Heavy-Ion Phenomenology

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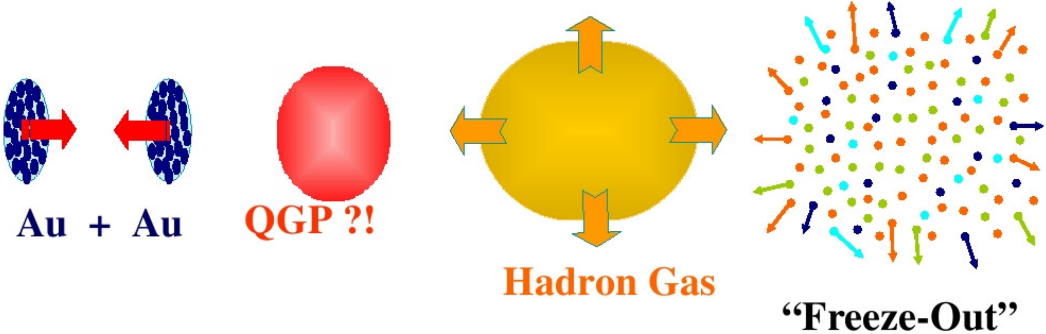
Outline

- Introduction: experimental pillars for theory picture of heavy-ion collisions
- Theory toolbox: QFT, transport, hydrodynamics
- Fluctuations of conserved charges
- Electromagnetic Probes

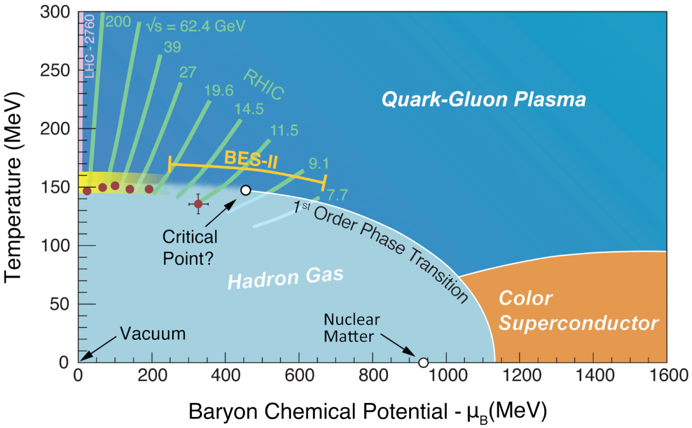
Introduction: QCD medium created in HICs

Ultrarelativistic Heavy-Ion Collisions

- ultra-relativistic collisions of heavy nuclei
- creates hot and dense fireball behaving like a strongly coupled medium
- early thermalization, starting in QGP phase
- rapidly expanding and cooling
- (cross-over) transition to hadron-resonance gas ($T_{\text{pc}} \simeq 150\text{-}160\,\text{MeV}$)



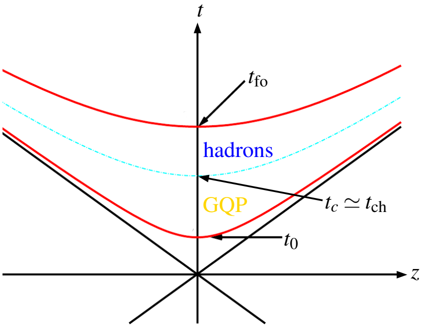
QCD Phase Diagram

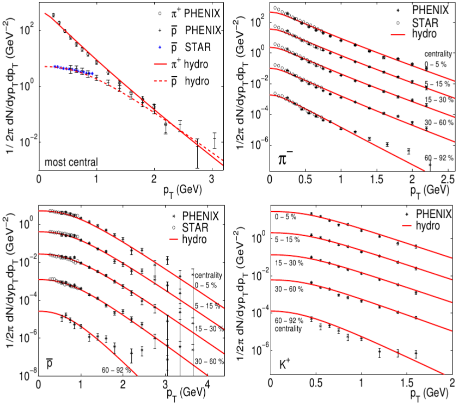


[Fig. from A. Aprahamian et al. Reaching for the horizon]

Collective flow of the fireball (Hydrodynamics)

- hydrodynamical model for ultra-relativistic heavy-ion collisions
  - after short formation time ($t_0 \lesssim 1\,\text{fm}/c$)
  - QGP in local thermal equilibrium → hadronization at $T_{\text{pc}} \simeq$ 150-160 MeV
  - chemical freeze-out: (inelastic collisions cease) $T_{\text{ch}} \simeq$ 150-160 MeV
  - thermal freeze-out: (also elastic scatterings cease) $T \sim 100$ MeV

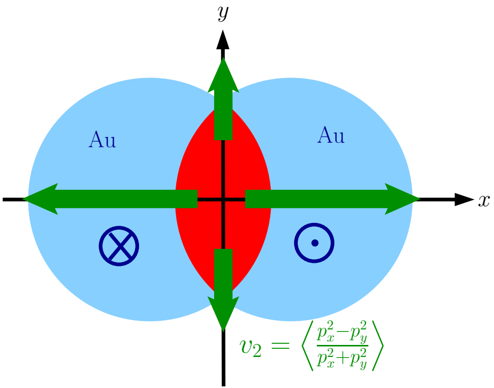


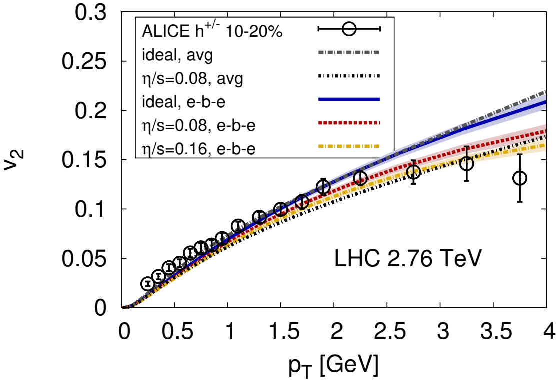


[KH03]

Hydrodynamical Behavior

- particle spectra compatible with collective flow (hydrodynamical expansion)
- elliptic flow as signature of pressure
- (nearly) ideal hydrodynamics $\eta/s \simeq 1\text{-}2 \times 1/4\pi$

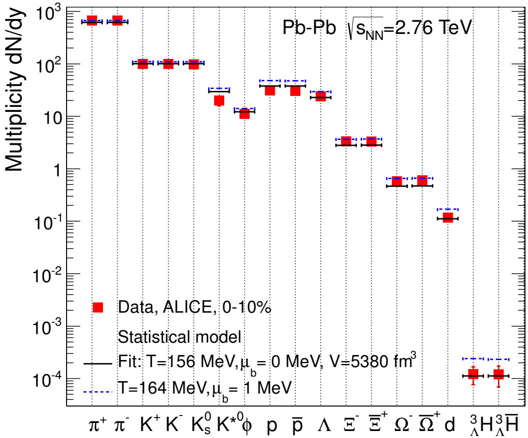




[SJG11]

Chemical freeze-out: Statistical hadronization model

- hadron abundancies: can be described by (grand-)canonical hadron-resonance-gas model ($T_{\text{ch}} \simeq T_{\text{pc}}, \mu_B = 0$)
- even light (anti-hyper-)nuclei follow the systematics!



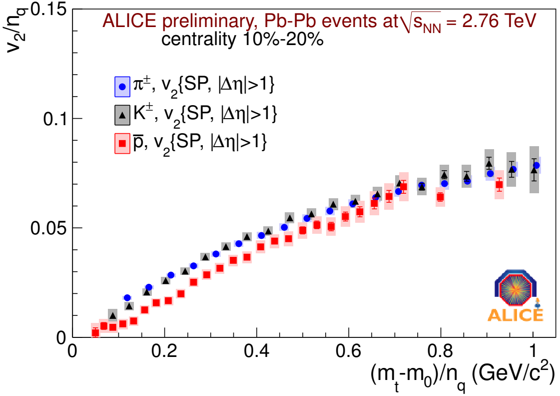
thermal hadronization model: J. Stachel et al [SABMR14]

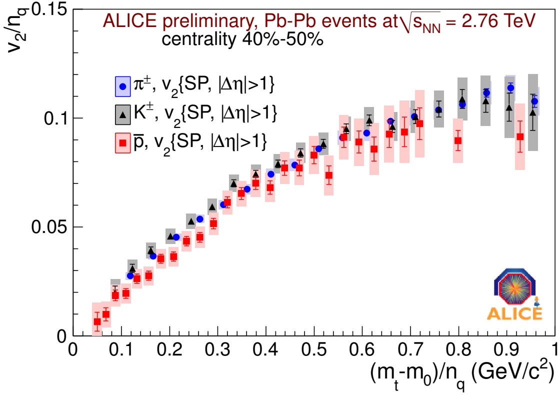
Constituent-quark-number scaling of v_2

- v_2 scales with number of constituent quarks

$$v_2^{(\text{had})}(p_T^{(\text{had})}) = n_q \, v_2^{(q)}(p_T^{(\text{had})}/n_q)$$

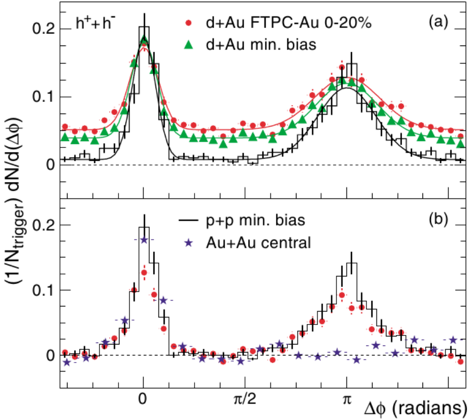
- indicates recombination of quarks in medium around T_{pc}
- "coalescence" of partonic degrees of freedom!

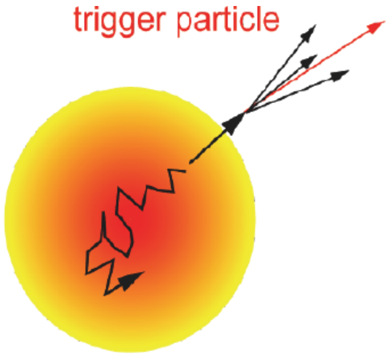




[Krz11]

Jet Quenching





- high p_T: jets going through medium suppressed
- high-density medium $\Rightarrow \rho > \rho_{\text{krit}}$
- energy loss due to elastic scattering and gluon bremsstrahlung
- more on heavy-ion phenomenology: [FHK+11]

Theory toolbox: QFT, Transport, Hydrodynamics

$$\mathscr{L}_{\text{QCD}} = \mathscr{L}_{\psi,A_\mu} + \mathscr{L}_G := \sum_{i \in \{u,d,s,c,b,t\}} \bar{\psi}_{i,j} \left(\mathrm{i}\gamma^\mu (D_\mu)^j{}_k - m_i \delta^j_k \right) \psi^k_i - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a, \quad \hat{D}_\mu = \partial_\mu + \mathrm{i} g \hat{T}^a A^a_\mu(x)$$

$$\mathscr{L}_{\psi,A_\mu} = \sum_{i \in \{u,d\}} [\bar{\psi}_{i,R} (i\gamma^\mu D_\mu) \psi_{i,R} + \bar{\psi}_{i,L} (i\gamma^\mu D_\mu) \psi_{i,L}] - \sum_{i \in \{u,d\}} m_i [\bar{\psi}_{i,R} \psi_{i,L} + \bar{\psi}_{i,L} \psi_{i,R}]$$

- asymptotic freedom: “running coupling” small at high energy scales
- non-perturbative at low energy scales
- confinement: only color-neutral objects observable (hadrons: mesons, baryons,...)
- lattice-QCD: Euclidean QCD, equilibrium many-body properties
- to describe dynamics: effective models based on “accidental” symmetries of QCD
- light-quark sector (u+d quarks): approximate chiral symmetry $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$

Quark-Meson linear-σ Model

CRC-TR 211

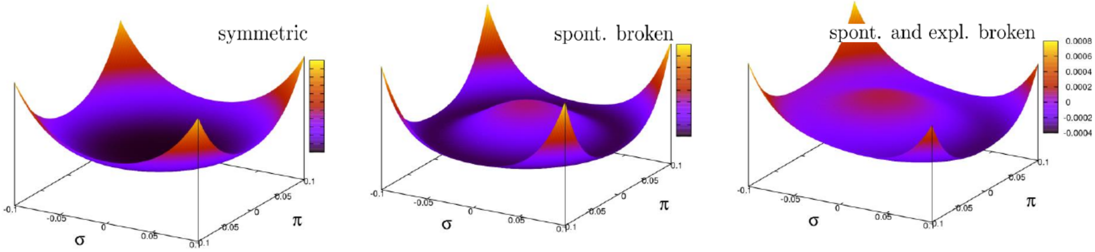
- from Meistrenko (PhD Thesis): [MHG21]
- $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$ linear-σ model
- mesons: σ, $\vec{\pi}$, quarks: $\psi = (u, d)$

$$\mathscr{L} = \bar{\psi}\left[\mathrm{i}\not{\partial} - g\left(\sigma + i\gamma_5 \vec{\pi}\cdot\vec{\tau}\right)\right]\psi + \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi}\right) - \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 + f_\pi m_\pi^2 \sigma + U_0$$

parameter	value	description
λ	20	coupling constant for σ and $\vec{\pi}$
g	$2-5$	coupling constant between $\sigma, \vec{\pi}$ and ψ
f_π	93 MeV	pion decay constant
m_π	138 MeV	pion mass
ν^2	$f_\pi^2 - m_\pi^2/\lambda$	field shift term
U_0	$m_\pi^4/(4\lambda) - f_\pi^2 m_\pi^2$	ground state

Quark-Meson linear-σ Model: meson potential

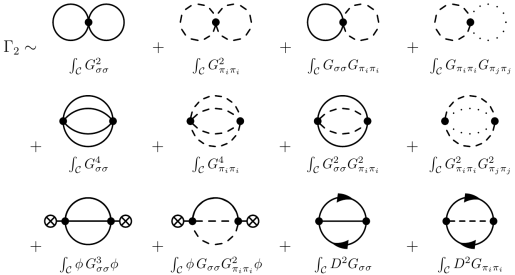
$$\mathscr{L} = \bar{\psi}\left[\mathrm{i}\not{\partial} - g\left(\sigma + i\gamma_5 \vec{\pi}\cdot\vec{\tau}\right)\right]\psi + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 + f_\pi m_\pi^2 \sigma + U_0$$



Quark-Meson linear-σ Model: 2PI action

$$\mathscr{L} = \bar{\psi}\left[\mathrm{i}\partial\!\!\!/ - g\left(\sigma + i\gamma_5 \vec{\pi}\cdot\vec{\tau}\right)\right]\psi + \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi}\right) - \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 + f_\pi m_\pi^2 \sigma + U_0,$$

$$\Gamma[\sigma,\vec{\pi},G,D] = S[\sigma,\vec{\pi}] + \frac{\mathrm{i}}{2}\mathrm{Tr}\ln G^{-1} + \frac{\mathrm{i}}{2} G_0^{-1} G - \mathrm{i}\,\mathrm{Tr}\ln D^{-1} - \mathrm{i}\,\mathrm{Tr}\, D_0^{-1} D + \Gamma_2[\sigma,\vec{\pi},G,D]$$



Equations of motion: Kadanoff-Baym equations for Green's functions + mean-field equations

$$\frac{\delta \Gamma}{\delta \sigma} = \frac{\delta \Gamma}{\delta \vec{\pi}} = \frac{\delta \Gamma}{\delta G} = \frac{\delta \Gamma}{\delta D} = 0.$$

Quark-Meson linear-σ Model: off-equilibrium equations

- real-time Keldysh contour ⇒ 2PI/Kadanoff Baym ⇒ transport equation (spatially homogeneous)
- Mean-field equation

$$\partial_t^2 \phi + D(t) + J(t) = 0, J(t) := \lambda \left(\phi^2 - v^2 + 3G_{\sigma\sigma}^{11} + \sum_i G_{\pi_i\pi_i}^{11} \right) \phi - f_\pi m_\pi^2 + g \langle \bar{\psi}\psi \rangle$$

- transport equations for meson- and quark-phase-space-distribution functions

$$\partial_t f^\sigma(t, \vec{p}_1) = \mathscr{C}^{b.}_{\sigma\sigma\leftrightarrow\sigma\sigma} + \sum_i \mathscr{C}^{b.}_{\sigma\pi_i\leftrightarrow\sigma\pi_i} + \sum_i \mathscr{C}^{b.}_{\sigma\sigma\leftrightarrow\pi_i\pi_i} + \mathscr{C}^{b.s.}_{\sigma\phi\leftrightarrow\sigma\sigma} + \sum_i \mathscr{C}^{b.s.}_{\sigma\phi\leftrightarrow\pi_i\pi_i} + \mathscr{C}^{f.s.}_{\sigma\leftrightarrow\psi\bar{\psi}},$$

$$\partial_t f^{\pi_i}(t, \vec{p}_1) = \mathscr{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_i\pi_i} + \sum_{j\neq i} \mathscr{C}^{b.}_{\pi_i\pi_j\leftrightarrow\pi_i\pi_j} + \sum_{j\neq i} \mathscr{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_j\pi_j} + \mathscr{C}^{b.}_{\pi_i\sigma\leftrightarrow\pi_i\sigma} + \mathscr{C}^{b.}_{\pi_i\pi_i\leftrightarrow\sigma\sigma} + \mathscr{C}^{b.s.}_{\pi_i\phi\leftrightarrow\pi_i\sigma} + \mathscr{C}^{f.s.}_{\pi_i\leftrightarrow\psi\bar{\psi}}$$

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$$\partial_t f^{\bar{\psi}}(t, \vec{p}_1) = \mathscr{C}^{f.s.}_{\bar{\psi}\psi\leftrightarrow\sigma} + \sum_i \mathscr{C}^{f.s.}_{\bar{\psi}\psi\leftrightarrow\pi_i},$$

- more on transport: [Buss:2011mx,Cassing:2021fkc]

Hydrodynamics

- ideal hydrodynamics: local thermal equilibrium

$$f^{(0)}(x,p) = g\exp[-\beta(x)u(x)\cdot p + \beta(x)\mu(x)]$$

- $u^\mu(x)$ with $u_\mu u^\mu \equiv 1$: fluid four-velocity, $\beta(x)$: inverse temperature, $\mu(x)$: chemical potential, $p^0 = \sqrt{m^2 + \vec{p}^2}$
- Boltzmann equation (collision term vanishes) ⇒ conservation of energy, momentum, and conserved charges

$$T^{\mu\nu}(x) = \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 p}{(2\pi)^3 p^0} p^\mu p^\nu f(x,p) = u^\mu u^\nu[\epsilon(x) + P(x)] - \eta^{\mu\nu} P(x),$$

$$N^\mu = \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 p}{(2\pi)^3 p^0} p^\mu f(x,p) = n(x) u^\mu(x),$$

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N^\mu = 0.$$

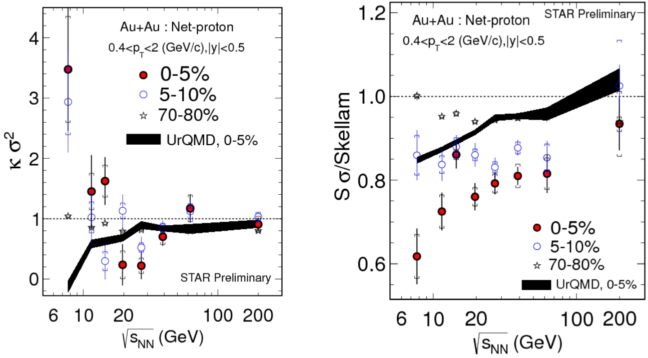
- to close system: equation of state $p = p(\epsilon, n)$
- extended to dissipative hydrodynamics: systematic expansion via moments of f more on hydro: [DR21]

Fluctuations of conserved charges

Quark-number susceptibilities

$$c_1 = \frac{N_{q,\text{net}}}{V T^3}, \quad c_2 = \frac{1}{V T^3} \left\langle \left(N_{q,\text{net}} - \langle N_{q,\text{net}} \rangle\right)^2 \right\rangle \equiv \frac{1}{V T^3} \sigma^2_{q,\text{net}}$$

$$c_3 = \frac{1}{V T^3} \left\langle \left(N_{q,\text{net}} - \langle N_{q,\text{net}} \rangle\right)^3 \right\rangle, \quad c_4 = \frac{1}{V T^3} \left[\left\langle \left(N_{q,\text{net}} - \langle N_{q,\text{net}} \rangle\right)^4 \right\rangle - 3\sigma^4_{q,\text{net}} \right],$$



$$\kappa\sigma^2 = c_4/c_2$$
$$S\sigma = c_3/c_2$$

[Luo16]

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$$\mathscr{L} = \bar{\psi}\left[\mathrm{i}\not{\partial} - g\left(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau}\right)\right]\psi + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 + f_\pi m_\pi^2\sigma + U_0,$$

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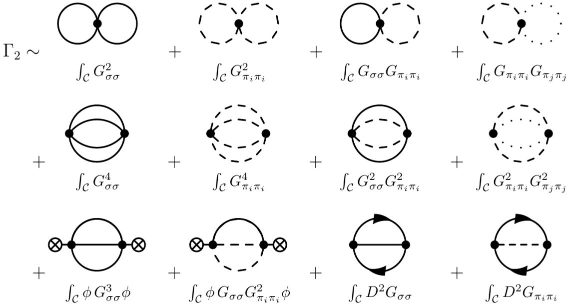
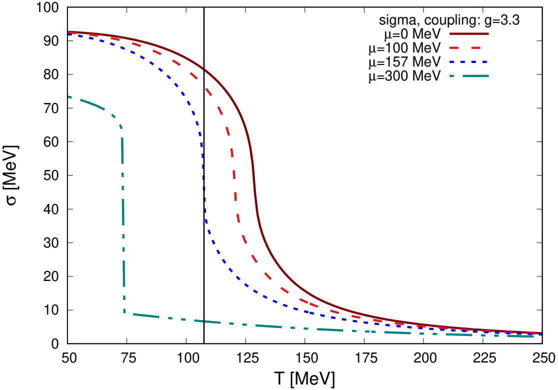


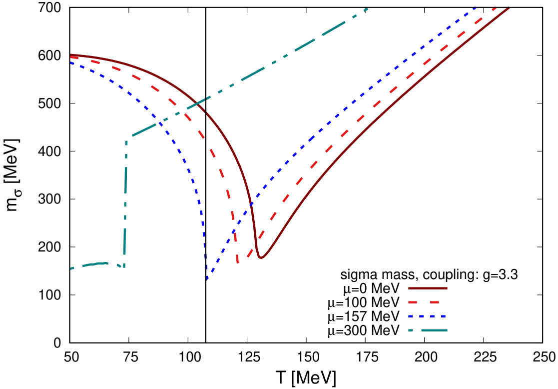
Equations of motion:

$$\frac{\delta\Gamma}{\delta\sigma} = \frac{\delta\Gamma}{\delta\vec{\pi}} = \frac{\delta\Gamma}{\delta G} = \frac{\delta\Gamma}{\delta D} = 0.$$

Quark-Meson linear-σ Model: equilibrium order parameter

$$\Omega_{\text{eff}}[\sigma,\vec{\pi},G,D] = -\frac{1}{\beta V}\mathrm{i}\Gamma[\sigma,\vec{\pi},G,D], \quad \frac{\partial\Omega_{\text{eff}}}{\partial\sigma} \stackrel{!}{=} 0, \quad M_\sigma^2 = \frac{\partial^2\Omega_{\text{eff}}}{\partial\sigma^2}, \quad M_\pi^2 = \frac{\partial^2\Omega_{\text{eff}}}{\partial\pi_i^2}$$





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$$+ \mathscr{C}^{b.s.}_{\pi_i\phi\leftrightarrow\pi_i\sigma} + \mathscr{C}^{f.s.}_{\pi_i\leftrightarrow\psi\bar{\psi}}$$

$$\partial_t f^{\psi}(t, \vec{p}_1) = \mathscr{C}^{f.s.}_{\psi\bar{\psi}\leftrightarrow\sigma} + \sum_i \mathscr{C}^{f.s.}_{\psi\bar{\psi}\leftrightarrow\pi_i}$$

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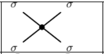
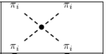
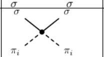
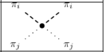
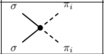
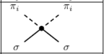
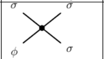
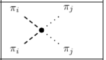
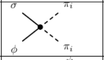
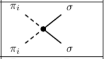
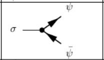
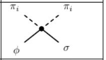
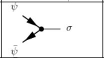
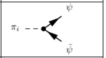
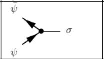
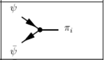
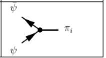
Quark-Meson linear-σ Model: collision terms

collision integral	diagram	collision integral	diagram
$C^{b.}_{\sigma\sigma\leftrightarrow\sigma\sigma}$		$C^{b.}_{\pi_i\pi_i\leftrightarrow\pi_i\pi_i}$	
$C^{b.}_{\sigma\pi_i\leftrightarrow\sigma\pi_i}$		$C^{b.}_{\pi_i\pi_j\leftrightarrow\pi_i\pi_j}$	
$C^{b.}_{\sigma\sigma\leftrightarrow\pi_i\pi_i}$		$C^{b.}_{\pi_i\sigma\leftrightarrow\pi_i\sigma}$	
$C^{b.s.}_{\sigma\phi\leftrightarrow\sigma\sigma}$		$C^{b.}_{\pi_i\pi_i\leftrightarrow\pi_j\pi_j}$	
$C^{b.s.}_{\sigma\phi\leftrightarrow\pi_i\pi_i}$		$C^{b.}_{\pi_i\pi_i\leftrightarrow\sigma\sigma}$	
$C^{f.s.}_{\sigma\leftrightarrow\psi\bar{\psi}}$		$C^{b.s.}_{\pi_i\phi\leftrightarrow\pi_i\sigma}$	
$C^{f.s.}_{\psi\bar{\psi}\leftrightarrow\sigma}$		$C^{f.s.}_{\pi_i\leftrightarrow\psi\bar{\psi}}$	
$C^{f.s.}_{\bar{\psi}\psi\leftrightarrow\sigma}$		$C^{f.s.}_{\psi\bar{\psi}\leftrightarrow\pi_i}$	
		$C^{f.s.}_{\bar{\psi}\psi\leftrightarrow\pi_i}$	

Quark-Meson linear-σ Model: expanding-fireball geometry

- Friedmann-Lemaître-Robertson-Walker metric (spatially flat)

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)(\mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2), \quad H = \dot{a}/a$$

- expanding fireball with radius $R(t) = R_0 + v_e t$, $\dot{a}/a = \dot{R}/R$
- mean-field equation

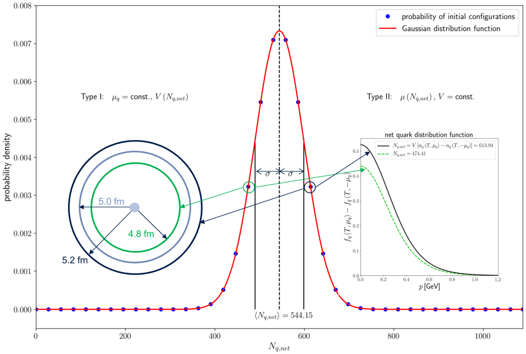
$$\partial_t^2 \phi + 3H\partial_t \phi + D(t) + J(t) = 0$$

- Boltzmann equation

$$\left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right) f = \mathscr{I}$$

Initialization of net-quark numbers

- goal: time-evolution of net-quark number fluctuations
- ensembles with fluctuating initial conditions



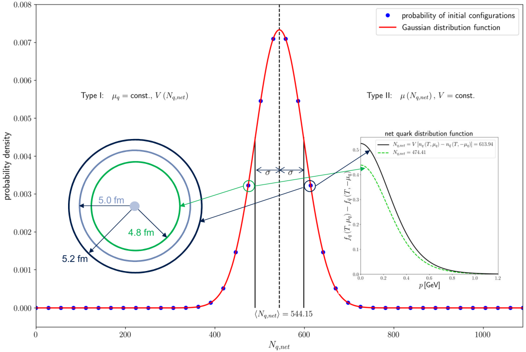
- mean net-quark number

$$\langle N_{\text{q,net}} \rangle = \frac{4\pi}{3} R_0^2 \int \frac{\mathrm{d}^3 p}{(2\pi)^3} [f_q(T,\mu_q) - f_q(T,-\mu_q)]$$

- standard deviation: $\sigma_{\text{q,net}} = \langle N_{\text{q,net}} \rangle / 10$
- choose $M = 200$-1000 values for $N_{\text{q,net}}$
- initialize type I or type II for each $N_{\text{q,net}}$

Observables

- goal: time-evolution of net-quark number fluctuations
- ensembles with fluctuating initial conditions

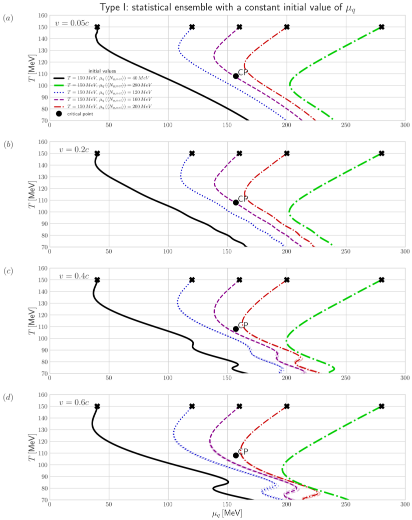


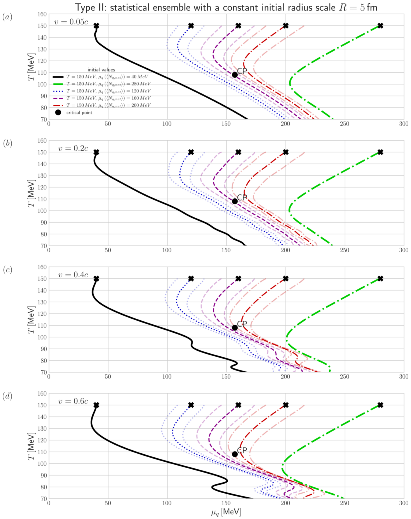
- ensembles with fluctuating initial conditions

$$\langle O \rangle = \frac{p_0 O_0 + p_M O_M}{2} + \sum_{k=1}^{M-1} p_k O_k$$

- cumulant ratios $R_{3,1} = c_3/c_1$, $R_{4,2} = c_1/c_2 = \kappa\sigma^2$, $c_1 = \langle m \rangle$, $c_2 = \tilde{m}_2 = \sigma^2$, $c_3 = \tilde{m}_3$, $c_4 = \tilde{m}_4 - 3\tilde{m}_2^2$

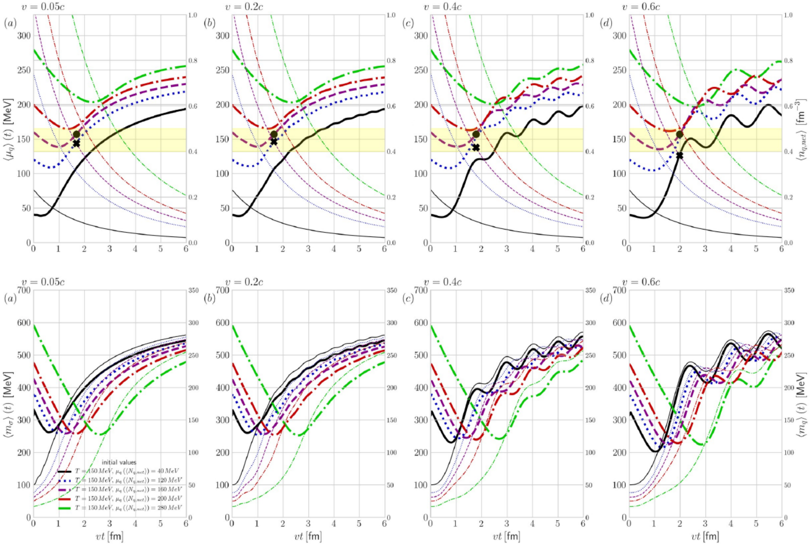
“Trajectories” in phase diagram



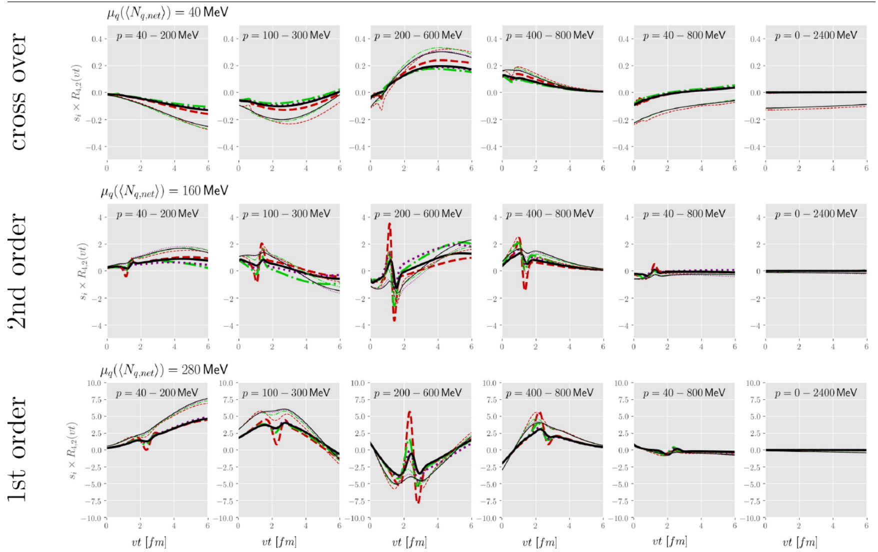


“Trajectories” in phase diagram

Type II: statistical ensemble with a constant initial radius scale $R = 5\,\mathrm{fm}$



"Trajectories" in phase diagram



$T_{\text{ini}} = 160$ MeV

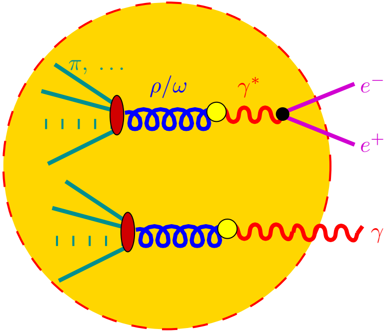
$\langle N \rangle_{q,\text{net}} = 123, 544, 1158$

- $s_1 = 10^5/\langle N_{q,net}\rangle^2$, $\sigma_{q,net} = \langle N_{q,net}\rangle/10$, $v = 0.05$
- $s_1 = 10^5/\langle N_{q,net}\rangle^2$, $\sigma_{q,net} = \langle N_{q,net}\rangle/10$, $v = 0.2$
- $s_1 = 10^5/\langle N_{q,net}\rangle^2$, $\sigma_{q,net} = \langle N_{q,net}\rangle/10$, $v = 0.4$
- $s_1 = 10^5/\langle N_{q,net}\rangle^2$, $\sigma_{q,net} = \langle N_{q,net}\rangle/10$, $v = 0.6$
- $s_2 = 10^4/\langle N_{q,net}\rangle^2$, $\sigma_{q,net} = \langle N_{q,net}\rangle/5$, $v = 0.05$
- $s_2 = 10^4/\langle N_{q,net}\rangle^2$, $\sigma_{q,net} = \langle N_{q,net}\rangle/5$, $v = 0.2$
- $s_2 = 10^4/\langle N_{q,net}\rangle^2$, $\sigma_{q,net} = \langle N_{q,net}\rangle/5$, $v = 0.4$
- $s_2 = 10^4/\langle N_{q,net}\rangle^2$, $\sigma_{q,net} = \langle N_{q,net}\rangle/5$, $v = 0.6$

Electromagnetic Probes

Electromagnetic probes in heavy-ion collisions

- $\gamma, \ell^{\pm}$: no strong interactions
- reflect whole "history" of collision:
  - from pre-equilibrium phase
  - from thermalized medium
    QGP and hot hadron gas
  - from VM decays after thermal freezeout



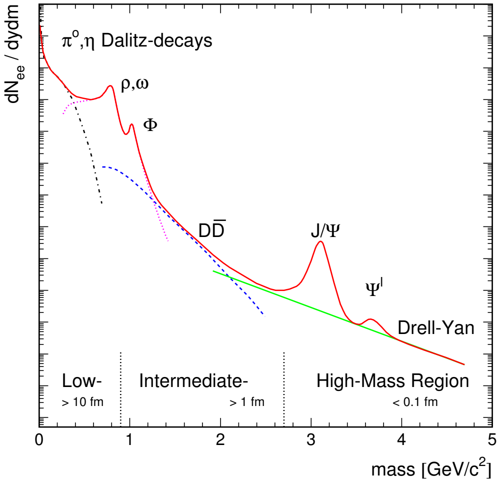


Fig. by A. Drees

Electromagnetic probes from thermal source

- retarded electromagnetic-current-correlation function

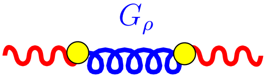
$$\Pi^{\mu\nu}_{\text{em},i} = \mathrm{i}\int \mathrm{d}^4 x \; \exp(\mathrm{i} q x)\Theta(x^0)\left\langle \left[j^{\mu}_{\text{em},i}(x), j^{\nu}_{\text{em},i}(0)\right]\right\rangle$$

- McLerran-Toimela formula [MT85, GK91]

$$q_0 \frac{\mathrm{d}N_\gamma}{\mathrm{d}^4 x \mathrm{d}^3 \vec{q}} = -\frac{\alpha_{\text{em}}}{2\pi^2} g^{\mu\nu} \operatorname{Im} \Pi^{(\text{ret})}_{\mu\nu}(q,u)\Big|_{q_0=|\vec{q}|} f_B(q\cdot u)$$

$$\frac{\mathrm{d}N_{e^+e^-}}{\mathrm{d}^4 x \mathrm{d}^4 q} = -g^{\mu\nu}\frac{\alpha^2}{3q^2\pi^3} \operatorname{Im} \Pi^{(\text{ret})}_{\mu\nu}(q,u)\Big|_{q^2=M^2_{e^+e^-}} f_B(q\cdot u)$$

- Lorentz covariant (dependent on four-velocity of fluid cell, u)
- $q \cdot u = E_{\text{cm}}$: Doppler blue shift of q_T spectra!
- to lowest order in α: $4\pi\alpha\Pi_{\mu\nu} \simeq \Sigma^{(\gamma)}_{\mu\nu}$
- vector-meson dominance model:

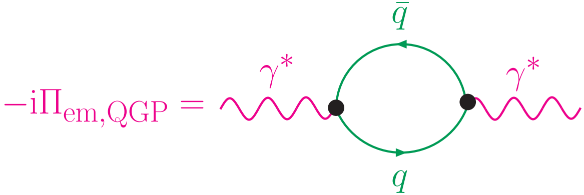
$\Sigma^{\gamma}_{\mu\nu} =$ 

- $\ell^+\ell^-$-inv.-mass spectra $\Rightarrow$ in-med. spectral functions of vector mesons (ρ, ω, ϕ)!

- General: McLerran-Toimela formula

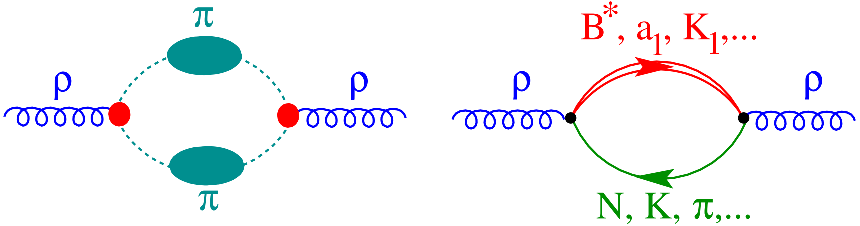
$$\frac{\mathrm{d}N^{(\mathrm{MT})}_{l^+l^-}}{\mathrm{d}^4x\,\mathrm{d}^4q} = -\frac{\alpha^2}{3\pi^3}\frac{L(M^2)}{M^2} g_{\mu\nu} \mathrm{Im} \sum_i \Pi^{\mu\nu}_{\mathrm{em},i}(M,\vec{q}) f_B(q\cdot u)$$

- in QGP phase: $q\bar{q}$ annihilation
- hard-thermal-loop improved em. current-current correlator



Hadronic many-body theory

- hadronic many-body theory (HMBT) for vector mesons
  [Ko et al, Chanfray et al, Herrmann et al, Rapp et al, …]
- $\pi\pi$ interactions and baryonic excitations
- effective hadronic models, implementing symmetries
- parameters fixed from phenomenology (photon absorption at nucleons and nuclei, $\pi N \rightarrow \rho N$)
- evaluated at finite temperature and density
- self-energies $\Rightarrow$ mass shift and broadening in the medium



- Baryons important, even at low **net** baryon density $n_B - n_{\bar{B}}$
- reason: $n_B + n_{\bar{B}}$ relevant (CP inv. of strong interactions)

Bulk evolution with transport and coarse graining

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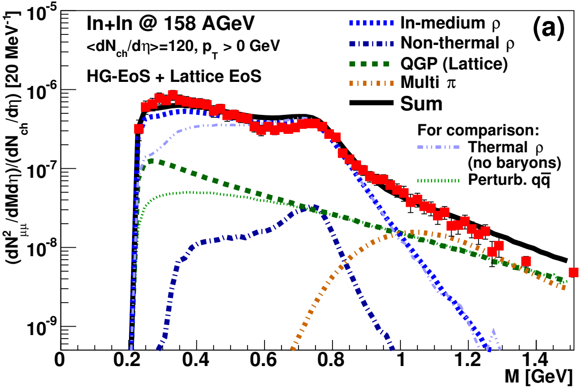
- established transport models for bulk evolution
  - e.g., UrQMD, GiBUU, BAMPS, (p)HSD,...
  - solve Boltzmann equation for hadrons and/or partons
- dilemma: need medium-modified dilepton/photon emission rates
- usually available only in equilibrium QFT calculations
- ways out:
  - (ideal) hydrodynamics ⇒ local thermal equilibrium ⇒ use equilibrium rates
  - transport-hydro hybrid model: treat early stage with transport, then coarse grain ⇒ switch to hydro ⇒ switch back to transport (Cooper-Frye "particlization")
- here: UrQMD transport for entire bulk evolution
  ⇒ use coarse graining in space-time cells ⇒ extract $T, \mu_B, \mu_\pi, \ldots$ ⇒ use equilibrium rates locally

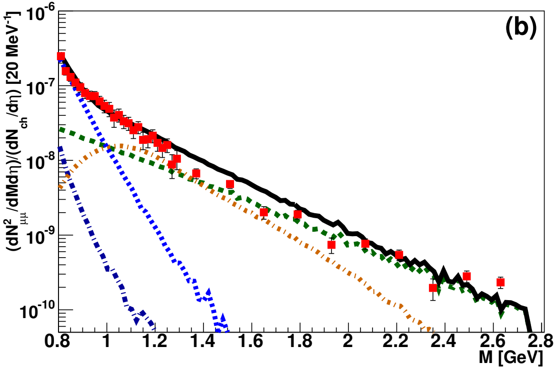
CGUrQMD: In+In (158 *A*GeV) (SPS/NA60)

- dimuon spectra from In + In(158 *A*GeV) → $\mu^+\mu^-$ (NA60) [EHWB15]
- min-bias data ($\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}y = 120$)



CGUrQMD: In+In (158 AGeV) (SPS/NA60)

- dimuon spectra from $\text{In}+\text{In}(158\,A\text{GeV}) \rightarrow \mu^+\mu^-$ (NA60) [EHWB15]
- min-bias data ($\mathrm{d}N_{\text{ch}}/\mathrm{d}y = 120$)
- higher IMR: provides averaged true temperature $\langle T \rangle_{1.5\,\text{GeV} \lesssim M \lesssim 2.4\,\text{GeV}} = 205\text{-}230\,\text{MeV}$
- clearly above $T_c \simeq 150\text{-}160\,\text{MeV}$ (no blueshifts in the invariant-mass spectra!)



- more on electromagnetic probes: [RW00, RWH10]

Conclusion and Outlook

Conclusion

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- QCD medium created in heavy-ion collisions ⇒ can be described as collectively moving fluid
  - p_T spectra, anisotropic flow, v_2
  - high-density medium: jet quenching
  - at highest beam energies: particle/light (anti-) nuclei ⟷ chemical freeze-out close to T_{pc}
  - electromagnetic probes: medium modifications of hadrons
  - (not covered in this lecture) heavy quarks: interaction strength ⟷ transport coefficients; quarkonia: screening, dissociation vs. regeneration
- Theory toolbox
  - fundamental level: QCD ⟷ effective (hadronic) QFT models (chiral symmetry,...)
  - many-body QFT: equilibrium ⇒ "imaginary time"/Matsubara formalism/lQCD/hadronic many-body calculations; non-equilibrium ⇒ "real-time"/Schwinger-Keldysh/2PI/Kadanoff-Baym equations
  - coarse graining I: gradient ($\hbar$) expansion ⇒ transport models (on- and off-shell)
  - coarse graning II: expansion around local thermal equilibrium/method of moments ⇒ derivation of transport coefficients (shear+bulk viscosity, electric conductivity, diffusion constants,...)
- Further "applications"
  - nuclear astro physics: neutron stars, neutron-star mergers/kilonovae

Outlook

- many open questions
  - phase diagram: is there a confinement-deconfinement 1st-order phasetransition line with critical endpoint? $\Rightarrow$ kinetics of “grand-canonical fluctuations” of conserved charges?
  - equation of state? $\Rightarrow$ neutron stars/kilonovae?
  - do we understand hadronization? $\Rightarrow$ kinetic theory vs. “naive coalescence”?
  - “spin transport/hydro”? $\Leftrightarrow$ polarization measurements (Λ, ϕ mesons at RHIC?)
  - initial state? early “off-equilibrium” phase of “fireball evolution”?

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