Triumph of the Symmetry

The (electro–weak) standard model and the discovery of the $W^{\pm}$– and Z–Bosons

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Content

- Relativistic quantum theory and gauge symmetry
- Phenomenology of weak interactions
- The (unified) theory of electromagnetic and weak interactions
- The discovery of $W^{\pm}$ and Z
- Status quo of experimental standard–model tests

Relativistic quantum theory

- Problems in "first quantization":
- No single particle wave function for free particles with
  - energy bounded from below: $E = \pm\sqrt{\vec{p}^2 + m^2}$
  - and "conserved current" with positive definite "charge"
    $\Rightarrow$ No probability interpretation for single particle wave functions
- Reason: Uncertainty relation $\Delta x \Delta p \geq 1/2$
  In principle particle can be sharply localized (small Δx) but then $\Delta p > m$
- Particles can be produced and annihilated $\Rightarrow$ need many particle theory!
- The way out: "second quantization" = "quantum field theory"
- Reinterpret "negative energy solutions" of relativ. wave equations as anti–particles
- micro–causality

$$[\hat{O}_1(t_1, \vec{x}_1), \hat{O}_2(t_2, \vec{x}_2)]_- = 0 \text{ for } |t_1 - t_2| < |\vec{x}_1 - \vec{x}_2|$$

Measurements of observables cannot influence each other if this would need faster than light travel of signals!

- existence of a lowest energy state (vacuum)
  Hamiltonian bounded from below
- Pauli 1940 Spin–statistics theorem: Particles with integer (half integer) spin must be bosons (fermions)

Field–Quantization

- Starting point: Hamilton's principle

$$S = \int \mathrm{d}^4x \mathscr{L}(\phi, \partial_\mu \phi)$$

- Lagrangian $\mathscr{L}$: Polynomial of fields $\phi(x)$ and its four–dim. gradient $\partial_\mu \phi(x)$
- Lorentz invariance $\Leftrightarrow$ $\mathscr{L}$ scalar field
- Locality $\Leftrightarrow$ $\mathscr{L}$ depends only on one space–time point x

- Classical Equations of motion

$$\delta S = 0 \Rightarrow \frac{\partial \mathscr{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)}$$

Quantization

- Hamilton formalism: Canonical field momenta

$$\Pi = \frac{\partial \mathscr{L}}{\partial(\partial_t \phi)}$$

- Quantization: Classical Fields $\phi \rightarrow$ Field operators $\hat{\phi}$
- Equal time commutators (bosons) or anti–comutators (fermions):

$$\begin{cases} \left[\hat{\phi}(t,\vec{x}),\hat{\Pi}(t,\vec{y})\right]_- = \mathrm{i}\delta^{(3)}(\vec{x}-\vec{y}) & \text{for bosons} \\ \left[\hat{\phi}(t,\vec{x}),\hat{\Pi}(t,\vec{y})\right]_+ = \mathrm{i}\delta^{(3)}(\vec{x}-\vec{y}) & \text{for fermions} \end{cases}$$

- Example: Free scalar field: $\mathscr{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi$
- Equations of motion: $(\partial_t^2 - \Delta)\phi^{(*)} = 0$ (Klein–Gordon equation)
- Quantization: Field operators in momentum basis:

$$\hat{\phi}(x) = \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\frac{1}{2\omega}\left[\hat{a}(\vec{p})\exp(-\mathrm{i}\omega t + \mathrm{i}\vec{p}\vec{x}) + \hat{b}^\dagger(\vec{p})\exp(\mathrm{i}\omega t - \mathrm{i}\vec{p}\vec{x})\right]$$

- $\hat{a}(\vec{p})$ annihilates particle with momentum $\vec{p}$
  $\hat{b}^\dagger(\vec{p})$ creates anti–particle with momentum $\vec{p}$

Symmetries

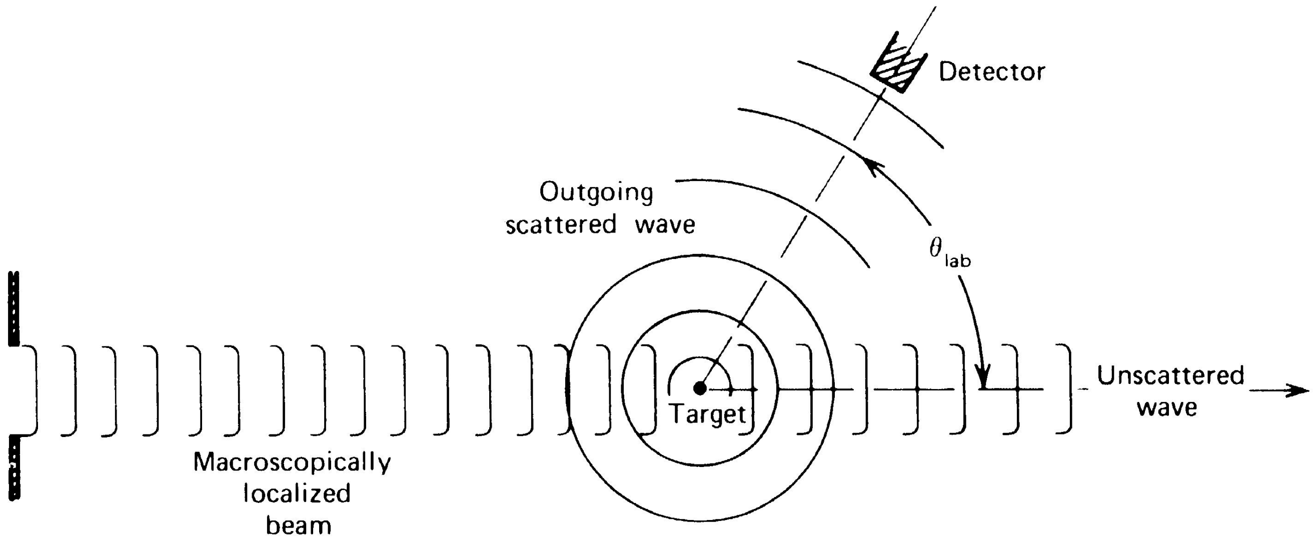
- Noether's theorem: If the action is invariant under an infinitesimal transformation: $\phi'(x') = \phi(x) + \delta\phi(x)$, $x' = x + \delta x$ then there exists a current j^μ which is conserved: $\partial_t j^0 + \vec{\nabla}\vec{j} = \partial_\mu j^\mu = 0$
- Space–time symmetries:

Symmetry	Conserved quantity
Translations in time	Energy
Translations in space	Momentum
Rotations	Angular Momentum

- Quantization: Need to chose ordering of field operators
- Physical input: Vacuum should be state of 0 energy and momentum

$$\begin{pmatrix} E \\ \vec{p} \end{pmatrix} = \begin{pmatrix} \int \mathrm{d}^3\vec{p}\,\omega(\vec{p}) \left[\underbrace{\hat{a}^\dagger(\vec{p})\hat{a}(\vec{p})}_{\text{density of particles}} + \underbrace{\hat{b}^\dagger(\vec{p})\hat{b}(\vec{p})}_{\text{density of anti–particles}} \right] \\ \int \mathrm{d}^3\vec{p}\,\vec{p} \left[\underbrace{\hat{a}^\dagger(\vec{p})\hat{a}(\vec{p})}_{\text{density of particles}} + \underbrace{\hat{b}^\dagger(\vec{p})\hat{b}(\vec{p})}_{\text{density of anti–particles}} \right] \end{pmatrix}$$

Interacting particles: Scattering

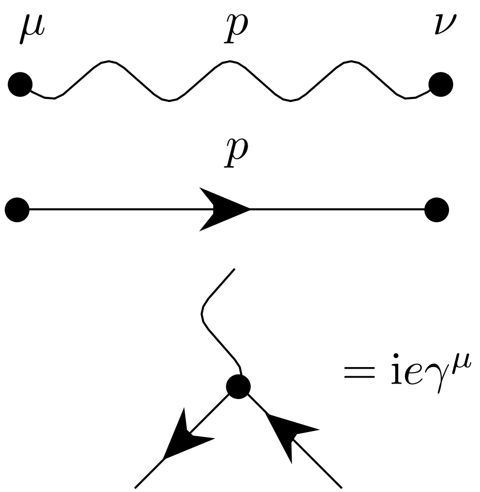


- Scattering cross section: $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\text{number of scattered particles per solid angle per time}}{\text{incomming particle flux}}$
- To calculate: Transition amplitude $T_{fi} = \left\langle f \left| \hat{T} \right| i \right\rangle$
- S(cattering)–Matrix $S_{fi} = \delta_{fi} + \mathrm{i}(2\pi)^4 \delta(P_f - P_i) \left\langle f \left| \hat{T} \right| i \right\rangle$
- $S_{fi} = \langle f, t \to \infty | \, i, t \to \infty \rangle$
  $|i, t \to \infty\rangle = \hat{S} \, |i, t \to -\infty\rangle$
- S_{fi} can be calculated only in perturbation theory
- $\hat{S}$ is unitary $\Rightarrow$ Overall normalization of probability time–independent!

Feynman–diagrams

- Logics of Model Building:
  (1) Find out symmetries ⇔ conservation of quantities in scattering experiments
  (2) Write down Lagrangian obeying the symmetries
  (3) Calculate cross sections, life times and check with experiment
- Invention by R.P. Feynman (1948) diagram rules
- From given $\mathscr{L}$ one derives diagram rules for perturbation series of scattering processes
- Example: QED

Internal lines: Propagators

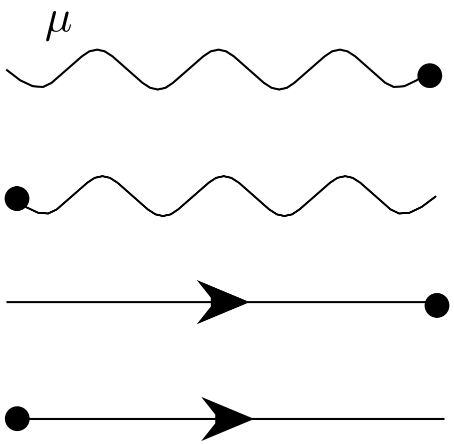


$= \mathrm{i}G_\gamma^{\mu\nu}(p)$

$= \mathrm{i}G_e(p)$

$= \mathrm{i}e\gamma^\mu$

External lines: Initial and final states



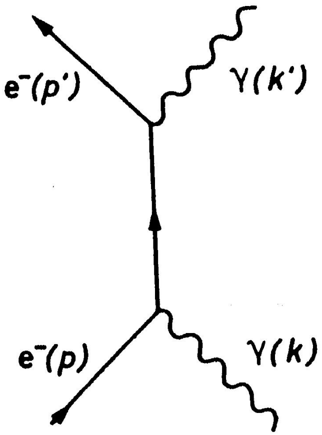
ϵ^μ

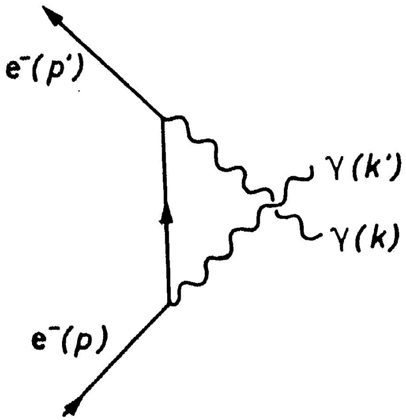
$(\epsilon^\mu)^*$

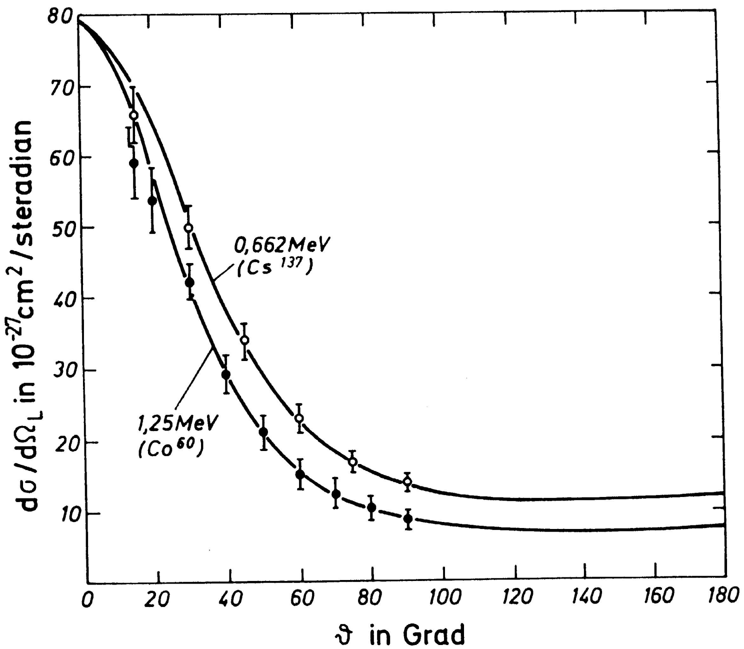
e^+ in final state or
e^- in initial state

e^+ in initial state or
e^- in final state

Compton–scattering







Compton–scattering: Lowest order perturbation theory (Klein–Nishina cross section)
Experimental values: Hofstadter, R. and McIntyre, J.A., Phys. Rev. **76**, 1269 (1949);
Evans, R.D., Handbuch der Physik **34**, Ed. S. Flügge, Springer, Berlin (1958)
Curves: Klein–Nishina cross section

QED-Lagrangian

- Four–vector potential A_μ: massless vector field
- couples to conserved electromagnetic current

$$\mathscr{L} = -\frac{1}{4}\underbrace{(\partial_\mu A_\nu - \partial_\nu A_\mu)}_{F_{\mu\nu}}(\partial^\mu A^\nu - \partial^\nu A^\mu) - j_\mu A^\mu$$

- Equations of motion $\Rightarrow$ Maxwell equations

$$(j^\mu) = \begin{pmatrix} \rho \\ \vec{j} \end{pmatrix}, \quad (A^\mu) = \begin{pmatrix} \Phi \\ \vec{A} \end{pmatrix}, \quad \vec{E} = -\partial_t \vec{A} - \nabla\Phi,\ \vec{B} = \nabla \times \vec{A}$$

- invariant under gauge transformations

$$A'_\mu = A_\mu + \partial_\mu \chi \Leftrightarrow \Phi' = \Phi + \partial_t \chi, \quad \vec{A}' = \vec{A} - \nabla\chi$$

- Electrons and positrons: Dirac particles $\Rightarrow$ current $j^\mu = -e\bar{\psi}\gamma^\mu\psi$

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(\mathrm{i}\partial_\mu + eA_\mu)\gamma^\mu\psi$$

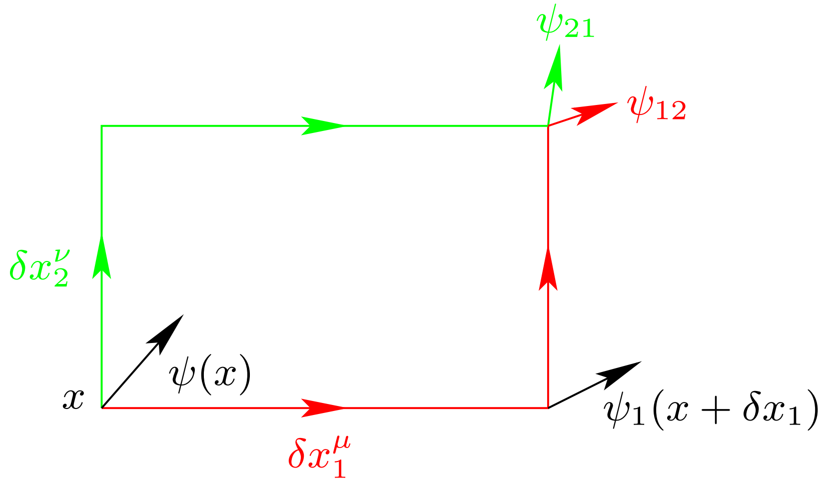
Gauge invariance

- Lagrangian invariant under local transformations

$$\psi'(x) = \exp[-\mathrm{i}e\chi(x)]\psi(x), \quad \bar{\psi}'(x) = \exp[+\mathrm{i}e\chi(x)]\bar{\psi}(x), \quad A'_\mu = A_\mu + \partial_\mu \chi(x)$$

- Vector field makes global phase–invariance of ψ local!
- Yang and Mills 1956 Physics invariant under local changes conventions (gauges) for “charge spaces”
- For each local gauge invariance $\Rightarrow$ one vector field = “gauge field”
- Veltmann and ’t Hooft 1971 (Nobelpreis 1999): Gauge theories renormalizable, i.e., have unitary S–matrix and a finite number of coupling–constants
- Now: Standardmodel of elementary particle physics is a gauge theory of the gauge group $SU(3)_{\text{strong}} \times SU(2)_{\text{weak}} \times U(1)_{\text{electro}}$

Geometry of gauge invariance



- Physicist at $x+\delta x_1$ defines "iso–spin" different from physicist at x
- Infinitesimal gauge transformation dependent on x necessary to compare wave functions at different space–time points

$$\psi_1(x+\delta x_1) = \psi(x) + \delta x_1^\mu[\partial_\mu \psi(x) + \mathrm{i}A_\mu^a(x)T^a]\psi(x) := \psi(x) + \delta x_1^\mu \mathscr{D}_\mu \psi(x)$$

- Definition of wave function depends on path of the "transport" from one space–time point to another:

$$\psi_{12}(x) - \psi_{21}(x) = \mathrm{i}g\delta x_1^\mu \delta x_2^\nu[\partial_\nu A_\mu^c - \partial_\mu A_\nu^c - g f^{cba} A_\nu^a A_\mu^c]T^c\psi(x) := \mathrm{i}g\delta x_1^\mu \delta x_2^\nu \mathcal{F}^{\mu\nu}(x)\psi(x)$$

- $\left[T^b, T^c\right]_- = \mathrm{i}f^{cba}T^c$
- For non–commutative groups: $\mathscr{F}^{\mu\nu}$ depends on coupling g!

A Brief History of weak interactions

1927	Ellis and Wooster	$^{210}_{83}\text{Bi} \xrightarrow{\beta} {}^{210}_{84}\text{Po}$: Violation of energy conservation?
1930	Wolfgang Pauli	Postulate of existence of Neutrinos (what we call $\bar{\nu}_e$ nowadays)
1933	Enrico Fermi	Four–fermion coupling theory for weak interactions
1953	Reines et al.	First direct experimental proof for existence of neutrinos
1956	Yang, Lee	Solution of the "ϑ–τ"–puzzle: "ϑ" and "τ" are one and the same particle, namely what we call K^+ nowadays; Weak interaction violates parity conservation
1957	Wu et al.	Direct experimental proof of parity non–conservation with polarized ^{60}Co
1957	Salam, Feynman et al.	Maximal violation of parity conservation, $V - A$–structure
1962	Ledermann et al	Two different sorts of neutrinos, discovery of ν_μ
1963	Cabibbo	Explanation for strangeness changing weak decays, "saving" universality of weak coupling constant $\rightarrow$ quark mixing
1973	Hasert et al	Discovery of neutral currents in reactions like $\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$, Neutral currents never change quark flavour

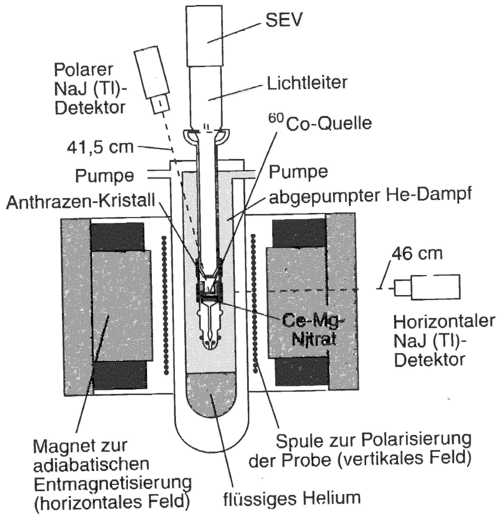
Fermi's theory of weak interactions

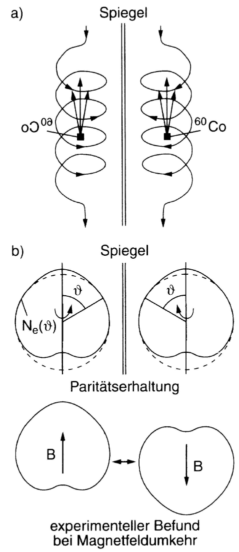
- Idea: Weak interaction involves always 4 fermions
- Analogy: Successful QED ⇒ Photon couples to current, is created and absorbed in reactions
- Instead of elementary photon ⇒ direct coupling of fermion currents

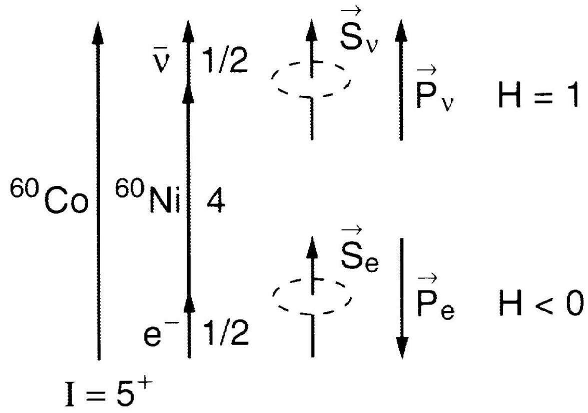
$$\mathscr{L}_{\text{int}} = -G(\bar{p}\gamma^{\mu}n)(\bar{e}\gamma_{\mu}\nu)$$

- Successful description of β–decay data, but not complete (Gamow–Teller transitions) ⇒ More Fermion currents needed, if one likes to stay with four–fermion couplings
- Bilinear forms of fermions: Systematics under parity transformations: scalar, pseudo–scalar, vector, axial vector, tensor
- "τ^{+}" $\to \pi^{+}\pi^{+}\pi^{-}$, "$\vartheta^{+}$" $\to \pi^{+}\pi^{0}$; life time and mass identical, but cannot be identical if parity is conserved
- Lee and Yang: Parity conservation is violated by weak interactions

The Wu experiment: Proof of P–violation







- Anti-neutrino: only right–handed
- helicity $= \vec{s}\vec{p}/(|\vec{s}||\vec{p}|) = 1$
- angular momentum conservation: $s_z^{(e)} = s_z^{(\bar{\nu})}$
- momentum conservation $\Rightarrow \vec{p}_e = -\vec{p}_{\bar{\nu}}$
- $\hat{p}_e = -\hat{s}_{\mathrm{Co}}$

The electro–weak standard model

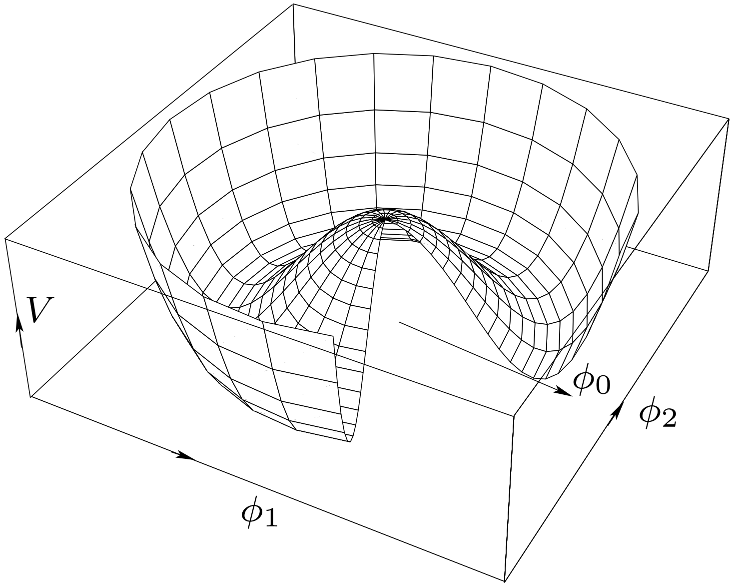
- Extension of Fermi–theory: Only left–handed particles (right–handed anti–particles) interact weakly:

$$\mathscr{L} = -\frac{G}{2}[\bar{\nu}\gamma^{\mu}(1-\gamma_5)e][\bar{n}\gamma_{\mu}(1-\gamma_5)p]$$

- Big theoretical flaws: non–renormalizable and non–unitary S–matrix
- Idea: Massive gauge bosons give at low energies four–fermion interactions like in Fermi–model $\Rightarrow$ masses for vector bosons $\approx 80 - 90\,\text{GeV}$
- Two charged heavy vector bosons: $W^{\pm}$
  One neutral heavy vector boson W^{0}
- but: Massterms for vector–mesons destroy gauge invariance
- Neutral current coupling depends on electric charge
- Gauge theory: $SU(2)$ gauge group: weak isospin (left–handed particles only!)
- Explanation: Neutral W^0 mixes with another gauge field B which couples to “weak hyper charge” $\Rightarrow$ Mass eigenstates are the massive Z^0 and the photon A
- Correct gauge group: $SU_W(2) \times U_Y(1)$:
  four gauge bosons spontaneously broken to $U_{\text{em}}(1)$
- Three massive vector bosons (charged W– and neutral Z–boson), one massless (photon)

The Higgs–mechanism and masses

- Remaining problems: Howto preserve gauge invariance and make W–bosons massive?
- Solution: Higgs (1961), Glashow, Salam, and Weinberg ($\approx$ 1967)
  Spontaneous breakdown of gauge–symmetry



- Potential symmetric under rotations in ϕ_1–ϕ_2–plane
- Stable ground state not symmetric $\Rightarrow$ degeneration of ground state
- $\phi = \exp[\mathrm{i}\vec{\chi}(x)\vec{T}]\phi_0(x))$

- Gauge–transformations local $\Rightarrow$ Can gauge the "polar" degrees of freedom away $\chi(x) = 0$
- Three $SU_W(2)$–bosons become massive ✔
- Photon stays massless ✔
- 1979 Nobel prize for Glashow, Salam, and Weinberg

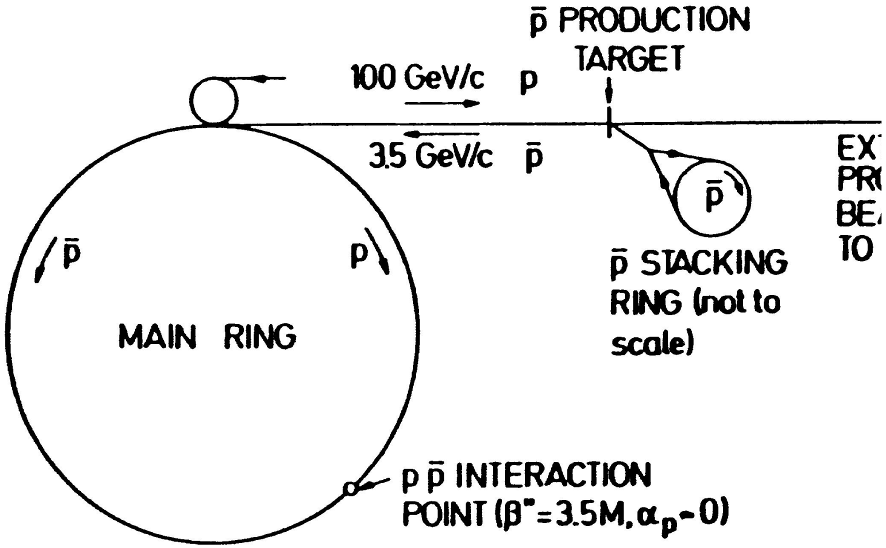
The discovery of the $W-$ and $Z-$bosons

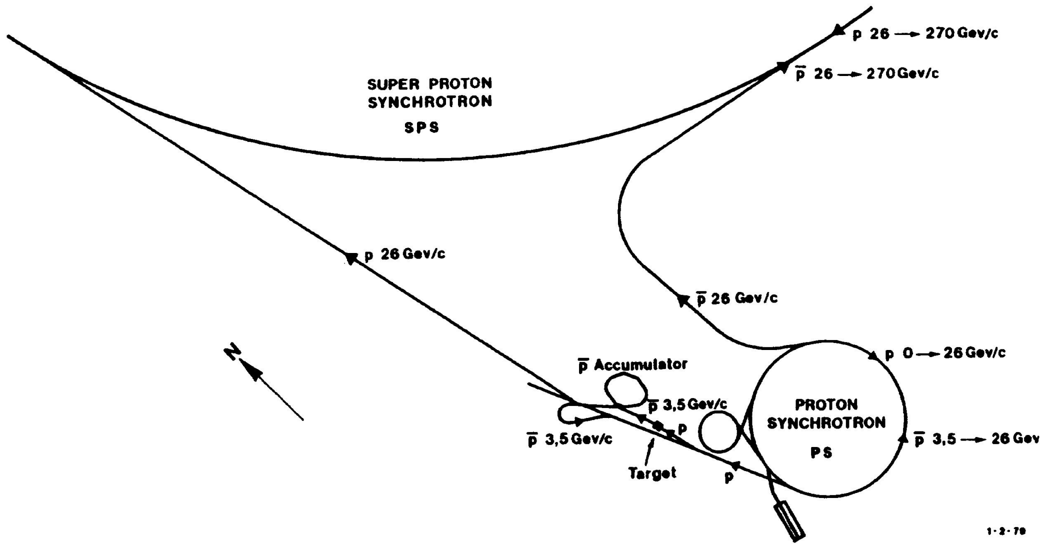
- 1983: Discovery of the $W-$ and $Z-$bosons by C. Rubbia and S. van der Meer (Nobel prize 1984)
- Need energy to produce W and Z: $m \approx 80 - -90\text{GeV} = \sqrt{s_{\text{min}}}$
- In 1983: Not available for e^+ and e^-;
  another reason $W^\pm$ can only be produced pairwise $\Rightarrow$ even more energy necessary
- p–$\bar{p}$ collisions in following reactions

$$u + \bar{u} \rightarrow Z^0,\ d + \bar{d} \rightarrow Z^0,\ d + \bar{u} \rightarrow W^-,\ u + \bar{d} \rightarrow W^+$$

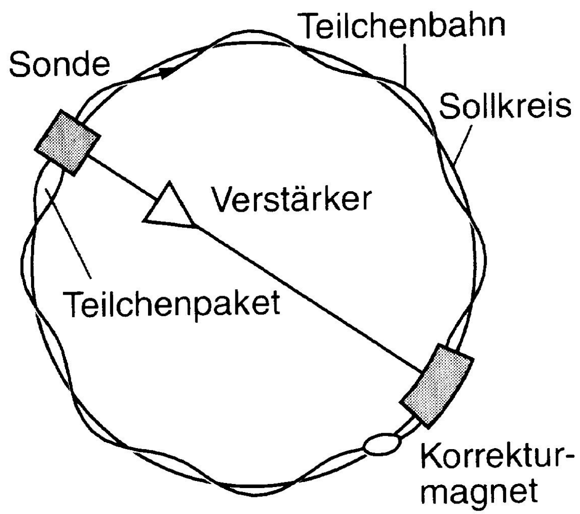
- (anti–) protons are bound states of quarks, each quark carries only fraction of full momentum
- From deep inelastic lepton–proton scattering: $\langle x_v \rangle \approx 0.12$, $\langle x_s \rangle \approx 0.04$ Quark fraction of momentum, half is carried by virtual gluons!

Experimental setup





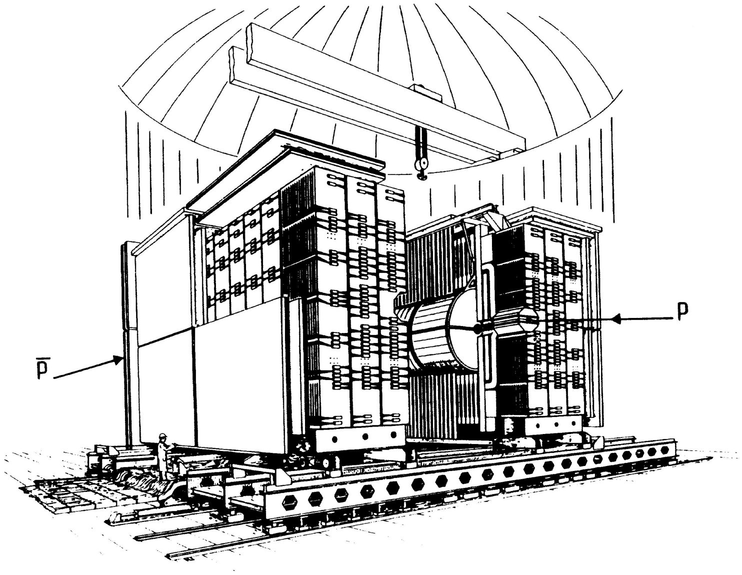
- Important invention bei van der Meer: Stochastic cooling



- Liouville's theorem: flow in phase space incompressible
- but: point–like objects with free space in between
- local density in phase space conserved but macroscopic density enhanced!

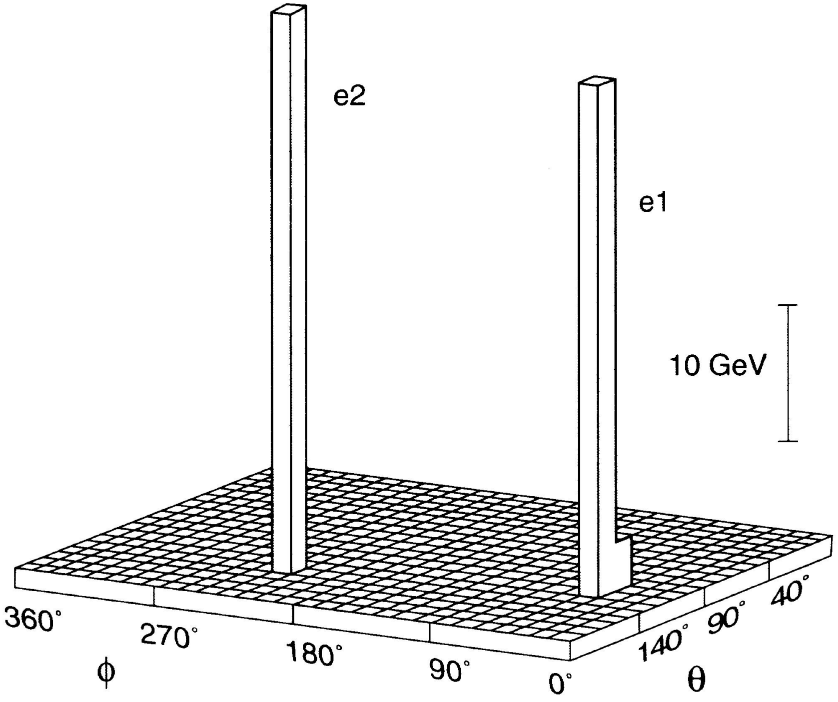
Detector

- Processes to be observed: $p + \bar{p} \to W^{\pm} + X$ and $W^{\pm} \to e^{\pm} + \nu_e$



- UA(1)–detector
- No chance to detect neutrino: measure missing momentum to track neutrinos
- Measure energy of charged particles with calorimeters

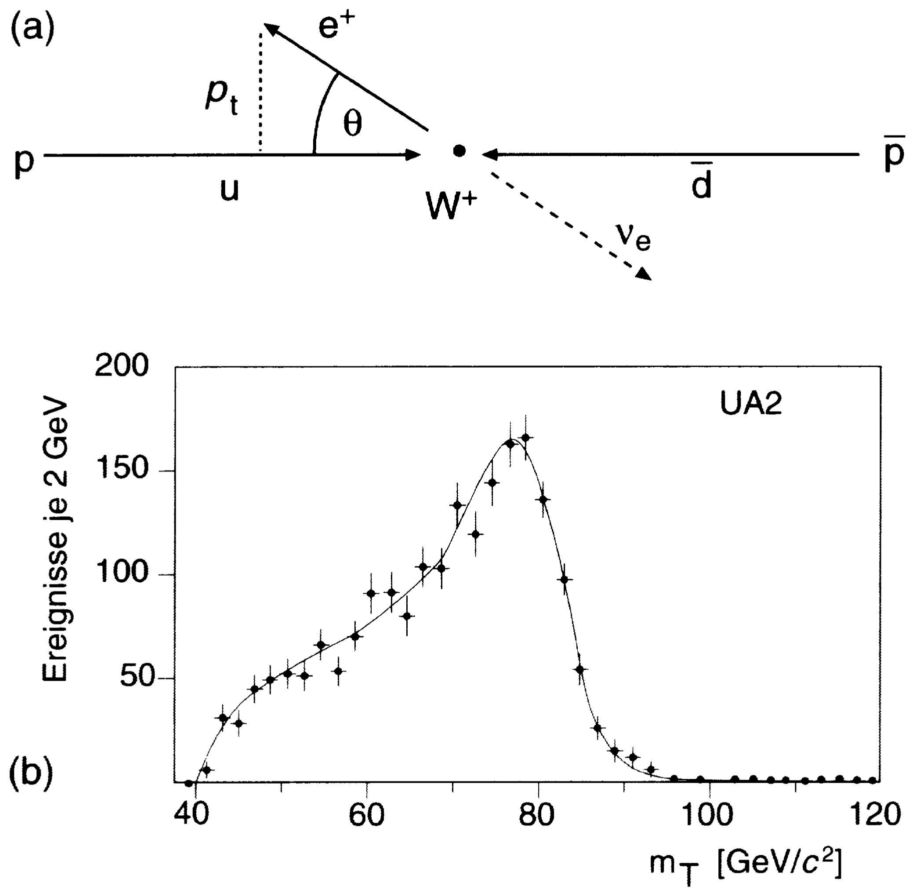
Discovery of Z^0



"Lego–diagram": polar angle θ and azimuthal angle ϕ for the decay $Z^0 \rightarrow e^+ + e^-$. The height is the energy of the particles which add to arround 90 GeV which is the Z^0–mass:

$m_Z = (91.1992 \pm 0.0026)\text{GeV}$

Discovery of $W^{\pm}$



- From $\frac{d\sigma}{dp_t} = \frac{d\sigma}{d(\cos\theta)} \frac{2p_t}{m_W c} \frac{1}{\sqrt{m_W^2 c^2/4 - p_t^2}}$
- Jacobi–maximum at $m_T c^2 = m_W c^2 = (80.419 \pm 0.056)$ GeV

Universality and "electro–weak mixing"

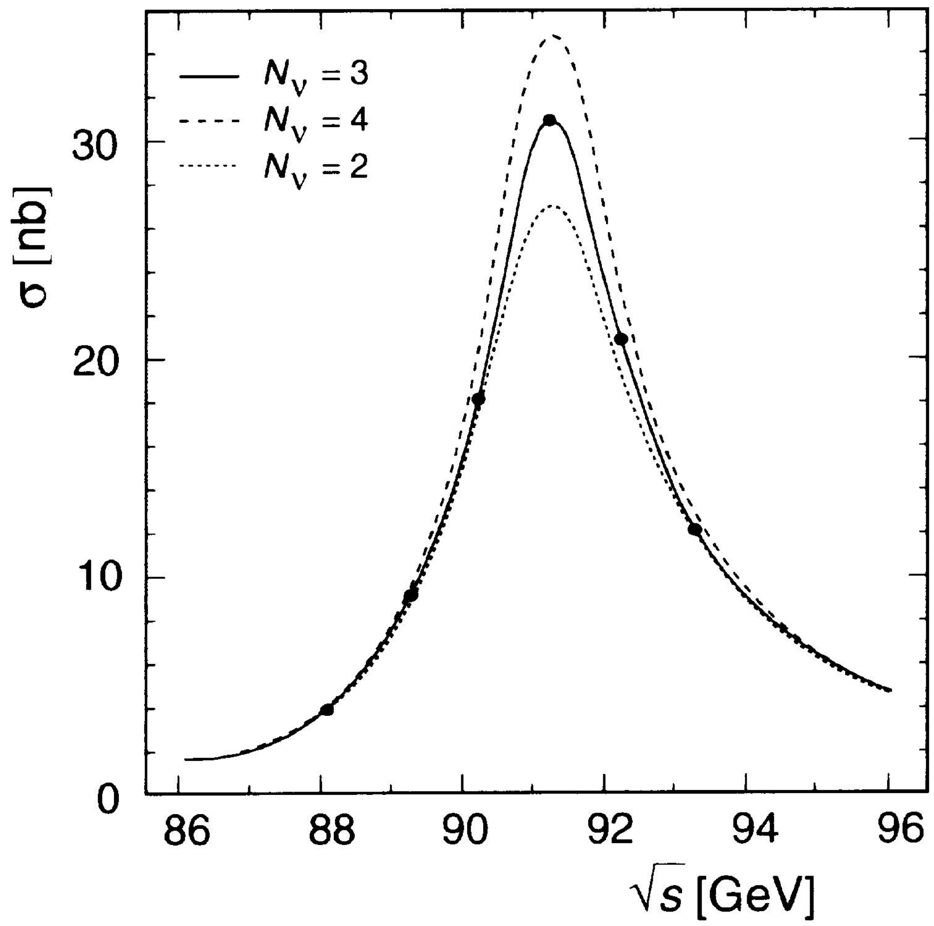
- Branching ratios from universality of charged current:
  $W^- \rightarrow (l^-, \bar{\nu}_l)$, $(\bar{u}, d')$, $(\bar{c}, s')$
  $\Rightarrow$: 1/9 of decays for each lepton, 6/9 in $(\bar{q}q)$
- Prediction for neutral current from $Z^0\gamma$–mixing:
  $g_L(f) = I_{3L}^{\text{weak}} - Q\sin^2\theta_W$, $g_L(f) = I_{3R}^{\text{weak}} - Q\sin^2\theta_W$
  Weak mixing angle: $\sin^2\theta_W = 0.23117(16)$

Decay mode	Fraction (in %)
$W^- \rightarrow (e^-, \bar{\nu}_e)$	10.66 ± 0.14
$W^- \rightarrow (\mu^-, \bar{\nu}_\mu)$	10.49 ± 0.20
$W^- \rightarrow (\tau^-, \bar{\nu}_\tau)$	10.4 ± 0.4
$W^- \rightarrow$ hadrons	68.5 ± 0.6

Decay mode	Ex. Fraction (in %)	Th. Fraction (in %)
$Z^0 \rightarrow (e^+, e^-)$	3.367 ± 0.005	3.445 ± 0.05
$Z^0 \rightarrow (\mu^+, \mu^-)$	3.367 ± 0.008	3.445 ± 0.05
$Z^0 \rightarrow (\tau^+, \tau^-)$	3.371 ± 0.009	3.445 ± 0.05
$Z^0 \rightarrow (\nu, \bar{\nu})^*$	20.02 ± 0.06	20.572 ± 0.2
$Z^0 \rightarrow$ hadrons	69.89 ± 0.07	69.092 ± 0.01

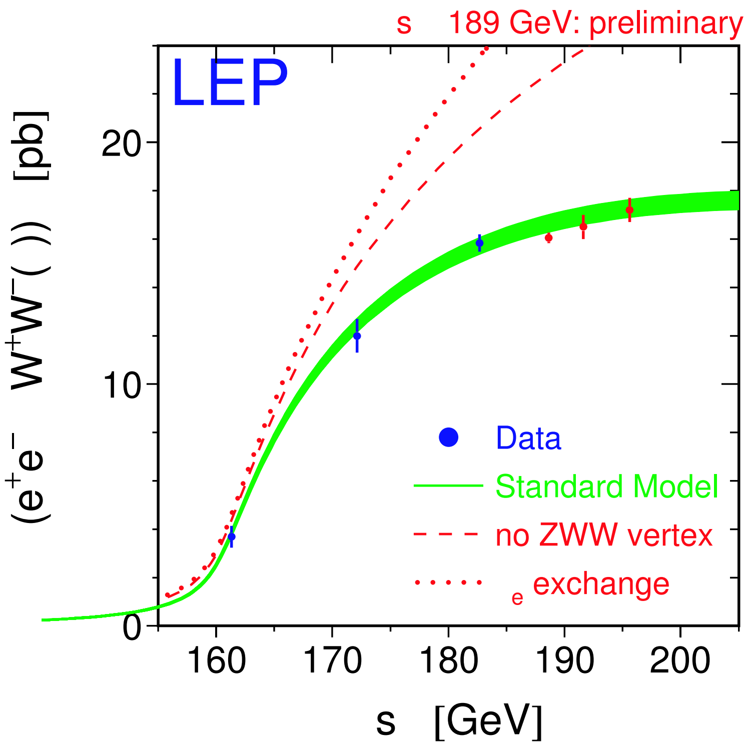
Total Z^0–width and number of families

- Cross section for $e^+e^- \to$ hadrons around the Z^0 resonance
- lines: Prediction according to standard model with N_ν families of massless neutrinos
- Experiment: OPAL@CERN



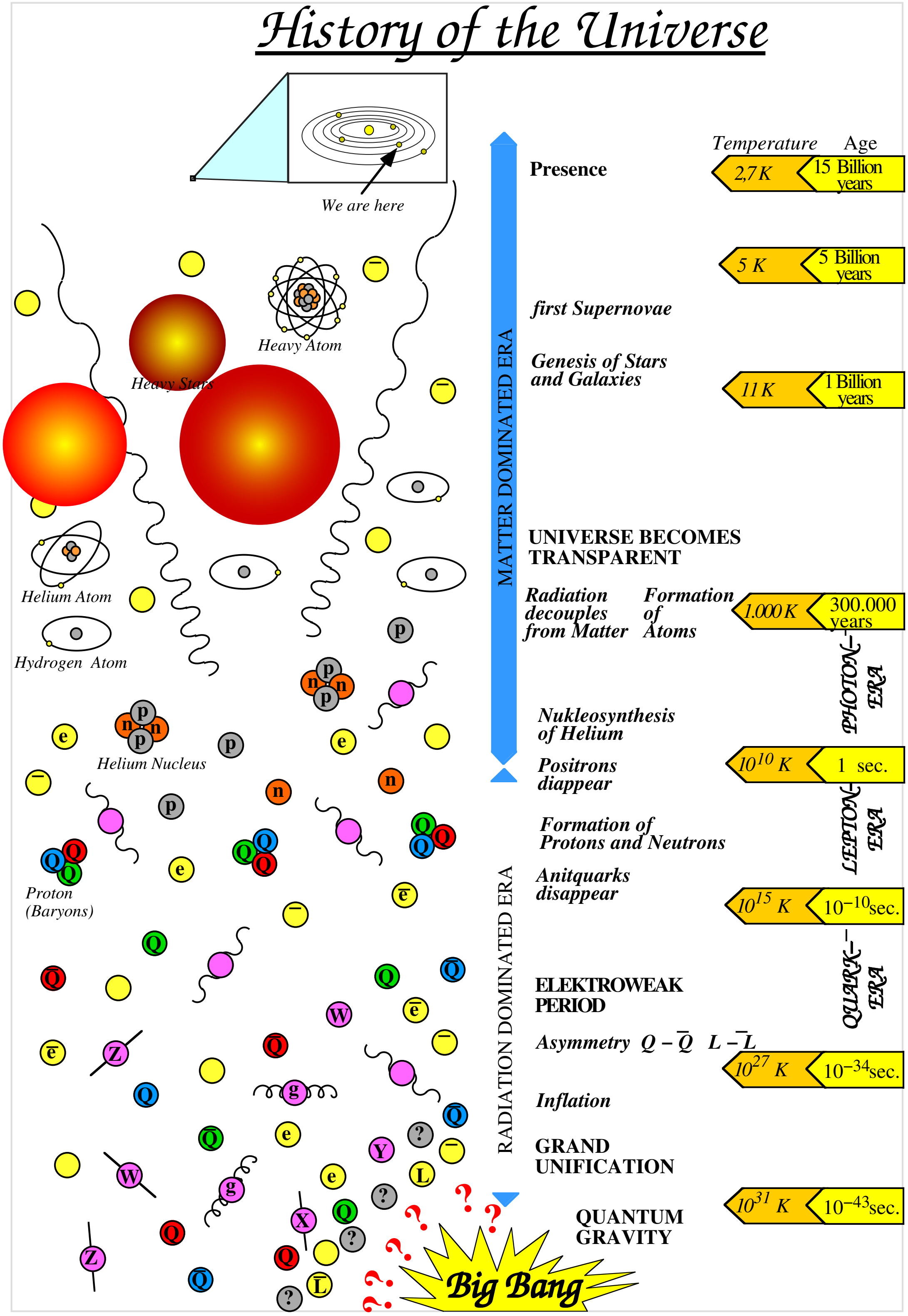
The non–abelian gauge structure

- Nowadays at LEP@CERN: $e^+e^- \to W^+W^-$ available



- From S. Bethge, Standard Model Physics at LEP, `hep-ex0001023`

Standard model and the Universe



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Conclusions and Outlook

- Great success: All observations described by standard model
- All particles observed except the Higgs $\rightarrow$ Tevatron@Fermilab, or LHC?
- but: 21 parameters for minimal model, with neutrino–oscillations (observed!) even more
- Why symmetry breaking as observed?
- CP–non–conservation understood?
- Enough to explain particle vs. anti–particle ratio in universe?
- How to include gravity?