Dynamics of the chiral phase transition

Hendrik van Hees, Christian Wesp, Carsten Greiner

Goethe-Universität Frankfurt

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Outline

1. Chiral Symmetry
2. Non-Equilibrium linear σ model
3. First Numerical Results
4. Conclusions and Outlook

Chiral Symmetry

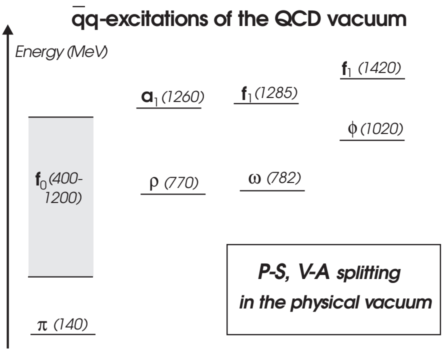
- Theory for strong interactions: QCD

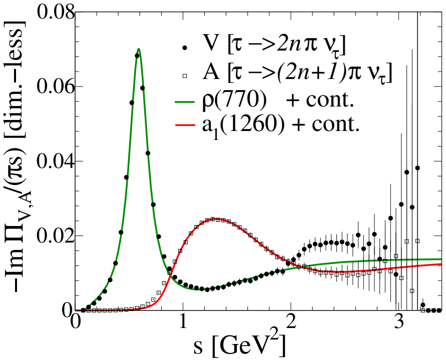
$$\mathscr{L}_{\text{QCD}} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \bar{\psi}(\mathrm{i}\not{D} - \hat{M})\psi$$

- Particle content:
  - ψ: Quarks, including flavor- and color degrees of freedom, $\hat{M} = \text{diag}(m_u, m_d, m_s, \ldots) =$ current quark masses
  - A_μ^a: gluons, gauge bosons of $\text{SU}(3)_{\text{color}}$
- Symmetries
  - fundamental building block: local $\text{SU}(3)_{\text{color}}$ symmetry
  - in light-quark sector: approximate chiral symmetry ($\hat{M} \rightarrow 0$)
  - chiral symmetry most important connection between QCD and effective hadronic models

Phenomenology and Chiral symmetry

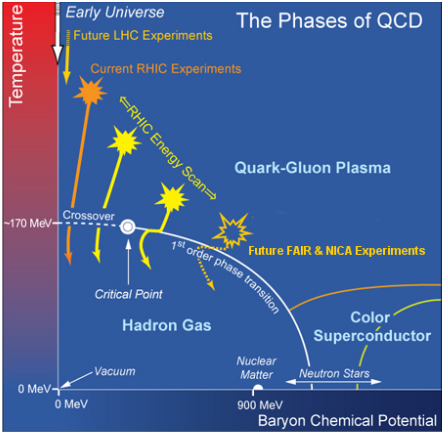
- In vacuum: Spontaneous breaking of chiral symmetry
- ⇒ mass splitting of chiral partners





The QCD-phase diagram

- at high temperature/density: restoration of chiral symmetry
- Lattice QCD ($\mu_B \simeq 0$): $T_c^\chi \simeq T_c^{\text{deconf}}$
- 1st-order phase transition at $\mu_B \neq 0$ observable?
- Signatures of critical endpoint? (critical fluctuations?)
- "fireballs" of finite extent and lifetime $\Rightarrow$ Non-equilibrium situation!



Quark-meson linear σ model

- quark-meson linear σ model
- chiral $\mathrm{SU}_L(2) \times \mathrm{SU}_R(2) \sim \mathrm{SO}(4)$ symmetry
- spontaneously broken to $\mathrm{SU}_V(2) \sim \mathrm{SO}(3)$
- mesons $\mathrm{SO}(4)$: σ (scalar) $\vec{\pi}$ (pseudoscalar)
- constituent quarks: $\mathrm{SU}_L(2) \times \mathrm{SU}_R(2)$

$$\mathscr{L} = \overline{\psi}[\mathrm{i}\partial\!\!\!/ - g(\sigma + \mathrm{i}\vec{\pi} \cdot \vec{\tau}\gamma_5)]\psi - \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi}) - U(\sigma, \vec{\pi})$$

- meson potential

$$U(\sigma, \vec{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 + f_\pi m_\pi^2 \sigma.$$

- explicit breaking of chiral symmetry

Mean-field and Vlasov equations

- Mean fields for σ and $\vec{\pi} \Rightarrow$ nonlinear Klein-Gordon equation:

$$\Box\sigma + \lambda^2(\sigma^2 + \vec{\pi}^2 - \nu^2)\sigma + g\left\langle\overline{\psi}\psi\right\rangle - f_\pi m_\pi^2 = 0,$$
$$\Box\vec{\pi} + \lambda^2(\sigma^2 + \vec{\pi}^2 - \nu^2)\vec{\pi} + g\left\langle\overline{\psi}\mathrm{i}\gamma_5\psi\right\rangle = 0.$$

- Vlasov equation for quark-phase-space distribution function

$$\left[\partial_t + \frac{\vec{p}}{E_q}\cdot\vec{\nabla}_r - (\vec{\nabla}_r E_q)\cdot\vec{\nabla}E_q\right] f(t,\vec{r},\vec{p}) = 0$$

- with $E_q = \sqrt{\vec{p}^2 + M_q(\vec{r})}, M_q^2 = g^2[\sigma^2 + \vec{\pi}^2]$

Equilibrium

- Quark/anti-quark distributions

$$f = \frac{1}{(2\pi)^3} f_{\mathrm{F}}[(E-\mu)/T], \quad \bar{f} = \frac{1}{(2\pi)^3} f_{\mathrm{F}}[(E+\mu)/T]$$

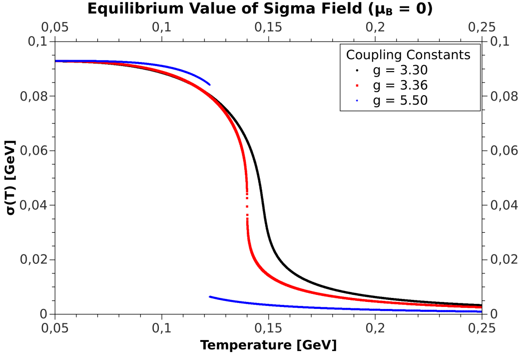
- scalar and pseudoscalar quark densities

$$\langle \overline{\psi}\psi \rangle = g\sigma \int \mathrm{d}^3\vec{p}\, \frac{f+\bar{f}}{E},$$

$$\langle \overline{\psi}\gamma_5\psi \rangle = g\vec{\pi} \int \mathrm{d}^3\vec{p}\, \frac{f+\bar{f}}{E}$$

Equilbrium

- evaluate $\langle \sigma \rangle (T)$ (dependent on quark densities)
- phase transitions: 1st, 2nd order, cross-over dependent on g:



- thermal fluctuations $\delta\sigma \propto T/(Vm_\sigma^2)$
- correlation length $1/\xi^2 = m_\sigma^2$
- critical point $m_\sigma \to 0 \ \Rightarrow \ \xi \to \infty$ (chiral limit!)

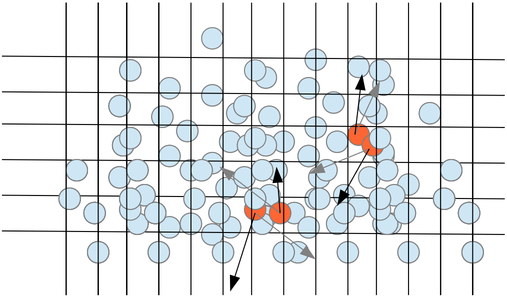
Non-Equilibrium

- test-particle ansatz

$$f(t,\vec{r},\vec{p}) = \frac{1}{N_{\text{test}}} \sum_i \delta^{(3)}[\vec{r} - \vec{r}_i(t)]\delta^{(3)}[\vec{p} - \vec{p}_i(t)]$$

- besides mean fields: binary collisions of quarks
- stochastic collision rates (as in BAMPS)

$$P_{22} = v_{\text{rel}} \frac{\sigma_{22}}{N_{\text{test}}} \frac{\Delta t}{\Delta^3 x}$$



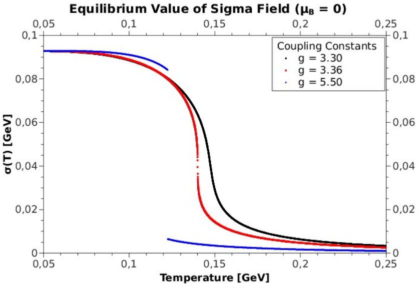
Heating or cooling via ghost-heat bath

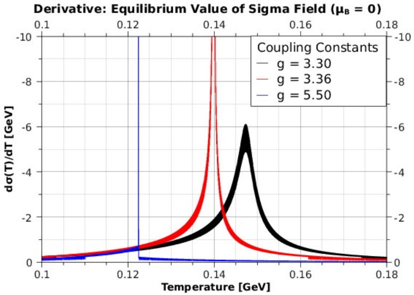
- simulate canonical heat bath $\Rightarrow$ particles can interact with thermal ghost particles in equilibrium
- energy exchange with heat bath
- collision rate

$$P_{22} = v_{\text{rel}} \sigma_{\text{therm}} n_{\text{ghost}}(T) \frac{\Delta t}{\Delta^3 \vec{r}}$$

- advantages:
  - energy-momentum conservation
  - enables "box calculations" $\Rightarrow$ equilibration of quark medium, heat-bath cooling, expanding droplets
  - no artificial spatial anisotropies
  - thermalization rate $\propto \sigma_{\text{bath}}/\sigma_{22}$

Initial Conditions





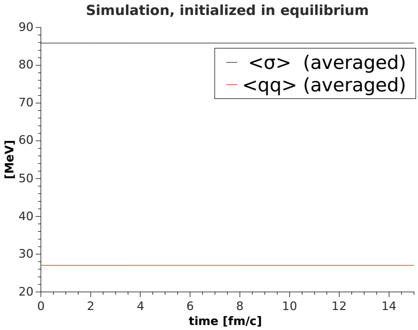
- σ-field: solving the nonlinear self-consistent equations $\partial_\mu \partial^\mu \sigma \equiv 0$:

$$\left[\lambda^2\left(\sigma_0^2 - \nu^2\right) + g^2 \int \mathrm{d}^3\vec{p}\, \frac{f(t,\vec{r},\vec{p},\sigma_0) + \bar{f}(t,\vec{r},\vec{p},\sigma_0)}{E(t,\vec{r},\vec{p})}\right] \sigma_0 = f_\pi m_\pi^2$$

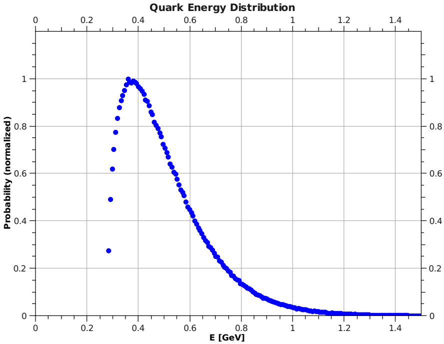
- $f_q(t,\vec{r},\vec{p},\sigma_0)$: Fermi distribution

Test: Equilibrium initialiazation

- σ and q thermal, $\pi = 0$.
- no spatial gradients, no anisotropy



σ field / Quark density



Quark energy

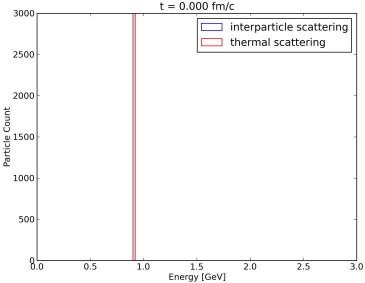
Test Scenario: Equilibration

Initial conditions:

$$f(\vec{x}, \vec{p}, t = 0) = \delta(|\vec{p}| - 800\text{MeV})$$

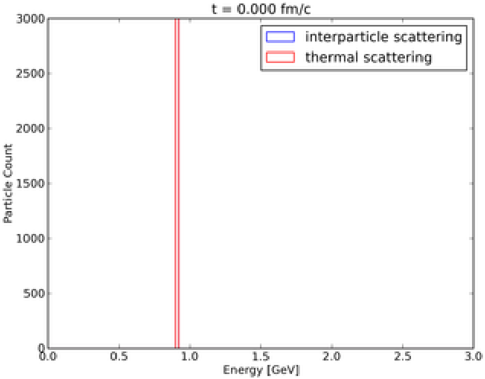
Comparison of equlibration:

- only mean-field interactions
- binary scattering
- scattering with heat bath



Note: mean-field scenario shows very slow or no equilibration!

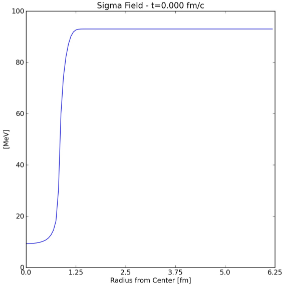
Thermalisation



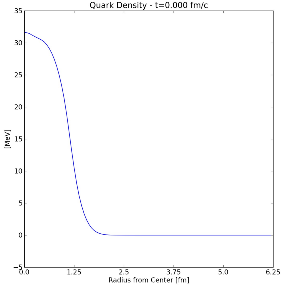
Test Scenario: Thermal Blob

- $\sigma(\vec{r})$ and $q(\vec{r})$ thermal, $\pi = 0$.
- spatial temperature / thermal 'blob'

$$T(\vec{r}) = \frac{T_{\text{init}}}{1 + \exp\left(\left(|\vec{r}| - R_0 \right) / \alpha \right)}$$

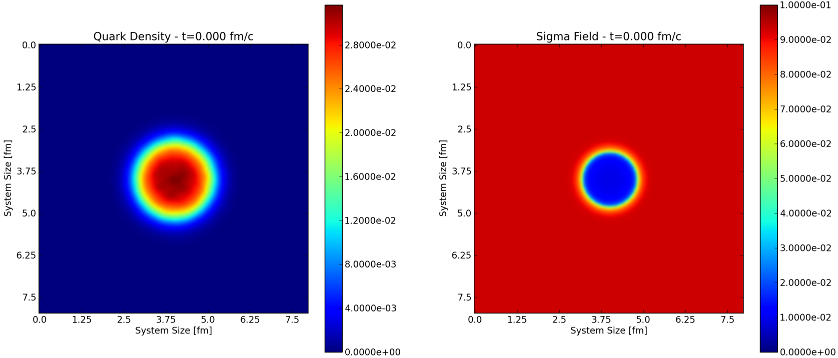


Sigma field



Quark density

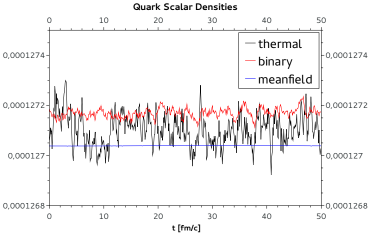
Thermal Blob Scenario



Fluctuations

Flucutations in $\langle \bar{\psi}\psi \rangle \rightarrow$ fluctuations in σ-field.

- mean field: only spatial fluctuations
- binary: spatial and global
- heat beath: spatial and global
- heat bath stronger due to canonical ensemble



Note: spatial fluctuations bigger than global one fluctuations.
How does the chararistics of fluctuations change at the phase transition?

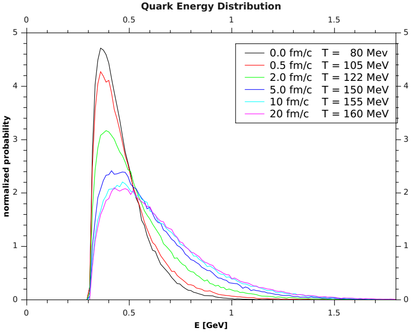
Heating and cooling through the phase transition

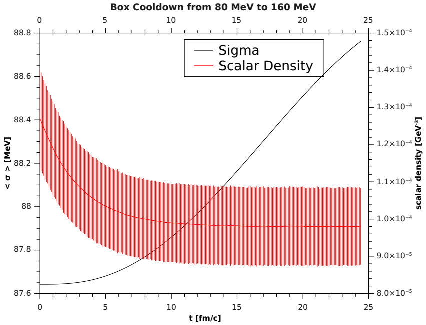
Box calculations

- system initialized in thermal and chemical equilibrium
- temperature is changed via heatbath
- from $T = 80$ MeV to $T = 160$ MeV (massive particles)
- from $T = 150$ MeV to $T = 80$ MeV (massles particles)
- $V = \text{const}, \quad N_q = \text{const}$

$\Rightarrow$ **Do we see a phase transition?**

Heating the Box



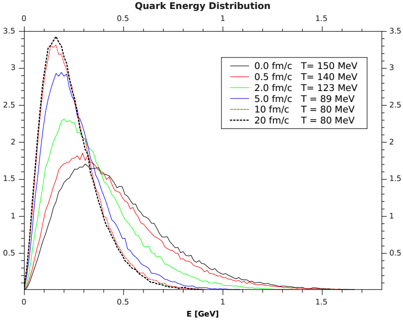


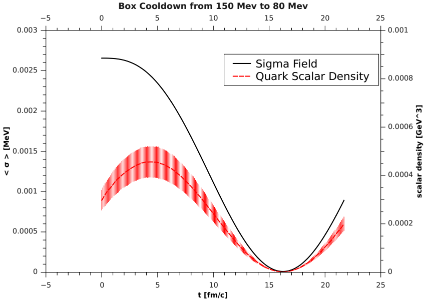
$$T : \nearrow \qquad \langle \bar{\psi}\psi \rangle : \searrow \qquad \sigma : \nearrow$$

We expected the opposite!

$\rightarrow$ No phase transition

Cooling the Box





$$T : \searrow \qquad \langle \bar{\psi}\psi \rangle : \nearrow \qquad \sigma : \searrow$$

Again, no phase transition.
σ-field starts to oscillate because of change in potential.

Non-Equilibrium effects of the density

with $\nabla\sigma = 0$ and $\pi = 0$:

$$\partial_t \, \sigma(t) + \lambda^2 \left(\sigma(t)^2 - \nu^2\right) \sigma(t) = -g\langle\bar{\psi}\psi\rangle + f_\pi m_\pi^2$$

for single-particle distribution-function:

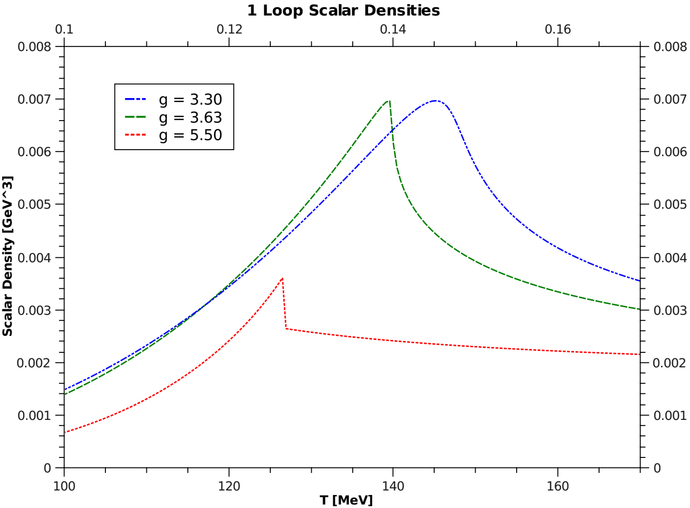
$$\begin{aligned}\langle\bar{\psi}\psi(\vec{r})\rangle &= g\sigma(\vec{r}) \int d^3\vec{p}\, \frac{f(\vec{r},\vec{p}) + \tilde{f}(\vec{r},\vec{p})}{E(\vec{r},\vec{p})} \\ &= g\sigma(\vec{r})\, \langle n(\vec{r},T)\rangle \left\langle \frac{1}{E(\vec{r},T)} \right\rangle\end{aligned}$$

for massless fermi-gas:

$$\langle n(T)\rangle = d_q \frac{3\,\zeta(3)}{4\pi^2} T^3 \qquad \left\langle \frac{1}{E(T)} \right\rangle = d_q \frac{\pi^2}{18\,\zeta(3)} T^{-1}$$

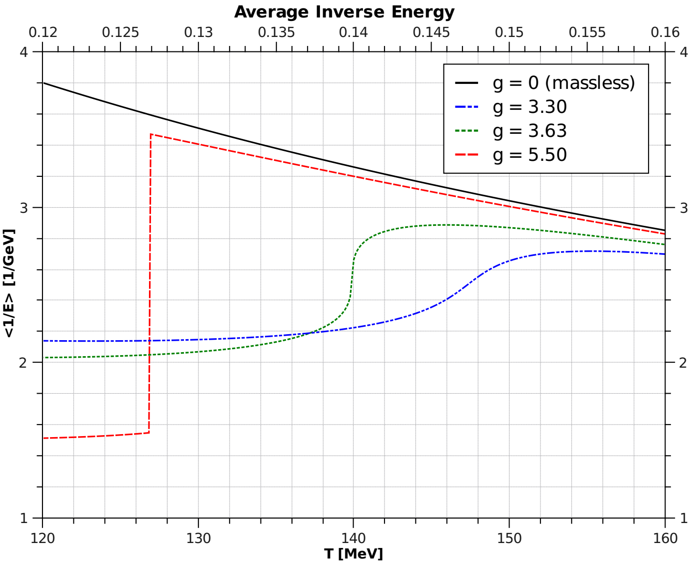
$$\langle n(T)\rangle \left\langle \frac{1}{E(T)} \right\rangle = \frac{1}{24} \frac{T_{\text{chem}}^3}{T_{\text{therm}}}$$

Non-Equilibrium effects of the density



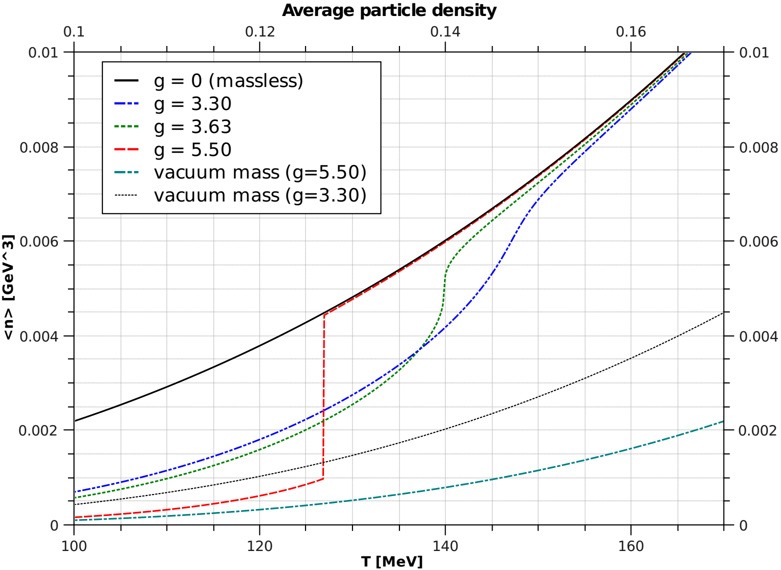
Equilibrium value

Non-Equilibrium effects of the density



Equilibrium value

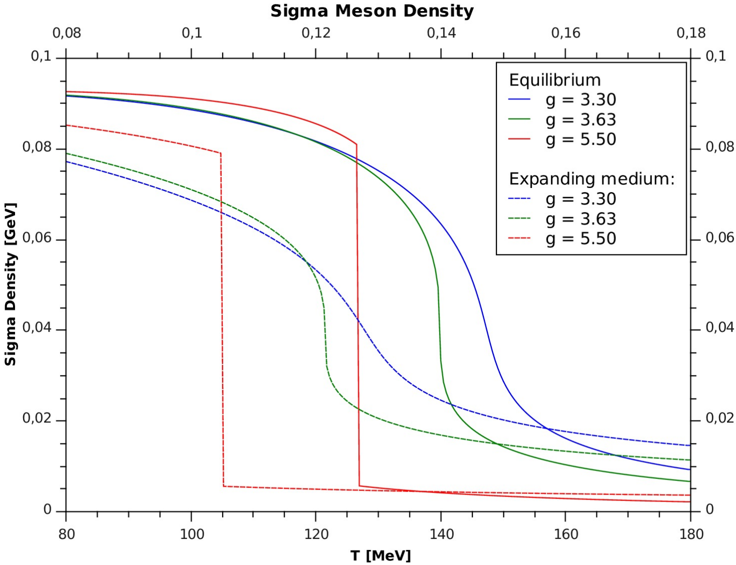
Non-Equilibrium effects of the density



Equilibrium value

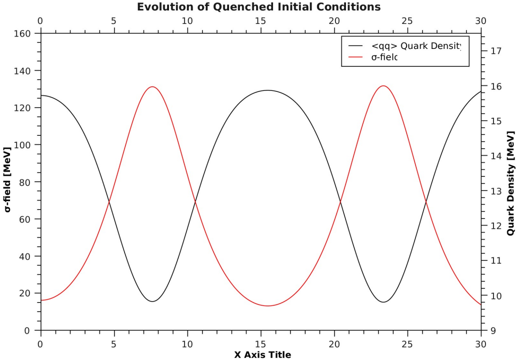
Non-Equilibrium effects of the density

Temperature shift of phase transition



Non-Equilibrium Quench

- initialize system in equilibrium (e.g. $T = 160\text{MeV}$)
- reinitilize quark energy and density (e.g. $T_q = 140\text{MeV}$)
- no spatial gradients



- damping of collective behavior?
- chemical equilibration? for study on the same model within a Langevin approach in a hydro background: [M. Nahrgang, S. Leupold, C. Herold, M. Bleicher, PRC **84**,024912 (2011); M. Nahrgang, S. Leupold, M. Bleicher, PLB **711**, 106 (2012)]

Non-Equilibrium effects of the density

Expansion scenario

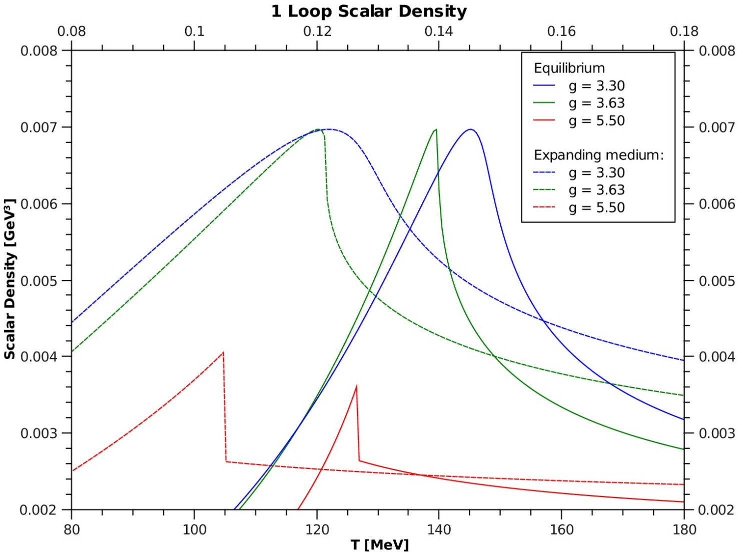
- initial thermal blob
- cooling and density thinning by expansion
- slow expansion (σ in equilibrium)

$$\begin{aligned} \text{no particle production:} \quad & n(t) \cdot V(t) && = n_0 \cdot V_0 \\ \text{adiabatic expansion:} \quad & T(t) V(t)^{\gamma-1} && = T_0 \, V_0^{\gamma-1} \end{aligned}$$

assuming an ideal gas: $\gamma = 5/3$

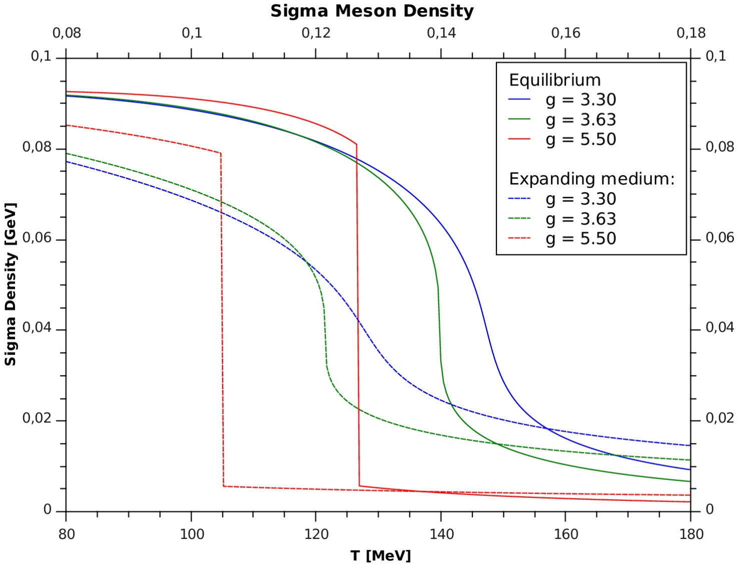
$$n(T) = n_0 \left(\frac{T}{T_0} \right)^{3/2}$$

Temperature shift of phase transition



Non-Equilibrium effects of the density

Temperature shift of phase transition



Conclusions and Outlook

- off-equilibrium study of a quark-meson linear σ model
  - mean-field approximation + binary qq collisions
  - coupling of quarks to a heat bath
  - in this "chemically frozen" scenario pseudo-phase transition behavior
  - no true phase transitions yet $\Rightarrow$ need full model with constistent quark-meson + mean field
- further developments
  - add quark-meson reactions
  - include chemical processes
  - investigate signatures of critical point in dynamical off-equilibrium environment (fluctuations)
  - foundations from non-equilibrium QFT (Kadanoff-Baym, 2PI, symmetries)
  - how to implement Polyakov loop? "Coarse-grained transport" for realization in hydro-Langevinapproach, see [C. Herold, M. Nahrgang, I. Mishustin, M. Bleicher, PRC **87**, 014907 (2013)]