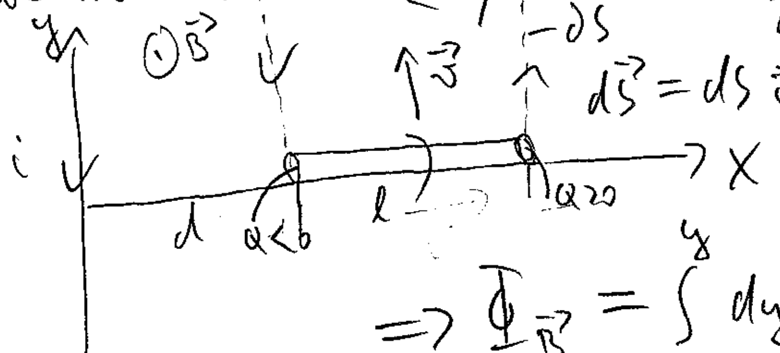
$$= + \frac{\mu_0 \xi i}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$\Rightarrow i_2 = -\frac{\mu_0 v i}{2\pi (R_1+R_2)} \ln\left(\frac{b}{a}\right) \quad (\text{where } v = \dot{\xi})$$

The negative sign tells us that the current through the circuit runs counter-clockwise (against our integration direction).
This is in accordance with Lenz's law since the force on the rod counteracts its motion.

$$\vec{B} = \frac{\mu_0 i}{2\pi X} \vec{i}_z \quad (\text{for } z=0)$$

(4)



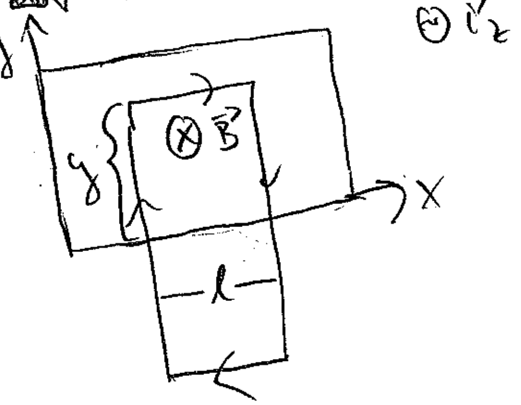
$$d\vec{S} = dS\, \vec{i}_z \; ; \; \vec{v} = v\, \vec{i}_y$$

$$\Rightarrow \Phi_{\vec{B}} = \int_0^{y} dy' \int_d^{d+l} dx \frac{\mu_0 i}{2\pi X} = \frac{\mu_0 i y}{2\pi} \ln\left(\frac{d+l}{d}\right)$$

$$\oint \vec{E}\, d\vec{r} = + \frac{\mu_0 i v}{2\pi} \ln\left(\frac{d+l}{d}\right) \quad (\text{because } \dot{y} = -v)$$

There is a voltage drop along the wire as indicated. In the steady state no current is flowing, but on the ends of the rod charges are setting of the indicated sign (due to the Lorentz force on the electrons)

(5)



$$\vec{B} = -B\vec{i}_z$$

$$\Phi_{\vec{B}} = +Bly$$

$$\Rightarrow \oint d\vec{r}\, \vec{E} = -Bl\dot{y} = Ri$$

$$\vec{F} = i l \vec{i}_x \times \vec{B} - mg\vec{i}_y$$

$$= (i l B - mg)\,\vec{i}_y$$

$$\Rightarrow m\ddot{y} = -\frac{B^2 l^2}{R}\dot{y} - mg$$

$$\text{or} \quad m\dot{v} = -\frac{B^2 l^2}{R} v - mg \quad (v = \dot{y})$$

Homogeneous equation

$$\dot{v}_H = -\frac{B^2 l^2}{mR} v_H \Rightarrow v_H = c \exp\left(-\frac{B^2 l^2}{mR} t\right)$$

Inhomogeneous eq. solved by special solution $v = \text{const} = v_\infty$

$$\Rightarrow \frac{B^2 l^2}{R} v_\infty = -mg$$

$$v_\infty = -\frac{mgR}{B^2 l^2}$$

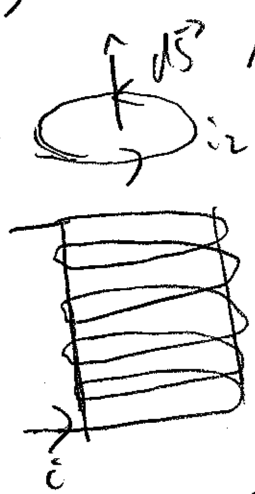
General solution

$$\Rightarrow v = v_\infty + v_H = -\frac{mgR}{B^2 l^2} + c\exp\left(-\frac{B^2 l^2}{mR} t\right)$$

$v(0) = 0$

$$\Rightarrow v = -\frac{mgR}{B^2 l^2}\left[1 - \exp\left(-\frac{B^2 l^2}{mR} t\right)\right]$$

(6)



$\uparrow \vec{B}$ for $\cos(\omega t) > 0$

$$i = i_0 \cos(\omega t)$$

$$\Rightarrow \vec{B} = B_0 \cos(\omega t)\, \vec{i}_z = B\, \vec{i}_z$$

$$\Phi_{\vec{B}} = A B_0 \cos(\omega t) = \pi a^2 B_0 \cos(\omega t)$$

$$\Rightarrow \oint \vec{E}\, d\vec{r} = i_2 R = -\dot{\Phi}_{\vec{B}} = +\pi a^2 B_0 \omega \sin(\omega t)$$

$$\Rightarrow i_2 = \frac{\pi a^2 B_0}{R} \omega \sin(\omega t)$$

Force on loop

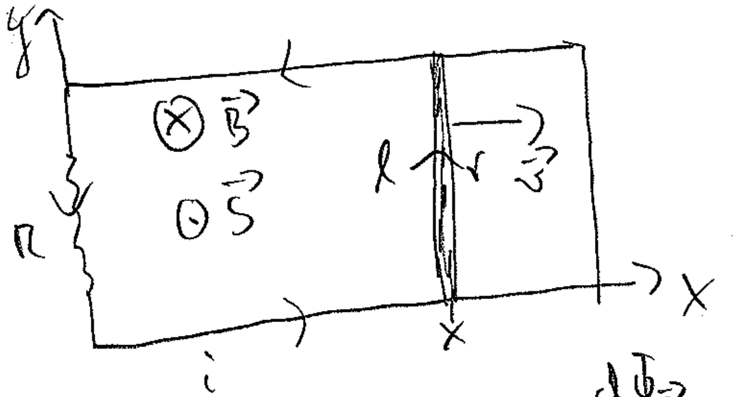
$$\vec{F} = i_2 \int_0^{2\pi} d\varphi\, \vec{i}_\varphi a \times \vec{B} = i_2 \int_0^{2\pi} d\varphi\, a B\, \vec{i}_\varrho = 0$$

But anyway the experiment works because the coil is finite in size and thus $\vec{B}$ not homogeneous. Particularly it has components $\perp \vec{i}_z$ which give the force to let it float! (3)

(7) See textbook

Exercises Chapt. 11

(1)



$$\oint \vec E\, d\vec r = i\,(R+r) = -\frac{d\Phi_{\vec B}}{dt} = -\frac{d}{dt}\left[-B\,l\,x\right] = B\,l\,v$$

(a) $$\vec F_{mag} = i\,l\,\vec i_y \times (-B\,\vec i_z) = -i\,l\,B\,\vec i_x = -\frac{l^2B^2v}{R+r}\,\vec i_x$$

Thus we need a force

$$\vec F_{ext} = -\vec F_{mag} = +\frac{l^2B^2v}{R+r}\,\vec i_x$$

(b) $$P_{mech} = \vec F_{ext}\,\vec v = \frac{l^2B^2v^2}{R+r}$$

(c) $$P_{el} = (R+r)\,i^2 = (R+r)\left(\frac{B\,l\,v}{R+r}\right)^2 = \frac{B^2l^2v^2}{R+r} = P_{mech}$$

Energy conservation: The heat produced per time is equal to the mechanical energy one puts into the system ☺.

(2)

(a) $$\oint \vec E\, d\vec r = i\,R = -\frac{d}{dt}\underbrace{\left(-B_0\sin(\omega t)\,\pi a^2\right)}_{\Phi_{\vec B}} = +B_0\,\omega\cos(\omega t)\,\pi a^2$$

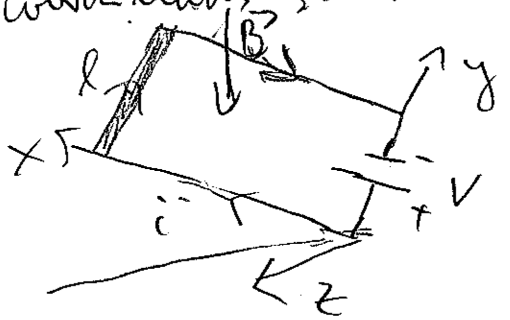
$\vec B = -B_0\sin(\omega t)\,\vec i_z$

$d\vec S \sim +\vec i_z$

$$i = \frac{B_0\,\omega\,\pi\,a^2}{R}\cos(\omega t)$$

(b) $i = i_{(a)}\cos\vartheta$ since now $d\vec S\cdot\vec i_z = \cos\vartheta$

(3) Coordinates as in problem 2



$\vec{S} = +lx\,\vec{i}_z$

now:

$$\vec{B} = B\cos\vartheta\,\vec{i}_z - B\sin\vartheta\,\vec{i}_x$$

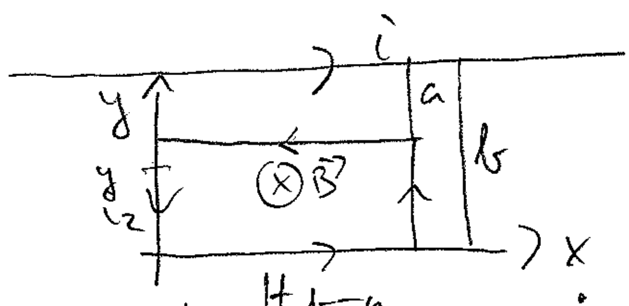
$$\Phi_{\vec{B}} = +Blx\cos\vartheta \Rightarrow \frac{d\Phi_{\vec{B}}}{dt} = -Blv\cos\vartheta$$

$$\oint \vec{E}\,d\vec{r} = V + iR = +Blv\cos\vartheta \quad (\text{since } \dot{x} = -v)$$

$$\Rightarrow i = +\frac{(Blv\cos\vartheta + V)}{R}$$

V switched $i = +\dfrac{Blv\cos\vartheta - V}{R}$ (See also alternative solution at the end)

(4)



$$\Phi_{\vec{B}} = +\int_0^H dx \int_0^{b-a} dy\,\frac{\mu_0 i}{2\pi(b-y)}$$

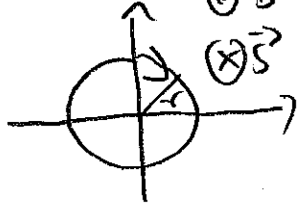
$$= -\frac{\mu_0 H i}{2\pi}\left[-\ln(b-y)\right]_0^{b-a}$$

$$= -\frac{\mu_0 H i}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$\oint d\vec{r}\,\vec{E} = i_2 R = -\frac{d\Phi_{\vec{B}}}{dt} = \frac{\mu_0 H \omega i_0}{2\pi}\cos(\omega t)\ln\left(\frac{b}{a}\right)$$

$$i_2 = \frac{\mu_0 H \omega i_0}{2\pi R}\cos(\omega t)\ln\left(\frac{b}{a}\right)$$

(5) $\odot\ \vec{B} = B_0 \vec{i}_z$ $\quad \otimes \vec{S}$

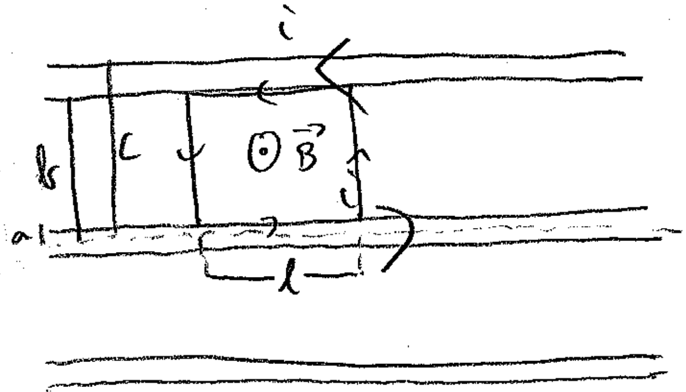


$$\Phi_{\vec{B}} = -\pi r^2 B_0$$

$$-\dot{\Phi}_{\vec{B}} = \oint_{\partial S} d\vec{r}\cdot\vec{E} = iR$$

$$i = -\frac{1}{R}\dot{\Phi}_{\vec{B}} = \frac{2\pi r \dot{r} B_0}{R} = \frac{4\pi(r_0+\alpha t^2)\alpha t B_0}{R}$$

(6) One can use the solution of Exercise 4 of Chapt. 10, but one must consider that there can be no time varying $\vec{B}$ field in such conductors, because there must be no $\vec{E}$ inside conductors. Thus the effective $\Phi_{\vec{B}}$ for the calculation of L is as indicated in the book:

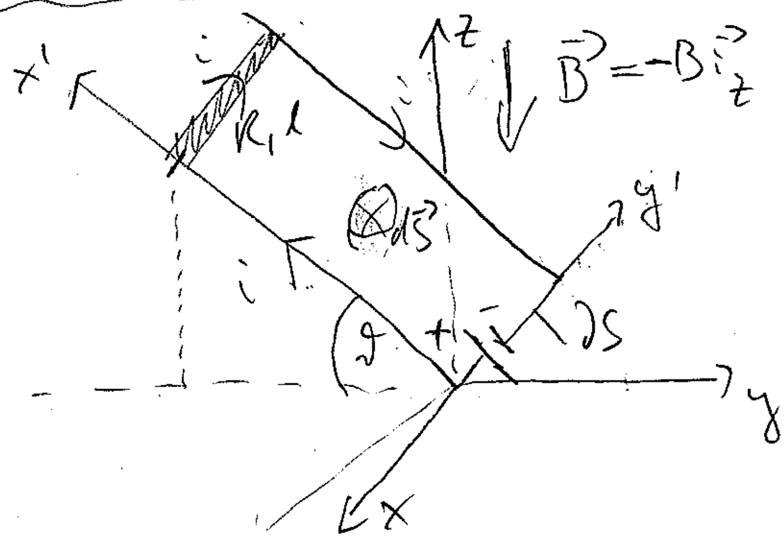


The $\vec{B}$ field in the free space between the cylinders is calculated as in exercise 4 (quasi stationary approximation). The with the orientation indicated

$$\Phi_{\vec{B}} = l\int_a^b dr\, B_\varphi(r) = \frac{l i \mu_0}{2\pi}\ln\left(\frac{b}{a}\right) = Li$$

$$\Rightarrow \boxed{\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)}$$

Alternative solution for Exercise 3 Chapt. 11 ①



$d\vec{S} = \vec{n}\, dS$

$$\vec{i}_{x'} = -\cos\vartheta\, \vec{i}_y + \sin\vartheta\, \vec{i}_z$$

$$\vec{i}_{y'} = -\vec{i}_x$$

$$\vec{n} = \vec{i}_{x'} \times \vec{i}_{y'} = (-\cos\vartheta\, \vec{i}_z - \sin\vartheta\, \vec{i}_y)$$

$$\Phi_{\vec{B}} = +x' l B \cos\vartheta$$

$$\dot{\Phi}_{\vec{B}} = -v l B \cos\vartheta$$

$$\oint_{\partial S} \vec{E}\, d\vec{r} = iR - V = +v l B \cos\vartheta$$

$$i = \frac{V + v l B \cos\vartheta}{R}$$

If we turn around the battery, we get a $-V$ instead of $+V$

$$i' = \frac{-V + v l B \cos\vartheta}{R}$$

where the assumed direction is always as indicated (in the page). In the book, for part (b) this direction is changed. So the results are in fact the same

Alternative solution to problem 2

The same as Exercise 3, but B flipped and V = 0

$$\Phi_{\vec{B}} = -x' l B \cos\vartheta$$

$$\frac{d\Phi_{\vec{B}}}{dt} = -\dot{x}' l B \cos\vartheta = -Ri$$

$$i = \frac{\dot{x}' l B \cos\vartheta}{R}$$

$$\vec{F}_{x'} = \left[-mg\vec{i}_z + (i l \vec{i}_{y'} \times B\vec{i}_z) \right] \vec{i}_{x'}$$

$$= -mg\sin\vartheta - i l B \cos\vartheta$$

$$= -mg\sin\vartheta - \frac{\dot{x}' l^2 B^2 \cos^2\vartheta}{R}$$

Terminal velocity

$$\boxed{v_{\text{terminal}} = -\dot{x}' = \frac{R\, mg\sin\vartheta}{l^2 B^2 \cos^2\vartheta}}$$