Heavy-Quark Kinetics in the Quark-Gluon Plasma

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with

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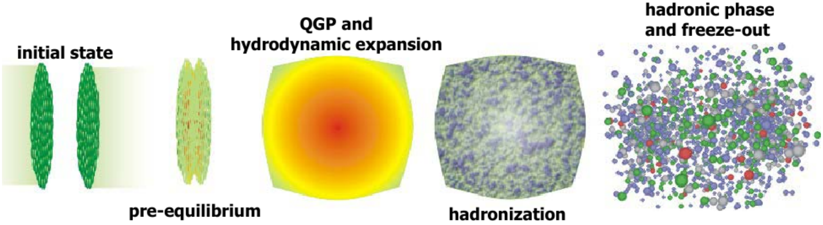
Outline

Heavy-Ion collisions in a Nutshell

- Theory of strong interactions: Quantum Chromo Dynamics, QCD
- At high enough densities/temperatures: hadrons dissolve into a Quark-Gluon Plasma (QGP)
- hope to create QGP in Heavy-Ion Collisions at RHIC (and LHC)
- RHIC: collide gold nuclei with energy of 200 GeV per nucleon:

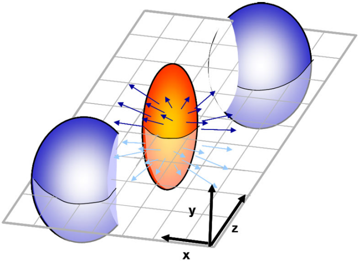


Evidence for QGP from heavy-ion observables

- particle p_T spectra show hydrodynamical behavior
- collective flow of matter in local thermal equilibrium
- nuclear modification factor ⇒ degree of thermalization

$$R_{AA}(p_T) = \frac{\mathrm{d}N_{\mathsf{AA}}/\mathrm{d}p_T}{N_{\mathsf{coll}}\mathrm{d}N_{pp}/\mathrm{d}p_T}$$

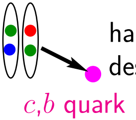
- no QGP ⇒ $R_{AA} = 1$; observed: $R_{AA} < 1$ (suppression) at high p_T
- in non-central collisions: anisotropic collective flow



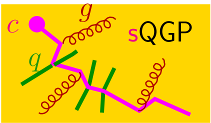
- initially reaction zone of elliptic shape
- pressure gradients: $\langle |p_x| \rangle > \langle |p_y| \rangle$
- measure of flow anisotropy:

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \langle \cos(2\phi_p) \rangle$$

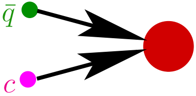
Heavy Quarks in Heavy-Ion collisions



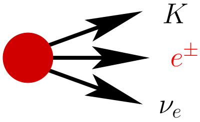
hard production of HQs
described by PDF's + pQCD (PYTHIA)



HQ rescattering in QGP: Langevin simulation
drag and diffusion coefficients from
microscopic model for HQ interactions in the sQGP



Hadronization to D,B mesons via
quark coalescence + fragmentation
V. Greco, C. M. Ko, R. Rapp, PLB **595**, 202 (2004)



semileptonic decay ⇒
"non-photonic" electron observables

Heavy-Quark diffusion

- Fokker Planck Equation

$$\frac{\partial f(t,\vec{p})}{\partial t} = \frac{\partial}{\partial p_i}\left[p_i A(t,p) + \frac{\partial}{\partial p_j} B_{ij}(t,\vec{p})\right] f(t,\vec{p})$$

- drag (friction) and diffusion coefficients

$$\begin{aligned} p_i A(t,\vec{p}) &= \left\langle p_i - p_i' \right\rangle \\ B_{ij}(t,\vec{p}) &= \frac{1}{2}\left\langle (p_i - p_i')(p_j - p_j') \right\rangle \\ &= B_0(t,p)\left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) + B_1(t,p)\frac{p_i p_j}{p^2} \end{aligned}$$

- transport coefficients defined via $\mathcal{M}$

$$\left\langle X(\vec{p}')\right\rangle = \frac{1}{\gamma_c}\frac{1}{2E_p}\int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3 2E_q}\int \frac{\mathrm{d}^3\vec{q}'}{(2\pi)^3 2E_{q'}}\int \frac{\mathrm{d}^3\vec{p}'}{(2\pi)^3 2E_{p'}} \sum |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(p+q-p'-q')\hat{f}(\vec{q})X(\vec{p}')$$

- correct equil. lim. $\Rightarrow$ Einstein relation: $B_1(t,p) = T(t) E_p A(t,p)$

Meaning of Fokker-Planck coefficients

- non-relativistic equation with constant $A = \gamma$ and $B_0 = D_1 = D$

$$\frac{\partial f}{\partial t} = \gamma \frac{\partial}{\partial \vec{p}}(\vec{p} f) + D \frac{\partial^2 f}{\partial \vec{p}^2}$$

- Green's function:

$$G(t, \vec{p}; \vec{p}_0) = \left\{ \frac{\gamma}{2\pi D[1 - \exp(-2\gamma t)]} \right\}^{3/2} \times \exp\left\{ -\frac{\gamma}{2D} \frac{[\vec{p} - \vec{p}_0 \exp(-\gamma t)]^2}{1 - \exp(-2\gamma t)} \right\}$$

- Gaussian with

$$\langle \vec{p}(t) \rangle = \vec{p}_0 \exp(-\gamma t),$$

$$\left\langle \vec{p}^2(t) \right\rangle - \langle \vec{p}(t) \rangle^2 = \frac{3D}{\gamma}[1 - \exp(-2\gamma t)] \underset{t \to 0}{\cong} 6Dt$$

- γ: friction (drag) coefficient; D: diffusion coefficient
- equilibrium limit for $t \to \infty$: $D = mT\gamma$
  (Einstein's dissipation-fluctuation relation)

Relativistic Langevin process

- Fokker-Planck equation equivalent to stochastic differential equation
- Langevin process: friction force + Gaussian random force
- in the (local) rest frame of the heat bath

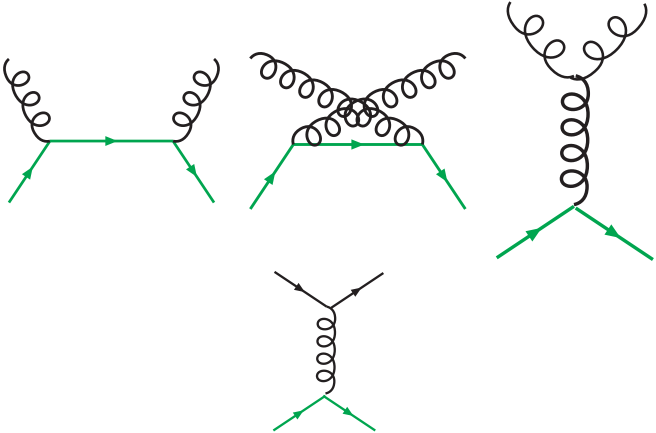
$$\mathrm{d}\vec{x} = \frac{\vec{p}}{E_p}\mathrm{d}t,$$

$$\mathrm{d}\vec{p} = -A\,\vec{p}\,\mathrm{d}t + \sqrt{2\mathrm{d}t}[\sqrt{B_0}P_\perp + \sqrt{B_1}P_\parallel]\vec{w}$$

- $\vec{w}$: normal-distributed random variable
- dependent on realization of stochastic process
- to guarantee correct equilibrium limit: Use Hänggi-Klimontovich calculus, i.e., use $B_{0/1}(t, \vec{p} + \mathrm{d}\vec{p})$
- for constant coefficients: Einstein dissipation-fluctuation relation $B_0 = B_1 = E_p T A$.
- to implement flow of the medium
  - use Lorentz boost to change into local "heat-bath frame"
  - use update rule in heat-bath frame
  - boost back into "lab frame"

Elastic pQCD processes

- Lowest-order matrix elements [Combridge 79]



- Debye-screening mass for t-channel gluon exch. $\mu_g = gT$, $\alpha_s = 0.4$
- not sufficient to understand RHIC data on "non-photonic" electrons

Non-perturbative interactions: effective resonance model

- General idea: Survival of D- and B-meson like resonances above T_c
- Chiral symmetry $\text{SU}_V(2) \otimes \text{SU}_A(2)$ in light-quark sector of QCD

$$\mathscr{L}_D^{(0)} = \sum_{i=1}^{2} [(\partial_\mu \Phi_i^\dagger)(\partial^\mu \Phi_i) - m_D^2 \Phi_i^\dagger \Phi_i] + \text{massive (pseudo-)vectors } D^*$$

- Φ_i: two doublets: pseudo-scalar $\sim \binom{\overline{D^0}}{D^-}$ and scalar
- Φ_i^*: two doublets: vector $\sim \binom{\overline{D^{0*}}}{D^{-*}}$ and pseudo-vector

$$\mathscr{L}_{qc}^{(0)} = \bar{q} \mathrm{i} \partial\!\!\!/ q + \bar{c}(\mathrm{i} \partial\!\!\!/ - m_c) c$$

- q: light-quark doublet $\sim \binom{u}{d}$
- c: singlet

Interactions

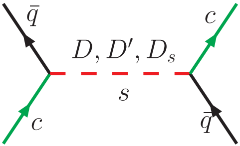
- Interactions determined by chiral symmetry
- For transversality of vector mesons:
  heavy-quark effective theory vertices

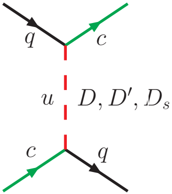
$$\begin{aligned}\mathscr{L}_{\text{int}} = &- G_S \left(\bar{q} \frac{1+\not{v}}{2} \Phi_1 c_v + \bar{q} \frac{1+\not{v}}{2} \mathrm{i}\gamma^5 \Phi_2 c_v + h.c. \right) \\ &- G_V \left(\bar{q} \frac{1+\not{v}}{2} \gamma^\mu \Phi_{1\mu}^* c_v + \bar{q} \frac{1+\not{v}}{2} \mathrm{i}\gamma^\mu \gamma^5 \Phi_{2\mu}^* c_v + h.c. \right)\end{aligned}$$

- v: four velocity of heavy quark
- in HQET: spin symmetry $\Rightarrow G_S = G_V$

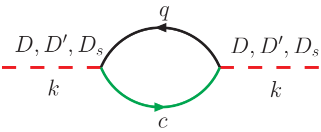
Resonance Scattering

- elastic heavy-light-(anti-)quark scattering



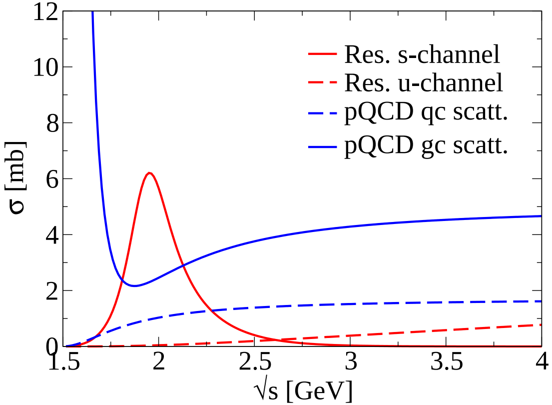


- D- and B-meson like resonances in sQGP



- parameters
  - $m_D = 2$ GeV, $\Gamma_D = 0.4 \ldots 0.75$ GeV
  - $m_B = 5$ GeV, $\Gamma_B = 0.4 \ldots 0.75$ GeV

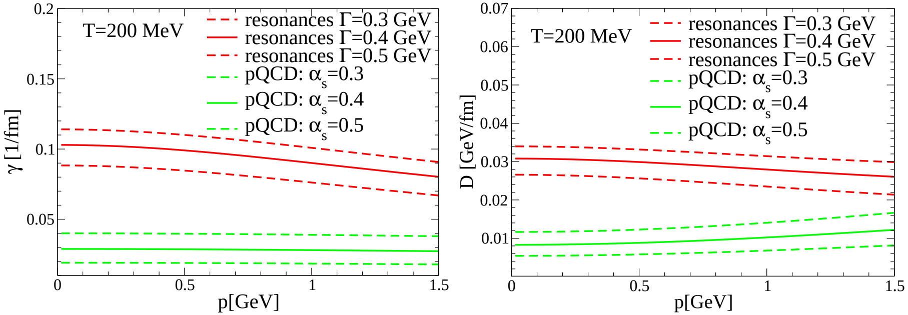
Cross sections



- total pQCD and resonance cross sections: comparable in size
- BUT pQCD forward peaked ↔ resonance isotropic
- resonance scattering more effective for friction and diffusion

Transport coefficients: pQCD vs. resonance scattering

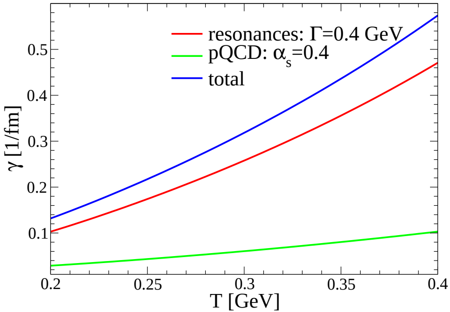
- three-momentum dependence

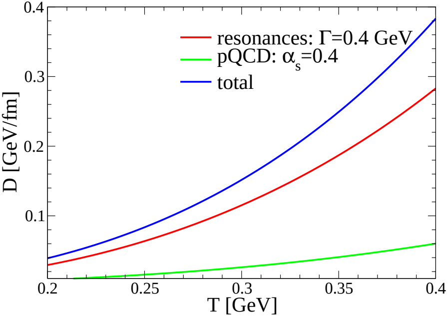


- resonance contributions factor $\sim 2 \ldots 3$ higher than pQCD!

Transport coefficients: pQCD vs. resonance scattering

- Temperature dependence





Time evolution of the fire ball

- Elliptic fire-ball parameterization
  fitted to hydrodynamical flow pattern [Kolb '00]

$$V(t) = \pi(z_0 + v_z t)a(t)b(t), \quad a,b\text{: half-axes of ellipse,}$$
$$v_{a,b} = v_\infty[1-\exp(-\alpha t)] \mp \Delta v[1-\exp(-\beta t)]$$

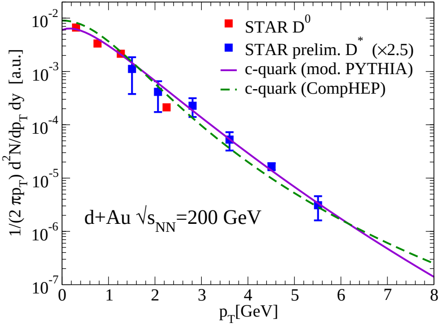
- Isentropic expansion: $S =$ const (fixed from N_{ch})
- QGP Equation of state:

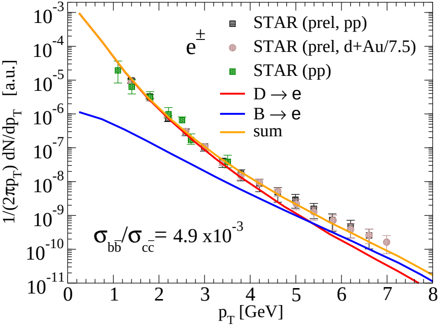
$$s = \frac{S}{V(t)} = \frac{4\pi^2}{90} T^3 (16 + 10.5 n_f^*), \quad n_f^* = 2.5$$

- obtain $T(t) \Rightarrow A(t,p)$, $B_0(t,p)$ and $B_1 = TEA$
- for semicentral collisions ($b = 7$ fm): $T_0 = 340$ MeV,
  QGP lifetime $\simeq 5$ fm$/c$.
- simulate FP equation as relativistic Langevin process

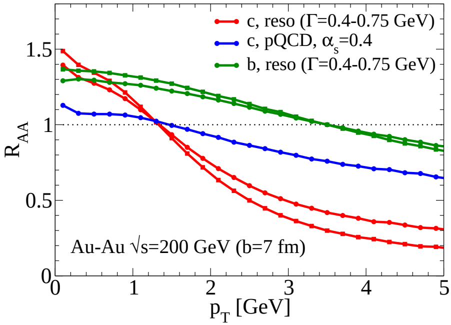
Initial conditions

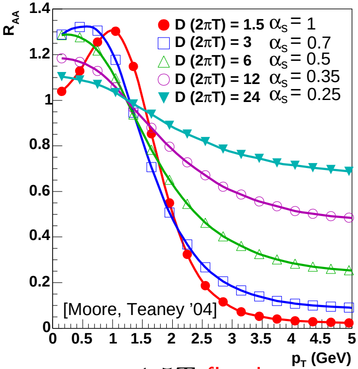
- need initial p_T-spectra of charm and bottom quarks
  - (modified) PYTHIA to describe exp. D meson spectra, assuming δ-function fragmentation
  - exp. non-photonic single-$e^{\pm}$ spectra: Fix bottom/charm ratio





Spectra and elliptic flow for heavy quarks

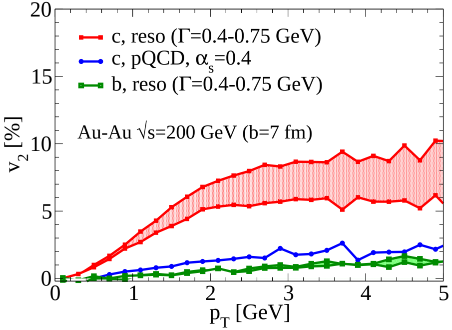


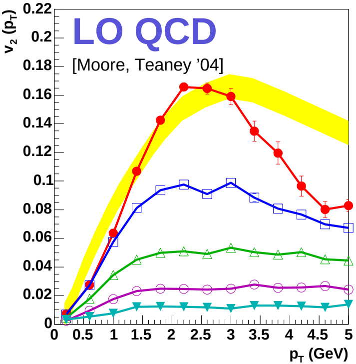


- $\mu_D = gT$, $\alpha_s = g^2/(4\pi) = 0.4$
- resonances $\Rightarrow$ c-quark thermalization without upscaling of cross sections
- Fireball parametrization consistent with hydro

- $\mu_D = 1.5T$ fixed
- spatial diff. coefficient: $D = D_s = \frac{T}{mA}$
- $2\pi T D \simeq \frac{3}{2\alpha_s^2}$

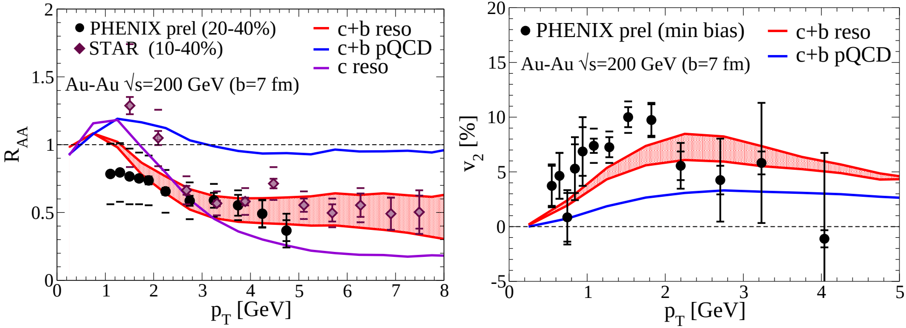
Spectra and elliptic flow for heavy quarks





Observables: p_T-spectra (R_{AA}), v_2

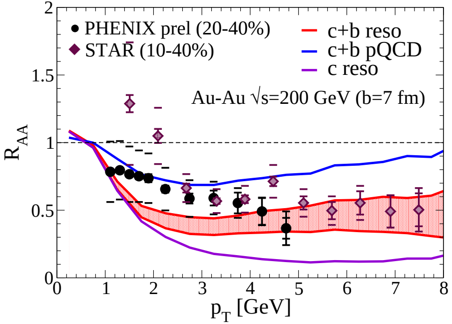
- Hadronization: Coalescence with light quarks + fragmentation
  $\Leftrightarrow c\bar{c}$, $b\bar{b}$ conserved
- single electrons from decay of D- and B-mesons

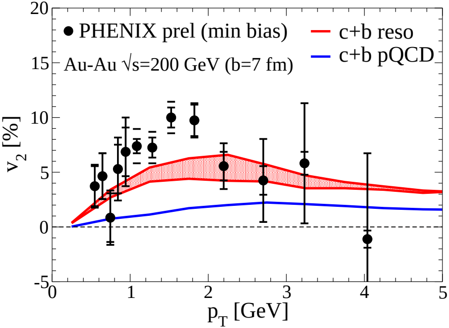


- Without further adjustments: data quite well described
  [HvH, V. Greco, R. Rapp, Phys. Rev. C **73**, 034913 (2006)]

Observables: p_T-spectra (R_{AA}), v_2

- Hadronization: Fragmentation only
- single electrons from decay of D- and B-mesons

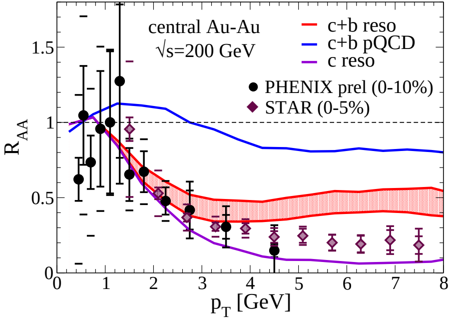




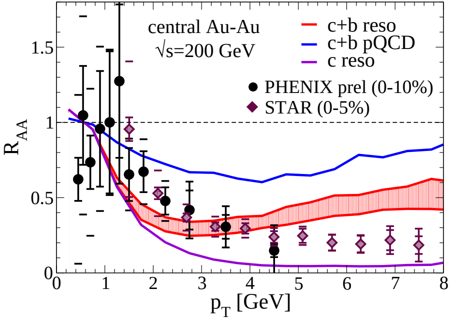
Observables: p_T-spectra (R_{AA}), v_2

- Central Collisions
- single electrons from decay of D- and B-mesons

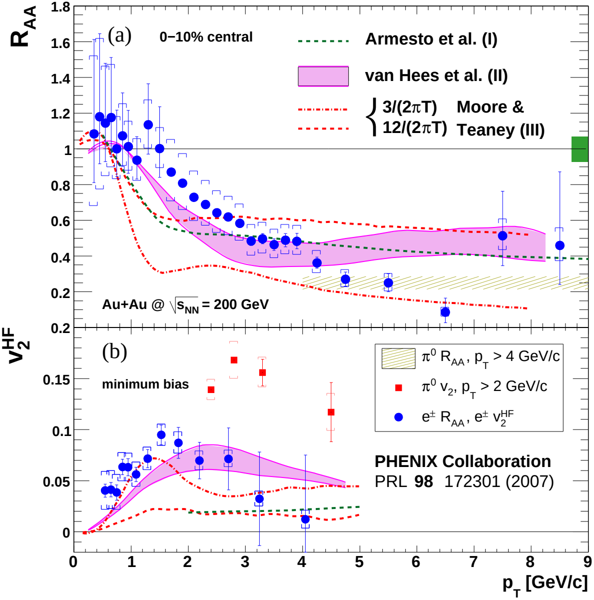
Coalescence+Fragmentation



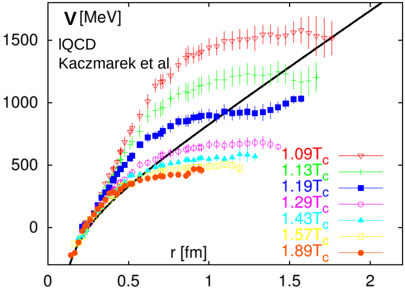
Fragmentation only



Comparison to newer data



Microscopic model: Static potentials from lattice QCD



- color-singlet free energy from lattice
- use internal energy

$$U_1(r,T) = F_1(r,T) - T\frac{\partial F_1(r,T)}{\partial T},$$

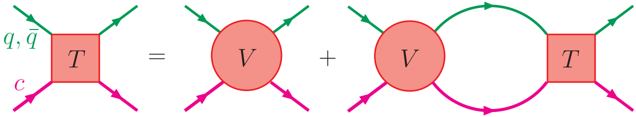
$$V_1(r,T) = U_1(r,T) - U_1(r \to \infty, T)$$

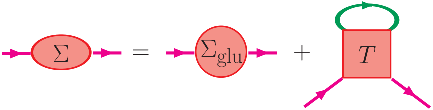
- Casimir scaling for other color channels [Nakamura et al 05; Döring et al 07]

$$V_{\bar{3}} = \frac{1}{2}V_1, \quad V_6 = -\frac{1}{4}V_1, \quad V_8 = -\frac{1}{8}V_1$$

T-matrix

- Brueckner many-body approach for elastic Qq, $Q\bar{q}$ scattering

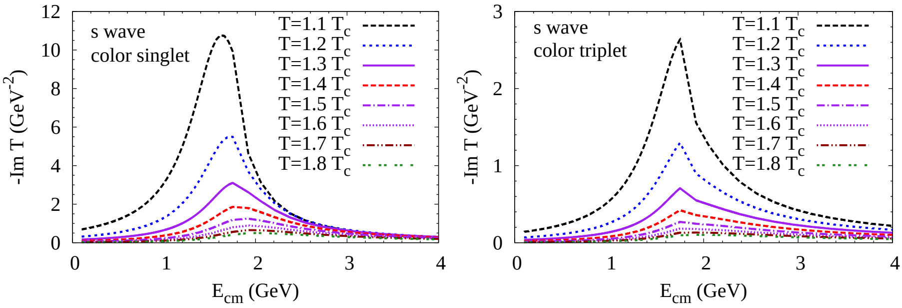




- reduction scheme: 4D Bethe-Salpeter → 3D Lipmann-Schwinger
- S- and P waves
- same scheme for light quarks (self consistent!)
- Relation to invariant matrix elements

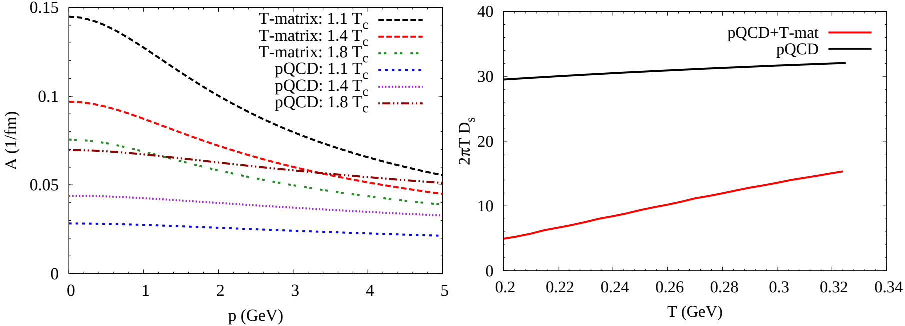
$$\sum |\mathcal{M}(s)|^2 \propto \sum_q d_a \left(|T_{a,l=0}(s)|^2 + 3|T_{a,l=1}(s)|^2 \cos\theta_{\text{cm}}\right)$$

T-matrix



- resonance formation at lower temperatures $T \simeq T_c$
- melting of resonances at higher T! $\Rightarrow$ sQGP
- P wave smaller
- resonances near T_c: natural connection to quark coalescence
  [Ravagli, Rapp 07]
- model-independent assessment of elastic Qq, $Q\bar{q}$ scattering
- problems: uncertainties in extracting potential from lQCD
  in-medium potential V vs. F?

Transport coefficients



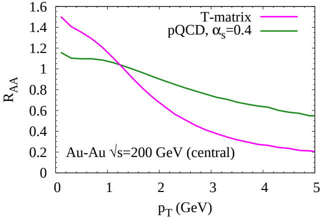
- from non-pert. interactions reach $A_{\text{non-pert}} \simeq 1/(7\ \text{fm}/c) \simeq 4 A_{\text{pQCD}}$
- A decreases with higher temperature
- higher density (over)compensated by melting of resonances!
- spatial diffusion coefficient

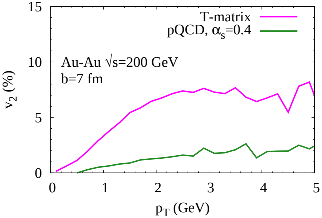
$$D_s = \frac{T}{mA}$$

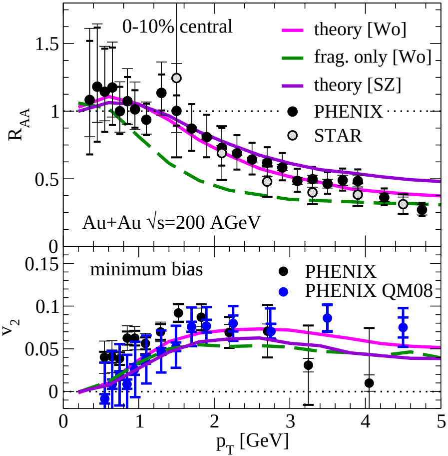
increases with temperature

Non-photonic electrons at RHIC

- same model for bottom
- quark coalescence+fragmentation $\rightarrow D/B \rightarrow e + X$





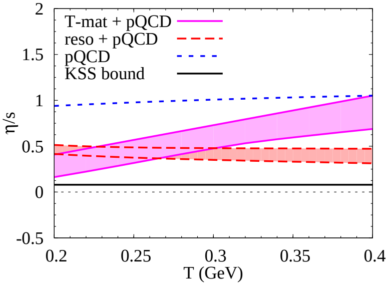


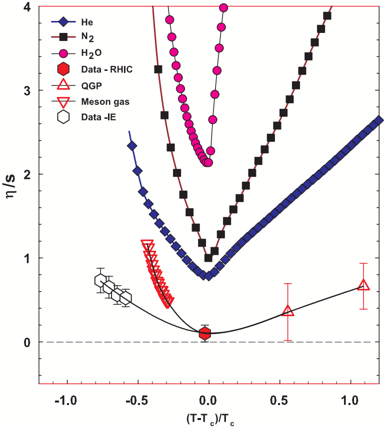
- coalescence crucial for explanation of data
- increases both, R_{AA} and $v_2 \Leftrightarrow$ “momentum kick” from light quarks!
- “resonance formation” towards T_c → coalescence natural!

Properties of the sQGP

- measure for coupling strength in plasma: η/s
- relation to spatial diffusion coefficient

$$\frac{\eta}{s} \simeq \frac{1}{2} T D_s \qquad \text{(AdS/CFT)}, \quad \frac{\eta}{s} \simeq \frac{1}{5} T D_s \qquad \text{(wQGP)}$$





Resonance-Recombination Model

- successes of quark-coalescence models in HI phenomenology
  - high baryon/meson ration in heavy-ion compared to pp collisions compared
  - Constituent-quark number scaling of v_2

$$v_{2,\text{had}}(p_T) = n_q v_{2,q}(p_T/n_q)$$

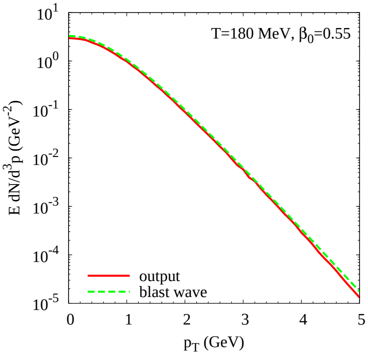
  - experiment: CQNS better for KE$_T$ than p_t
- problems with "naive" coalescence models
  - violates conservation laws (energy, momentum!)
  - violates 2^{nd} theorem of thermodynamics (entropy)
- Resonance structures close to T_c
  - transport process with $q\bar{q}(qq) \leftrightarrow R$

Resonance-Recombination Model

$$\frac{\partial}{\partial t} f_M(t,p) = -\frac{\Gamma}{\gamma_p} f_M(t,p) + g(p) \Rightarrow f_M^{(\text{eq})}(p) = \frac{\gamma_p}{\Gamma} g(p)$$

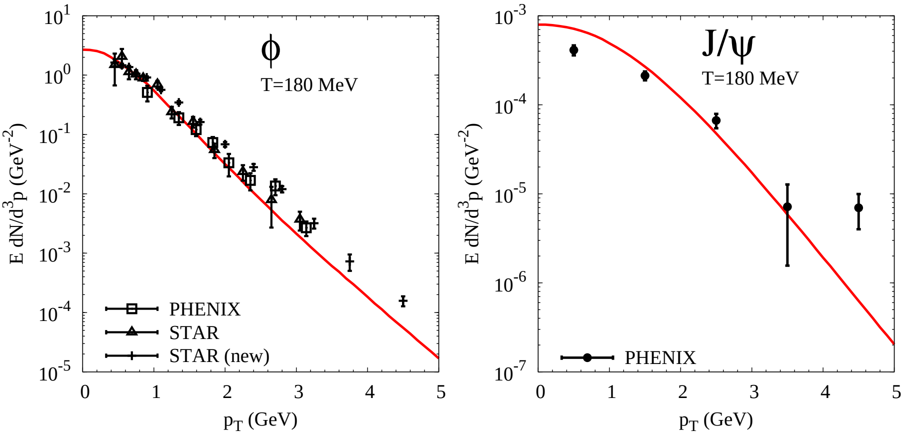
$$g(p) = \int \frac{\mathrm{d}^3 p_1 \mathrm{d}^3 p_2}{(2\pi)^6} \int \mathrm{d}^3 x f_q(x,p_1) f_{\bar{q}}(x,p_2) \sigma(s) v_{\text{rel}} \delta^{(3)}(p - p_1 - p_2)$$

$$\sigma(s) = g_\sigma \frac{4\pi}{k_{\text{cm}}^2} \frac{(\Gamma m)^2}{(s-m^2)^2 + (\Gamma m)^2}$$



Meson spectra

- q-$\bar{q}$ input: Langevin simulation
- meson output: resonance-recombination model

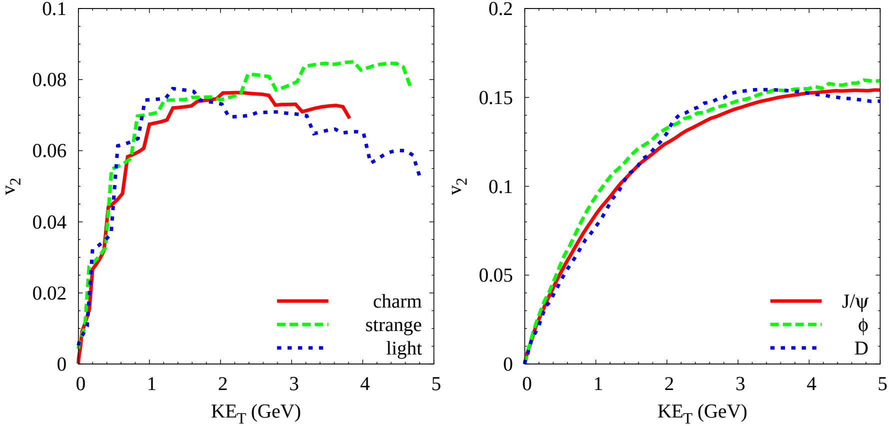


Constituent-quark number scaling

- usual coalescence models: factorization ansatz

$$f_q(p,x,\varphi) = f_q(p,x)[1 + 2v_2^q(p_T)\cos(2\varphi)]$$

- CQNS usually not robust with more realistic parametrizations of v_2
- here: q input from Langevin simulation



Summary and Outlook

- Summary
  - Heavy quarks in the sQGP
  - non-perturbative interactions
    - mechanism for strong coupling: resonance formation at $T \gtrsim T_c$
    - lQCD potentials parameter free
    - res. melt at higher temperatures ⇔ consistency betw. R_{AA} and v_2!
  - also provides "natural" mechanism for quark coalescence
  - resonance-recombination model
  - problems
    - extraction of V from lattice data
    - potential approach at finite T: F, V or combination?
- Outlook
  - include inelastic heavy-quark processes (gluon-radiation processes)
  - other heavy-quark observables like charmonium suppression/regeneration