Renormalization of Conserving Selfconsistent Dyson equations

Hendrik van Hees

GSI

Darmstadt

Motivation

- Thermodynamics of strongly interacting systems
- Conservation laws, detailed balance, thermodynamical consistency
- Finite width effects (resonance, damping, $\cdots$)

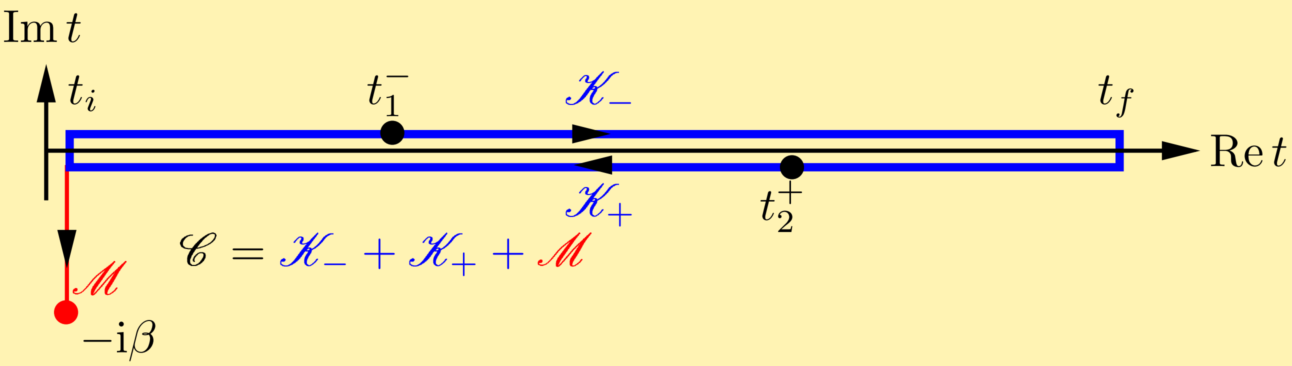
Concepts

- The Φ-derivable scheme
- Renormalization (example ϕ^4)
- Conclusions and outlook

Schwinger-Keldysh Formalism

- In equilibrium: $\boldsymbol{\rho} = \exp(-\beta \mathbf{H})/Z$ with $Z = \operatorname{Tr}\exp(-\beta \mathbf{H})$
- "Imaginary time" evolution

☞ Imaginary part of the time contour



- Correlation functions with real times: $\mathrm{i}G_{\mathscr{C}}(x_1^-, x_2^+)$
- Fields periodic (bosons) or anti-periodic (fermions)
- Introduce local and bilocal auxiliary sources:

$$Z[J,K] = N \int \mathrm{D}\phi \exp\left[\mathrm{i}S[\phi] + \mathrm{i}\left\{J_1\phi_1\right\}_1 + \left\{\frac{\mathrm{i}}{2}K_{12}\phi_1\phi_2\right\}_{12}\right]$$

- Generating functional for connected diagrams

$$Z[J,K] = \exp(\mathrm{i}W[J,K])$$

- The mean field and the connected Green's function

$$\underbrace{\varphi_1 = \frac{\delta W}{\delta J_1},\ G_{12} = -\frac{\delta^2 W}{\delta J_1 \delta J_2}}_{\text{standard quantum field theory}} \Rightarrow \frac{\delta W}{\delta K_{12}} = \frac{1}{2}[\varphi_1\varphi_2 + \mathrm{i}G_{12}]$$

- Legendre transformation for φ and G:

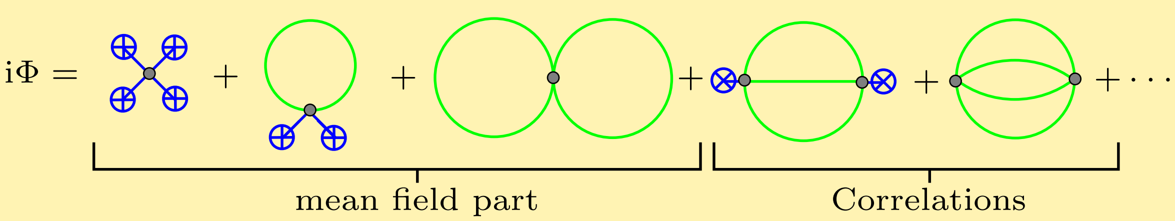
$$\begin{aligned}\Gamma[\varphi, G] &= W[J,K] - \{\varphi_1 J_1\}_1 - \frac{1}{2}\{(\varphi_1\varphi_2 + \mathrm{i}G_{12})K_{12}\}_{12} \\ &= S_0[\varphi] + \frac{\mathrm{i}}{2}\operatorname{Tr}\ln(-\mathrm{i}G^{-1}) + \frac{\mathrm{i}}{2}\left\{D_{12}^{-1}(G_{12} - D_{12})\right\}_{12} \\ &\quad + \Phi[\varphi, G] \Leftarrow \text{all closed 2PI interaction diagrams}\end{aligned}$$

“Diagrammar”

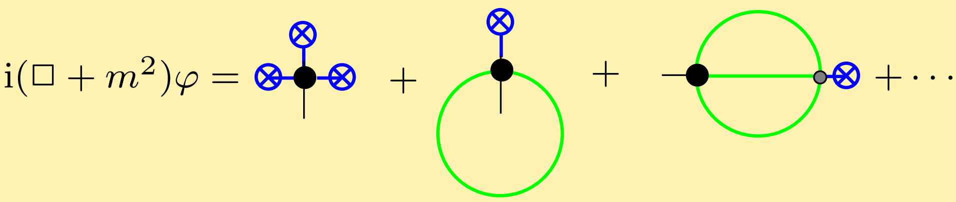
- ϕ^4-theory

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$$

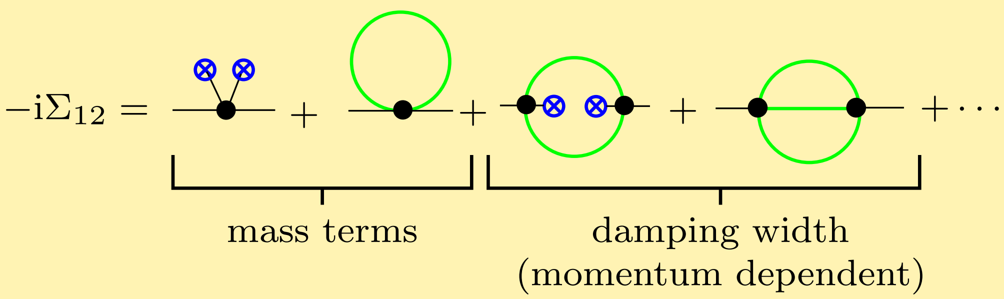
- 2PI Generating Functional



- Mean field equation of motion



- Self-energy



Properties of the Φ-derivable Approximations

Why using the Φ-functional?

- Truncation of the Series of diagrams for Φ

☞ Expectation values for currents are conserved
$\Rightarrow$ "Conserving Approximations"

- In equilibrium $\mathrm{i}\Gamma[\varphi, G] = \ln Z(\beta)$
(thermodynamical potential)
- consistent treatment of Dynamical quantities (real time formalism) and thermodynamical bulk properties (imaginary time formalism) like energy, pressure, entropy

- Real- and Imaginary-Time quantities "glued" together by Analytic properties from (anti-)periodicity conditions of the fields (KMS-condition)

- Self-consistent set of equations for self-energies and mean fields

Problem of Renormalization

Why renormalization?

- ☞ Diagrams UV-divergent
- ☞ Control the physical parameters in vacuum
- ☞ Temperature dependence from theory alone

How to renormalize self-consistent diagrams?

- ☞ In terms of perturbation theory: Resummation of all self-energy insertions in propagators
- ☞ Self-consistent diagrams with explicit nested and overlapping sub-divergences
- ☞ "Hidden" sub-divergences from self-consistency

How to manage it numerically?

- ☞ Power counting (Weinberg) valid for self-consistent diagrams
- ☞ At finite temperatures:
  Self-consistent scheme rendered finite with local counterterms independent of temperature
- ☞ Analytical properties ⇒ subtracted dispersion relations
- ☞ BPHZ-renormalization ⇒ Subtracting the integrands
- ☞ Advantage: Clear scheme how to subtract temperature independent sub-divergences
- ☞ Φ-functional ⇒ consistency of counterterms

Self-consistent Renormalization

First step: Vacuum

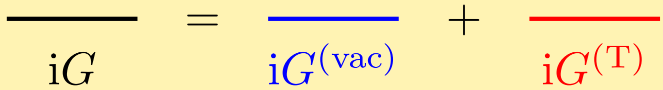
- Power-counting for self-consistent propagators as in perturbation theory: $\delta = 4 - E$
- Usual BPHZ-renormalization for wave function, mass and coupling constant renormalization
- In practice: Use Lehmann-representation and dimensional regularization

✔ Closed self-consistent finite Dyson-equations of motion

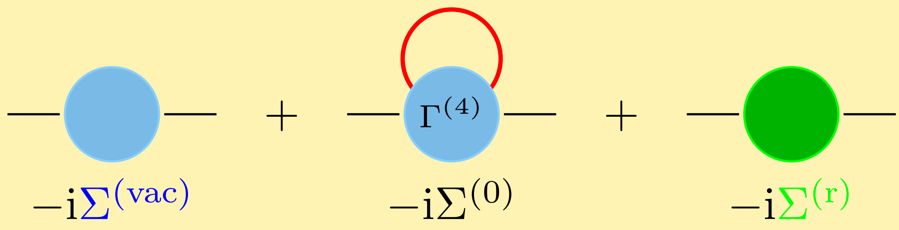
✔ Numerically treatable

Second step: Finite Temperature

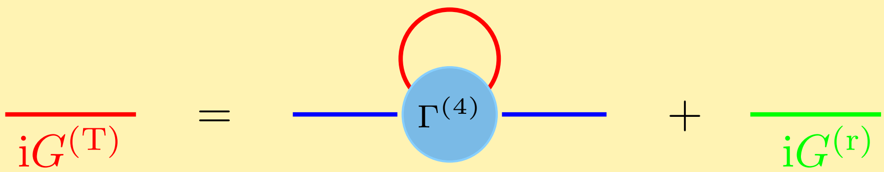
- Split propagator in vacuum and T-dependent part



- Expand self-energy around vacuum part



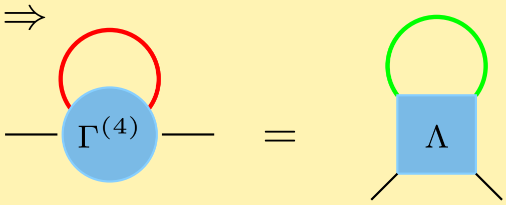
- Need further splitting of propagator



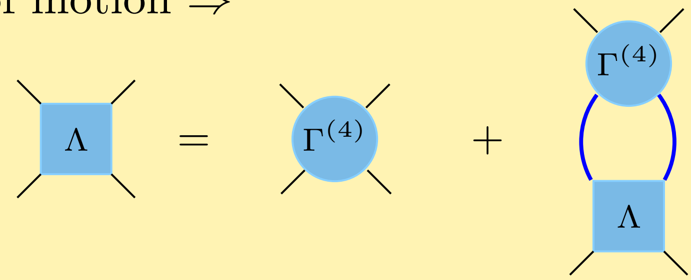
Self-consistent Renormalization

Third step: 4-point vertex renormalization

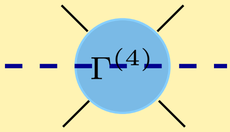
- Σ^0 linear in $G^{(\mathrm{r})}$ $\Rightarrow$



- Equation of motion $\Rightarrow$



☞ s-channel Bethe-Salpeter equation



cuts more than three lines!

$\Rightarrow$ “BPHZ Boxes” in ladder-diagrams do not cut inside $\Gamma^{(4)}$.

$\Rightarrow$ Asymptotics + BPHZ-formalism:

$$\Gamma^{(4)}(l,p) - \Gamma^{(4)}(l,0) \cong O(l^{-\alpha}) \text{ with } \alpha > 0$$

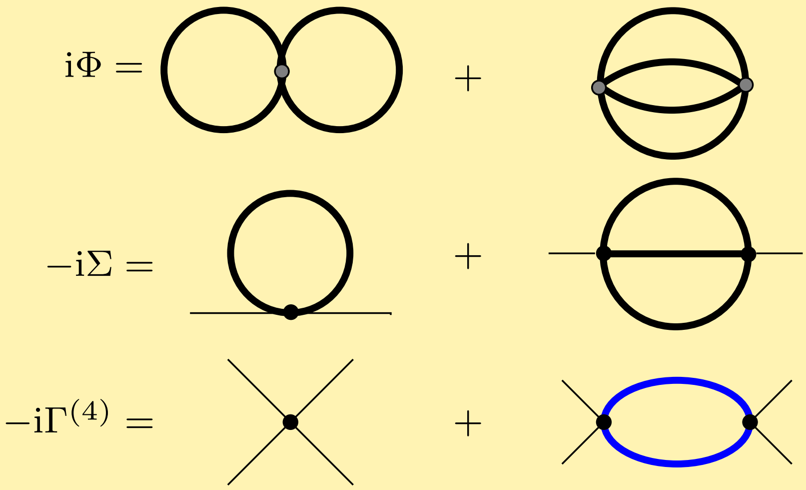
$\Rightarrow$ Renormalized eq. of motion for Λ:

$$\begin{aligned}\Lambda(p,q) = &\Lambda(0,0) + \Gamma^{(4)}(p,q) - \Gamma^{(4)}(0,0) \\ &+ \mathrm{i} \int \frac{\mathrm{d}^4 l}{(2\pi)^4} [\Gamma^{(4)}(p,l) - \Gamma^{(4)}(0,l)][G^{\mathrm{vac}}]^2(l)\Lambda(l,q) \\ &+ \mathrm{i} \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \Lambda(0,l)[G^{\mathrm{vac}}]^2(l)[\Gamma^{(4)}(l,q) - \Gamma^{(4)}(l,0)]\end{aligned}$$

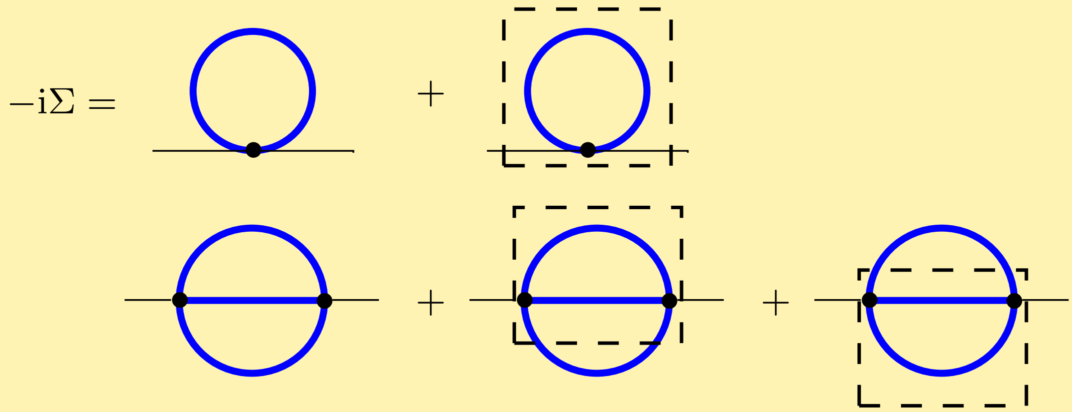
✔ Self-energy finite with vacuum counter terms

Example: Tadpole+Sunset

Approximation of the Φ-functional



Renormalization (vacuum)



+overall

- In practice: Use dispersionrelations for propagators

☞ Kernels, can be calculated analytically with standard formulae of dimensional regularization

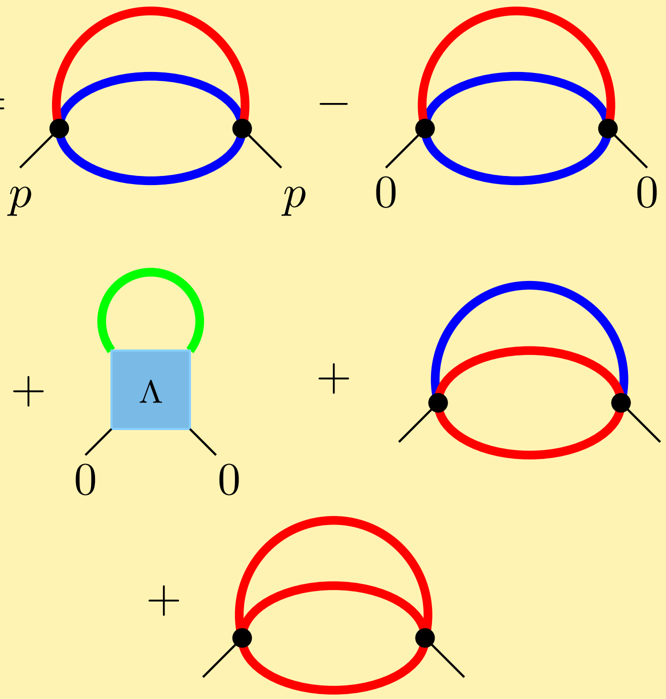
✔ Finite Self-consistent integral equations of motion ⇒ Solved iteratively

- Calculate also $\Gamma^{(4)}$ and $\Lambda(0, q)$

Example: Tadpole+Sunset

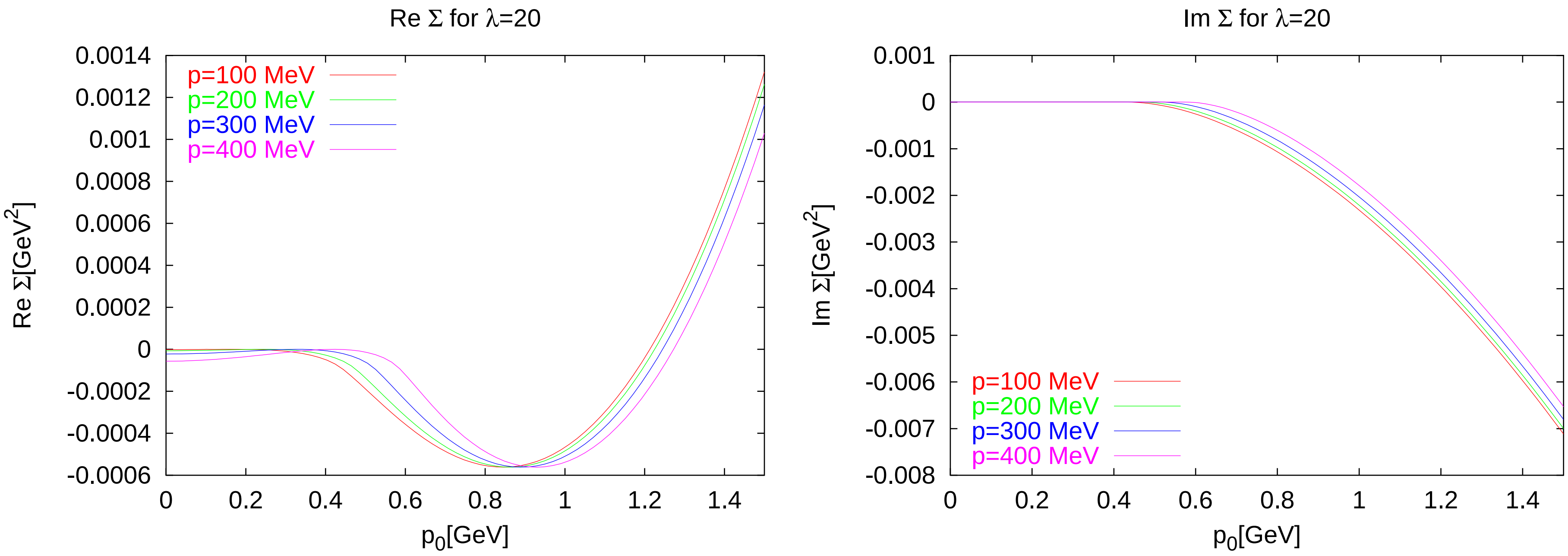
Renormalization (Finite Temperature)

$$-\mathrm{i}\Sigma^{(\mathrm{T})}(p) =$$



- Only finite integrals
- ✔ Numerics for three-dim integrals on a lattice in p_0 and $|\vec{p}|$
- ✔ Equations of motion solved iteratively

Results for "Sunset + Tadpole" at $T = 0$



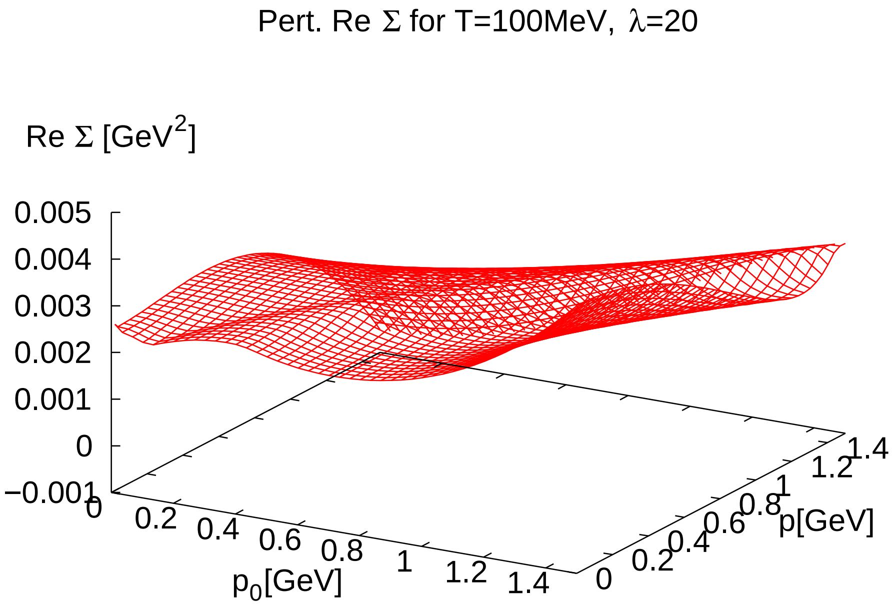
- Difference between perturbative and self-consistent calculation unvisible!

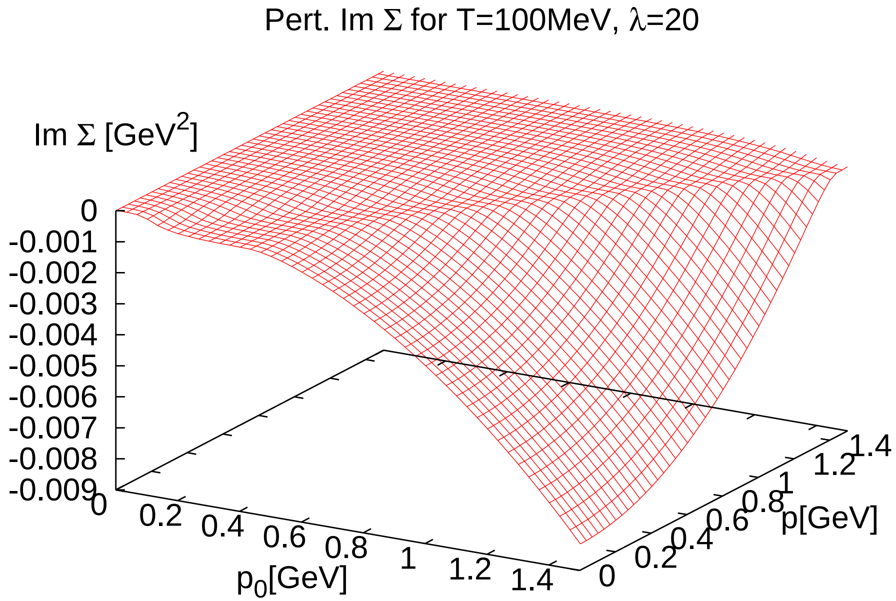
☞ Tadpole contribution "renormalized away" ⇒ on-shell renormalization scheme

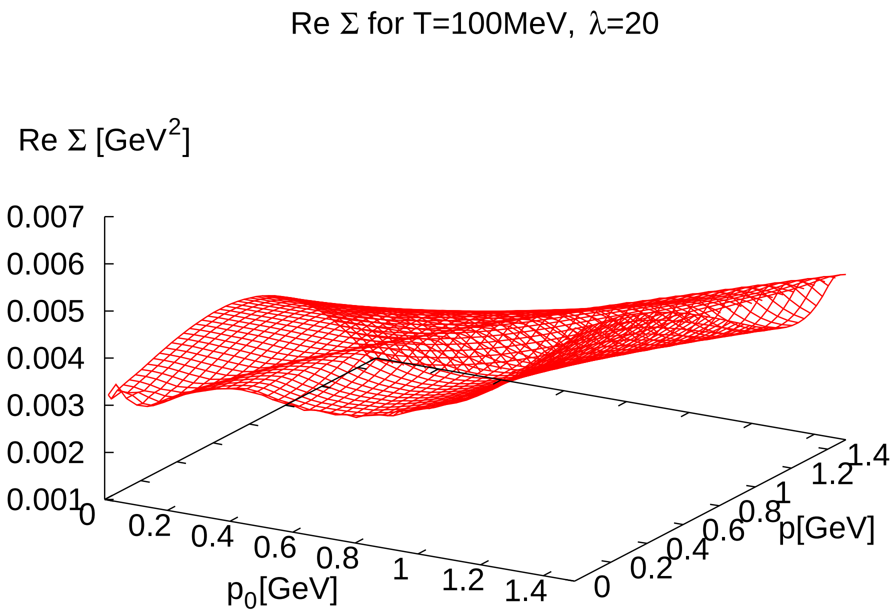
☞ Main contribution from the pole term of the propagator

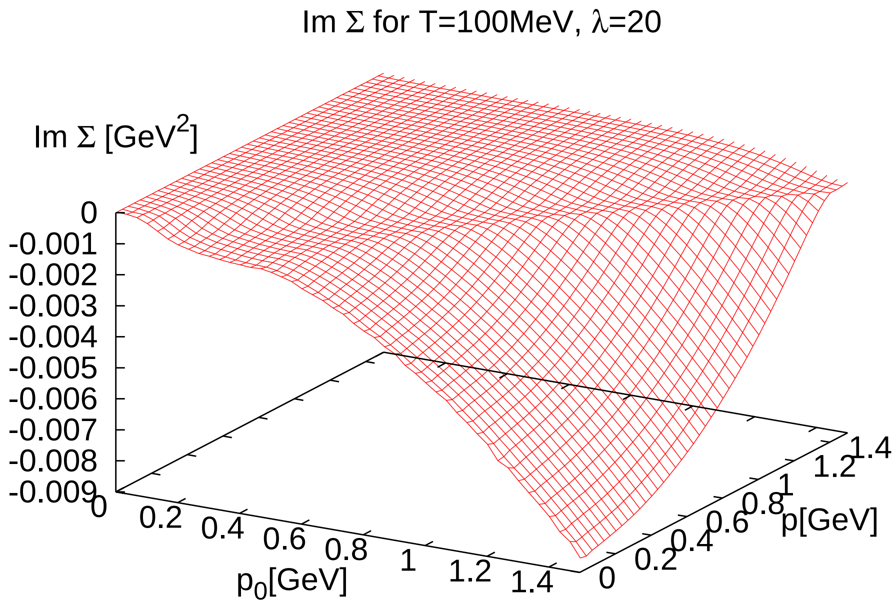
☞ Threshold for continues part of the spectral function $\sqrt{s} = 3m$!

Results for "Sunset + Tadpole" at $T > 0$









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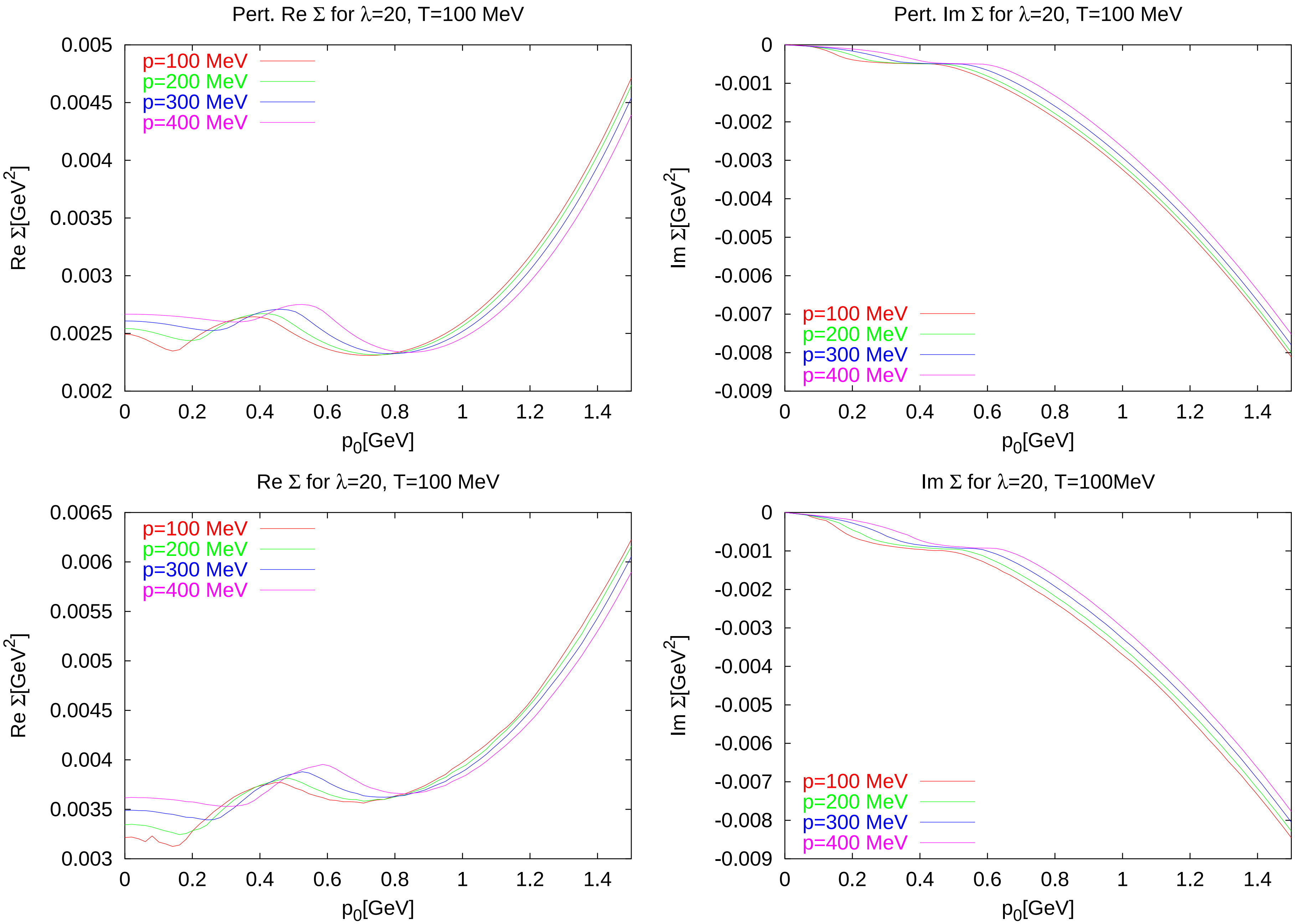

Conclusions and Outlook

Summary

- ✔ Self–consistent Φ–derivable schemes
- ✔ Renormalization:
  http://arXiv.org/abs/hep-ph/0107200
- ✔ Numerical treatment

Outlook

- ✔ "Toolbox" for application to realistic models
- ✔ Symmetry analysis for Φ-derivable approximations:
  PhD-thesis: http://theory.gsi.de/~vanhees
- ✔ Perspectives for self–consistent treatment of vector particles:
  http://arXiv.org/abs/hep-ph/0002087
- ✘ General gauge theories?
- ✘ QCD e.g. beyond HTL?
- ✔ Transport equations for particles with finite width
- ☞ See talk given by Jörn Knoll