

Polarization of dileptons in heavy-ion collisions

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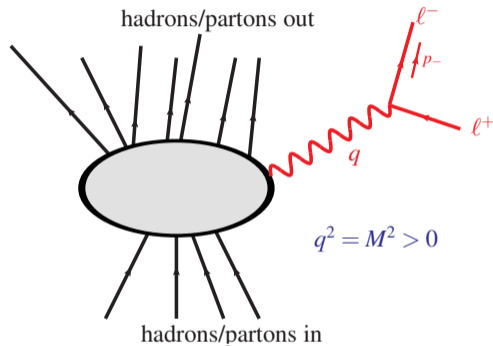
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- 1 Production rates for dileptons
- 2 Bulk evolution
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 - Dielectrons (SIS/HADES)
 - Dimuons (SPS/NA60)
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Production rates for dileptons



- Fermi's golden rule \Rightarrow transition-matrix element for process $|i\rangle \rightarrow |f'\rangle = |f^{\ell^+\ell^-}\rangle$
- QED Feynman rules

The McLerran-Toimela formula

- result (in **rest frame of dilepton!**)

$$\frac{dN_{\ell^+\ell^-}}{d^4x d^4q d^2\Omega_\ell} = \frac{\alpha^2}{32\pi^3} \frac{1}{M^4} \sqrt{1 - \frac{4m_\ell^2}{M^2}} L_{\mu\nu} \rho^{\mu\nu} n_B(u \cdot q)$$

- **spectral**, **thermal**, and **polarization of γ^*** information!
- u : four-velocity of fluid cell, $\vec{q}^{*2} = (u \cdot q)^2 - q^2$: γ^* -momentum in **fluid rest frame**
- $M^2 = q \cdot q$: invariant mass, $u \cdot q$: energy of dilepton in rest frame of fluid cell
- $d^2\Omega_\ell$: solid angle of lepton momentum, \vec{p}_- (in rest frame of virtual photon)
- $L_{\mu\nu} = 2[M^2\Theta_{\mu\nu} - 2(p_\mu^- q_\nu + p_\nu^- q_\mu) + 4p_\mu^- p_\nu^-]$: Lepton tensor
- in-medium electromagnetic **current-current correlation function**

$$i\Pi_{\text{ret}}^{\mu\nu}(q) := \int d^4x \exp(iq \cdot x) \langle [J_{\text{em}}^\mu(x), J_{\text{em}}^\nu(0)] \rangle_{T, \mu_B} \Theta(x^0)$$

- written in (local) **rest frame of the medium**:

$$\rho^{\mu\nu} = -2 \text{Im} \Pi_{\text{ret}}^{\mu\nu}(M^2, |\vec{q}^*|) = \rho_T(M, |\vec{q}^*|) \Theta_T^{\mu\nu} + \rho_L(M, |\vec{q}^*|) \Theta_L^{\mu\nu}$$

Radiation from thermal QGP: $q\bar{q}$ annihilation

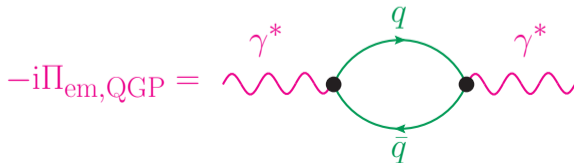
- McLerran-Toimela formula for γ^* (integrate over $d^2\Omega_\ell$)

$$\frac{dN_{\ell+\ell-}}{d^4x d^4q} = -\frac{\alpha^2}{3\pi^3} \frac{M^2 + 2m_\ell^2}{M^4} \sqrt{1 - \frac{4m_\ell^2}{M^2}} g_{\mu\nu} \text{Im} \Pi_{\text{ret}}^{\mu\nu}(M, \vec{q}) n_B(u \cdot q)$$

- in-medium em. current-current correlation function

$$i\Pi_{\text{ret}}^{\mu\nu}(q) := \int d^4x \exp(iq \cdot x) \langle [\mathbf{J}_{\text{em}}^\mu(x), \mathbf{J}_{\text{em}}^\nu(0)] \rangle_{T, \mu_B} \Theta(x^0)$$

- Feynman diagrams: photon self-energy
- in QGP phase: $q\bar{q}$ annihilation
- hard-thermal-loop improved em. current-current correlator

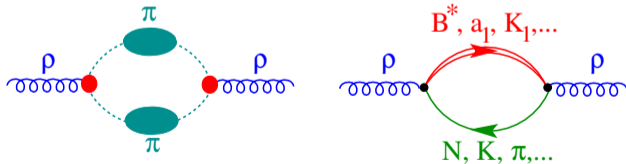


Hadronic many-body theory

- hadronic many-body theory (HMBT) of vector mesons

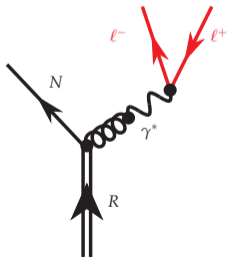
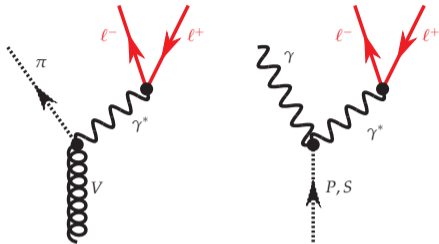
[Ko et al, Chanfray et al, Herrmann et al, Rapp et al, ...]

- $\pi\pi$ interactions and **hadronic excitations**
- effective hadronic models, implementing symmetries
- good approximation: **vector-meson dominance**, $J_{\text{em}}^\mu \propto \rho^\mu, \omega^\mu, \phi^\mu$
- dilepton/photon rates then $\propto \text{Im } D_{\text{VM}}$ (**VM-spectral functions**)
- parameters fixed by phenomenology
(photon absorption at nucleons and nuclei, $\pi N \rightarrow \rho N$)
- evaluated at **finite temperature and density**
- self-energies \Rightarrow **mass shift and broadening** in the medium

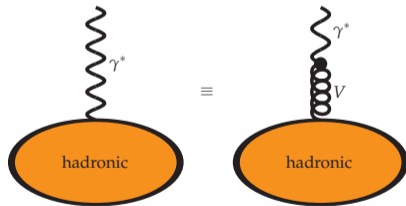


- Baryons** important, even at low **net** baryon density $n_B - n_{\bar{B}}$
- reason: $n_B + n_{\bar{B}}$ relevant (CP inv. of strong interactions)

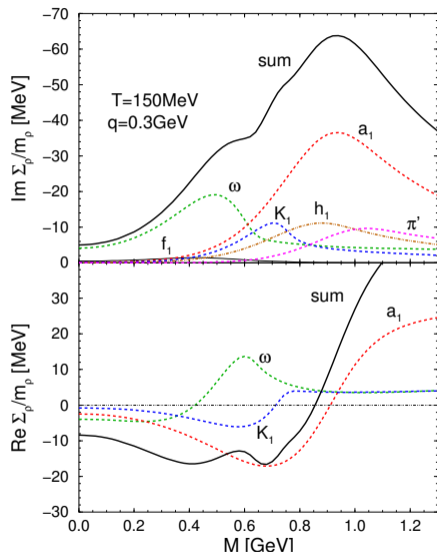
Dalitz decays



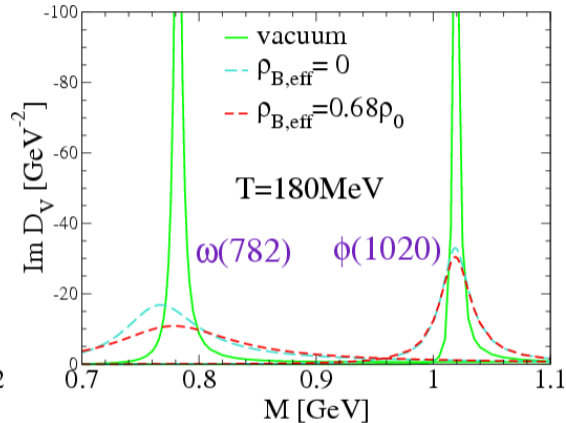
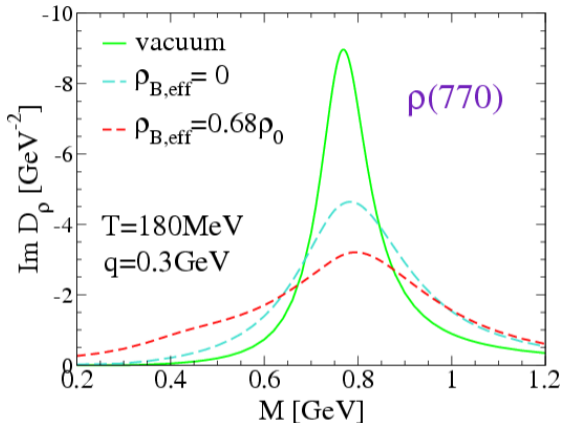
- **Dalitz decay:**
1 particle \rightarrow 3 particles
- V : $\omega \rightarrow \pi + \gamma^* \rightarrow \pi + l^+ + l^-$
- P, S : $\pi, \eta \rightarrow \gamma + \gamma^* \rightarrow \gamma + l^+ + l^-$
- R : Baryon resonances
 $\Delta, N^* \rightarrow N + V \rightarrow N + \gamma^* \rightarrow N + l^+ + l^-$
- vector-meson dominance



Meson contributions



In-medium spectral functions and baryon effects

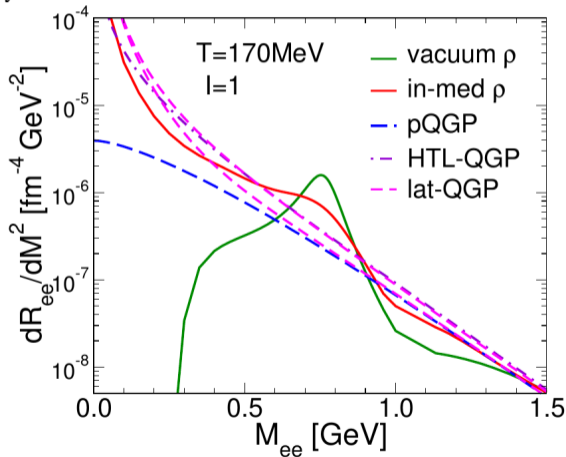


[RW99]

- **baryon effects** important
 - large contribution to broadening of the peak
 - responsible for most of the strength at small M

Dilepton rates: Hadron gas \leftrightarrow QGP

- in-medium **hadron gas** matches with **QGP**
- similar results also for γ rates
- “quark-hadron duality”?



Bulk evolution with transport and coarse graining

- established transport models for **bulk evolution**
 - e.g., **UrQMD**, GiBUU, BAMPS, (p)HSD,...
 - solve **Boltzmann equation** for hadrons and/or partons
- dilemma: need medium-modified **dilepton/photon emission rates**
- usually available only in **equilibrium QFT calculations**
- one way out:
 - **UrQMD transport** for entire bulk evolution
 - ⇒ use **coarse graining** in space-time cells ⇒ extract T, μ_B, μ_π, \dots
 - ⇒ use equilibrium rates locally
 - fit **temperature, chemical potentials, flow-velocity field** from anisotropic energy-momentum tensor [FMRS13]

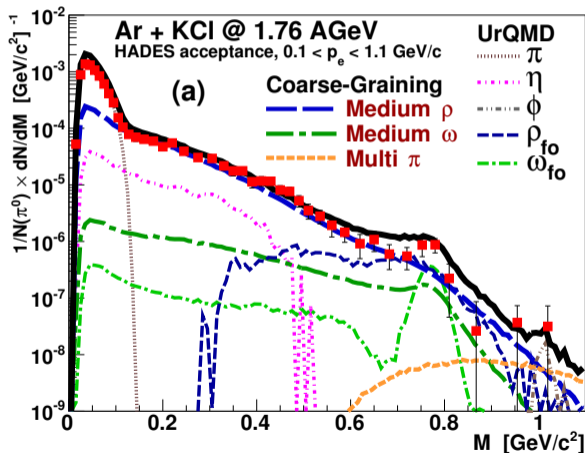
$$T^{\mu\nu} = (\epsilon + P_\perp)u^\mu u^\nu - P_\perp g^{\mu\nu} - (P_\parallel - P_\perp)V^\mu V^\nu$$

- thermal rates from **partonic/hadronic QFT become applicable**

Dielectrons (SIS/HADES)

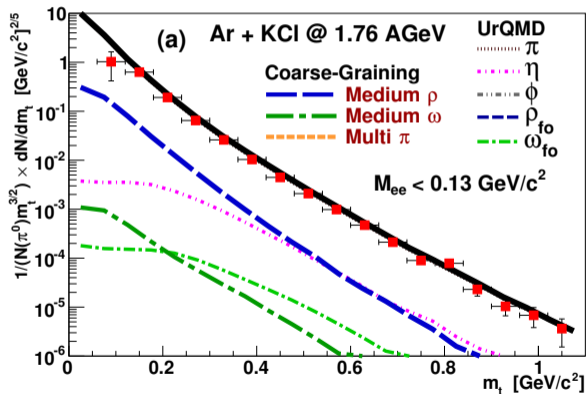
CGUrQMD: Ar+KCl (1.76 AGeV) (SIS/HADES)

- coarse-graining method works at low energies!
- UrQMD-medium evolution + RW-QFT rates



CGUrQMD: Ar+KCl (1.76 AGeV) (SIS/HADES)

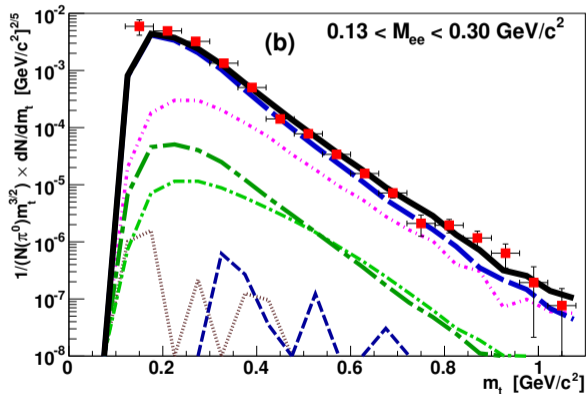
- dielectron spectra from Ar + KCl(1.76 AGeV) $\rightarrow e^+e^-$ (SIS/HADES)
- m_t spectra
- $M_{ee} < 0.13$ GeV



[S. Endres, HvH, J. Weil, M. Bleicher] [EHWB15b]

CGUrQMD: Ar+KCl (1.76 AGeV) (SIS/HADES)

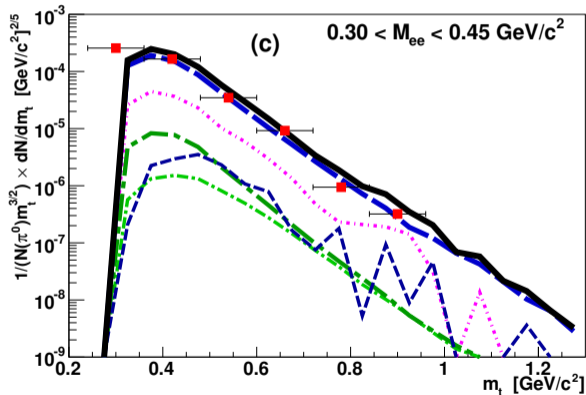
- dielectron spectra from Ar + KCl(1.76 AGeV) $\rightarrow e^+e^-$ (SIS/HADES)
- m_t spectra
- $0.13 \text{ GeV} < M_{ee} < 0.3 \text{ GeV}$



[S. Endres, HvH, J. Weil, M. Bleicher] [EHWB15b]

CGUrQMD: Ar+KCl (1.76 AGeV) (SIS/HADES)

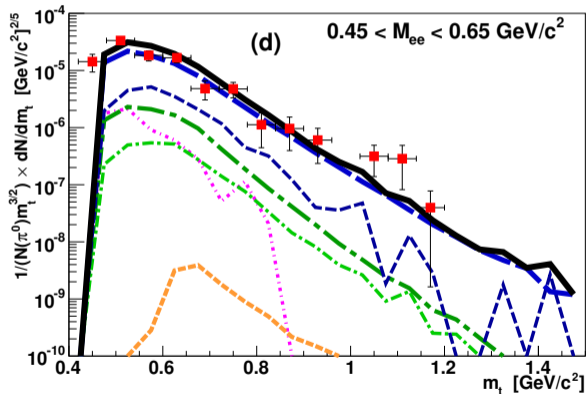
- dielectron spectra from Ar + KCl(1.76 AGeV) $\rightarrow e^+e^-$ (SIS/HADES)
- m_t spectra
- $0.3 \text{ GeV} M_{ee} < 0.45 \text{ GeV}$



[S. Endres, HvH, J. Weil, M. Bleicher] [EHWB15b]

CGUrQMD: Ar+KCl (1.76 AGeV) (SIS/HADES)

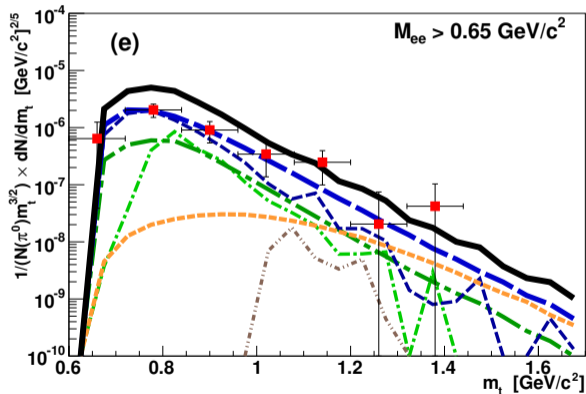
- dielectron spectra from Ar + KCl(1.76 AGeV) $\rightarrow e^+e^-$ (SIS/HADES)
- m_t spectra
- $0.45 \text{ GeV} < M_{ee} < 0.65 \text{ GeV}$



[S. Endres, HvH, J. Weil, M. Bleicher] [EHWB15b]

CGUrQMD: Ar+KCl (1.76 AGeV) (SIS/HADES)

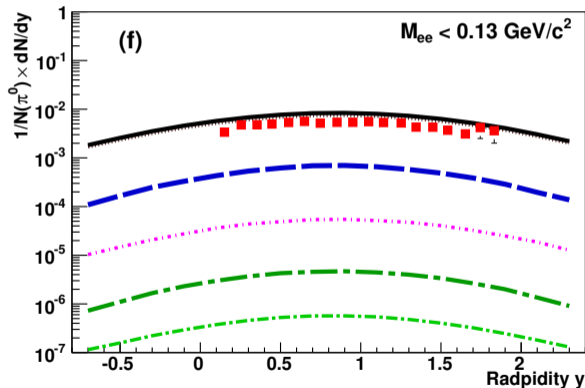
- dielectron spectra from Ar + KCl(1.76 AGeV) $\rightarrow e^+e^-$ (SIS/HADES)
- m_t spectra
- $M_{ee} > 0.65$ GeV



[S. Endres, HvH, J. Weil, M. Bleicher] [EHWB15b]

CGUrQMD: Ar+KCl (1.76 AGeV) (SIS/HADES)

- dielectron spectra from Ar + KCl(1.76 AGeV) $\rightarrow e^+e^-$ (SIS/HADES)
- m_t spectra
- rapidity spectrum ($M_{ee} < 0.13$ GeV)

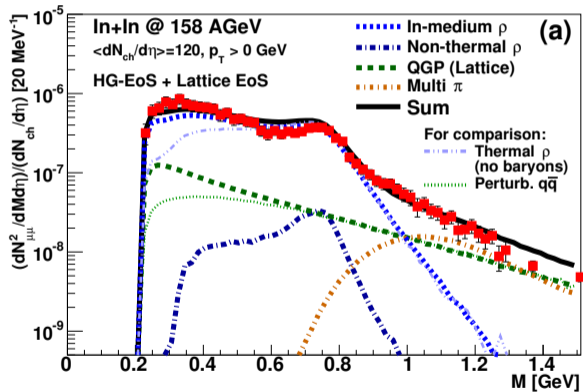


[S. Endres, HvH, J. Weil, M. Bleicher] [EHWB15b]

Dimuons (SPS/NA60)

CGUrQMD: In+In (158 AGeV) (SPS/NA60)

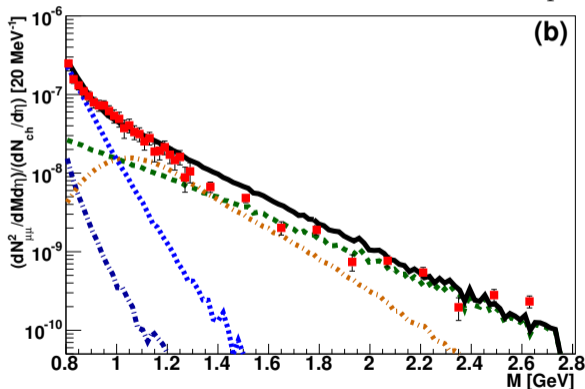
- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+\mu^-$ (NA60) [EHWB15b]
- min-bias data ($dN_{\text{ch}}/dy = 120$)



[S. Endres, HvH, J. Weil, M. Bleicher] [EHWB15a]

CGUrQMD: In+In (158 AGeV) (SPS/NA60)

- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+ \mu^-$ (NA60) [EHWB15b]
- min-bias data ($dN_{\text{ch}}/dy = 120$)
- higher IMR: provides **averaged true temperature** $\langle T \rangle_{1.5 \text{ GeV} \lesssim M \lesssim 2.4 \text{ GeV}} = 205\text{-}230 \text{ MeV}$
- clearly above $T_c \simeq 150\text{-}160 \text{ MeV}$ (no blueshifts in the **invariant-mass** spectra!)



[S. Endres, HvH, J. Weil, M. Bleicher] [EHWB15a]

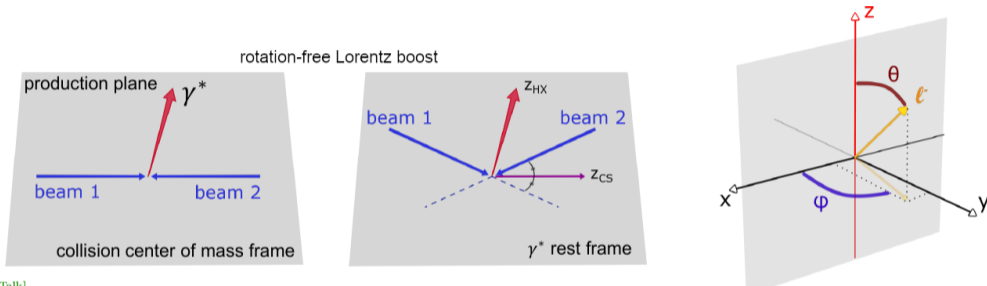
γ^* -polarization observables

F. Seck, B. Friman, T. Galatyuk, HvH, E. Speranza, R. Rapp, J. Wambach

[arXiv:2309.03189](https://arxiv.org/abs/2309.03189) [nucl-th]

- Helicity Frame HX, Collins-Soper Frame,... choice of “spin-quantization axis”

[P. Faccioli and C. Lourenço, Particle Polarization in High Energy Physics, Springer, Cham (2022)] [FL22]



[F. Seck QM23 Talk]

- in γ^* rest frame (general form for **parity-conserving decays of massive vector particles**)

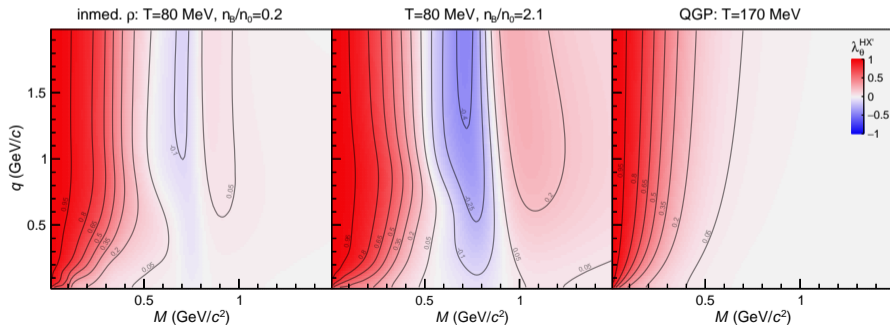
$$\frac{dN_{\ell+\ell^-}}{d^4x d^4q d^2\Omega_\ell} = \mathcal{N} \left(1 + \lambda_\theta \cos^2 \theta + \lambda_\phi \sin^2 \theta \cos(2\phi) + \lambda_{\theta\phi} \sin(2\theta) \cos \phi \right. \\ \left. + \lambda_\phi^\perp \sin^2 \theta \sin(2\phi) + \lambda_{\theta\phi}^\perp \sin(2\theta) \sin \phi \right)$$

- λ 's **depend on choice of “frame”!** Transformation via corresponding **rotations** of axes

HX' frame

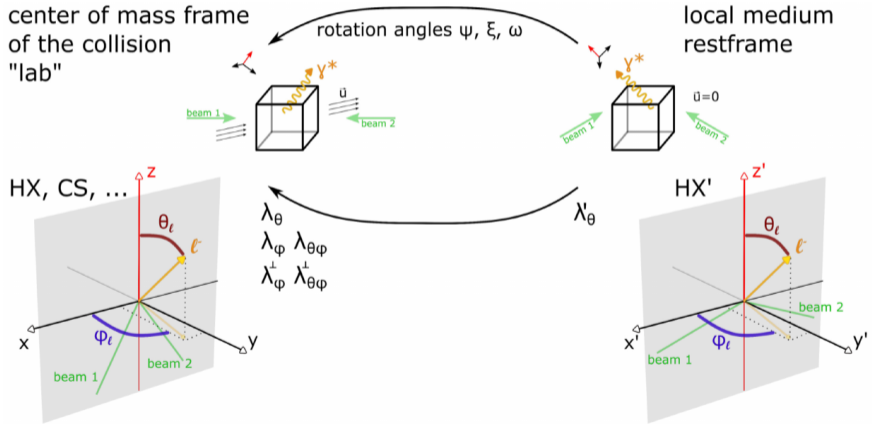
- for γ^* /dileptons from **thermal medium** \Rightarrow “preferred frame” (local) rest frame of heat bath
- use \vec{q} as quantization direction \Rightarrow **HX' frame**
- in γ^* rest frame: only preferred direction is \vec{u} \Rightarrow only $\lambda'_\theta \neq 0$:

$$\lambda'_\theta = \frac{2(M^2 - 4m_\ell^2)(\rho_T - \rho_L)}{2M^2(\rho_L + \rho_T) + 8m_\ell^2\rho_T} \stackrel{m_\ell \ll M}{\approx} \frac{\rho_T - \rho_L}{\rho_T + \rho_L}$$



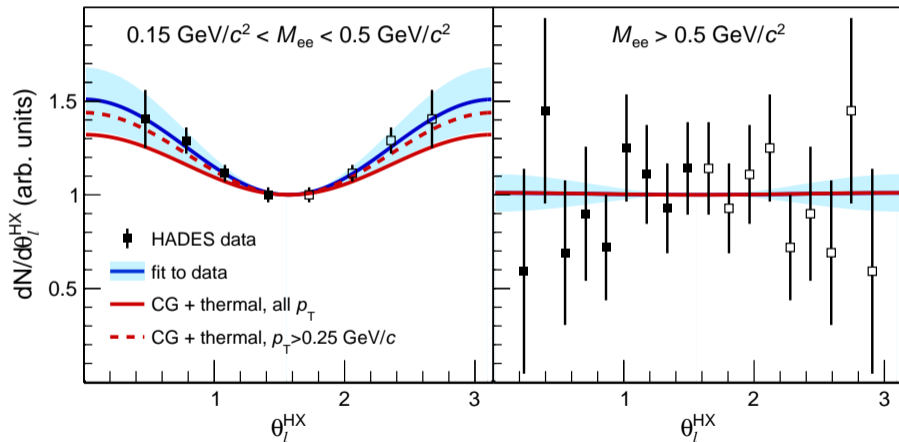
Dilepton polarization observables in heavy-ion collisions

- fireball described as fluid (blast-wave model or coarse-grained transport)
- transform for each fluid cell by rotation from HX' to HX (for HADES experiment) or CS frame (for NA60 experiment)



Comparison to data: HADES

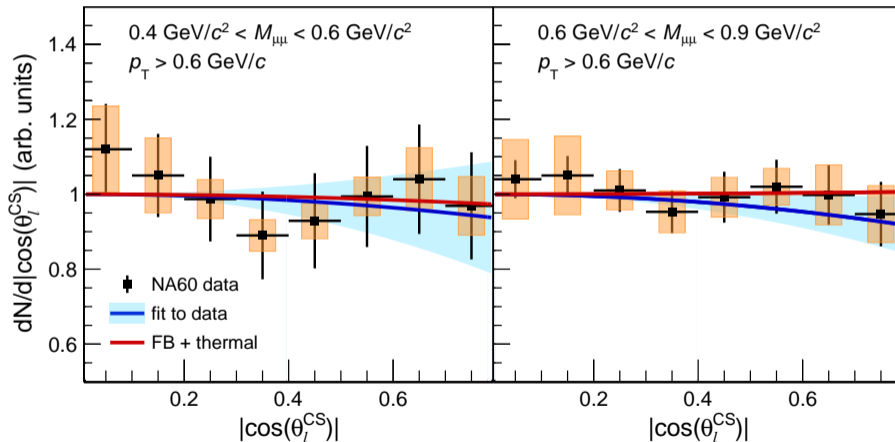
- $\theta^{(\text{HX})}$ distribution measured by HADES in Ar+KCl collisions at 1.76 AGeV ^[A+11]



[F. Seck, B. Friman, T. Galatyuk, HvH, E. Speranza, R. Rapp, J. Wambach, arXiv:2309.03189 [nucl-th]] [SFG⁺23]

Comparison to data: NA60

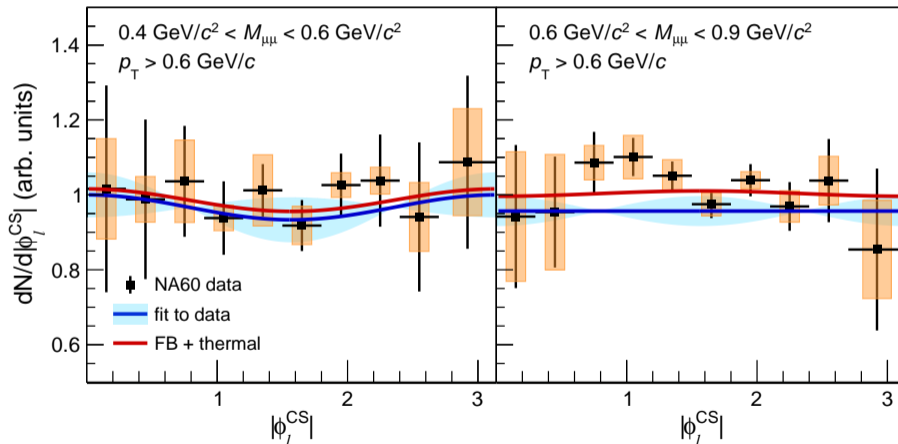
- $\lambda_\theta^{(\text{CS})}$ measured by NA60 in In+In collisions at 158A GeV [A⁺09]



[F. Seck, B. Friman, T. Galatyuk, HvH, E. Speranza, R. Rapp, J. Wambach, arXiv:2309.03189 [nucl-th]] [SFG⁺23]

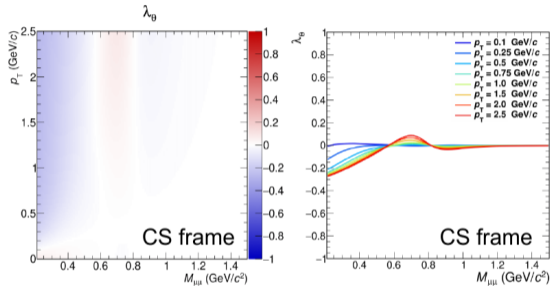
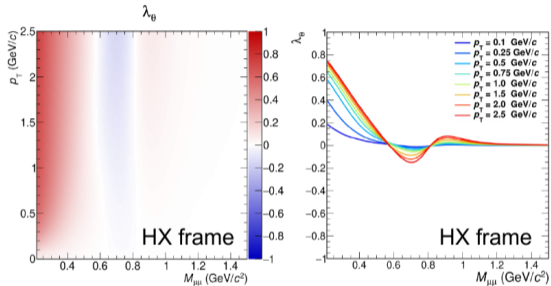
Comparison to data: NA60

- $\theta^{(CS)}$ and $\phi^{(CS)}$ distributions measured by NA60 in In+In collisions at 158A GeV [A⁺09]



[F. Seck, B. Friman, T. Galatyuk, HvH, E. Speranza, R. Rapp, J. Wambach, arXiv:2309.03189 [nucl-th]] [SFG⁺23]

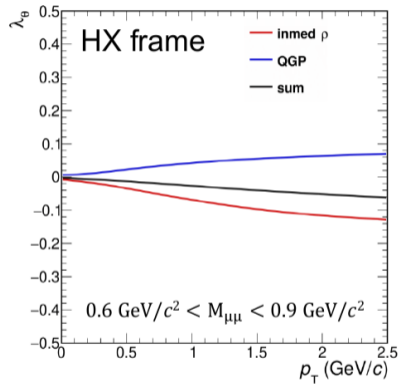
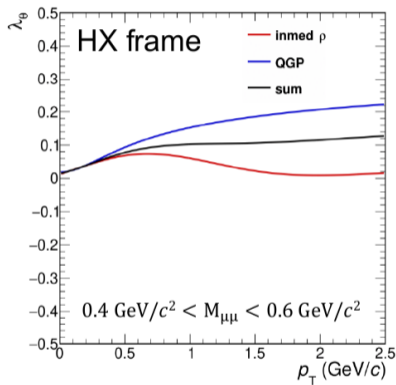
HX versus CS frame for NA60



[F. Seck, QM23 Talk]

- polarization coefficients pretty sensitive to choice of frame!

Outlook: more high-precision differential dilepton spectra



[F. Seck, QM23 Talk]

- sensitivity to kind of source!?!

Conclusions and Outlook

● General ideas

- em. probes \Leftrightarrow **in-medium electromagnetic current-correlation function**
- dual rates around T_c (compatible with χ symmetry restoration)
- **medium modifications of ρ , ω , ϕ**
- importance of **baryon-resonance interactions**

● Application to dileptons in HICs

- successful description from **SIS to RHIC energies**
- consistent description of **M and m_T spectra!**
- effective slope of M spectra ($1.5 \text{ GeV} < M < M_{J/\psi}$) **provides $\langle T \rangle$**
- **polarization observables** need more differential measurements of dilepton spectra
- models also here in good agreement with **existing data** (HADES Ar+KCl electrons, NA60 In+In muons)

● Outlook

- better understanding of **dilepton-production mechanisms** with **polarization observables**
- sensitivity to **phase diagram?**
- relation to **vorticity of medium** and/or **magnetic field?**

Transversal projectors

- for q^μ (four-momentum of a massive vector boson/virtual γ /dilepton with invariant mass, $M^2 = q \cdot q = q_\mu q^\mu$)

$$\Theta_{\mu\nu} = \eta_{\mu\nu} - \frac{q_\mu q_\nu}{M^2}.$$

- in **rest-frame of heat bath**, $(u^\mu) = (1, 0, 0, 0)$, this is further decomposed into 3D longitudinal and transverse parts

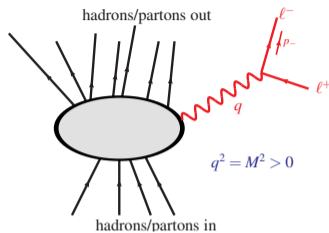
$$\Theta_{\text{T}}^{*\mu\nu} = \begin{pmatrix} 0 & \vec{0}^{\text{T}} \\ \vec{0} & (-\delta^{ij} + q^j q^k / \vec{q}^2) \end{pmatrix}, \quad \Theta_{\text{L}}^{*\mu\nu} = \Theta^{\mu\nu} - \Theta_{\text{T}}^{*\mu\nu}.$$

- in covariant form (valid in **any frame**)

$$\Theta_{\text{T}}^{\mu\nu} = \eta^{\mu\nu} - u^\mu u^\nu - \frac{q_\perp^\mu q_\perp^\nu}{q_\perp^2}, \quad \Theta_{\text{L}}^{\mu\nu} = \Theta^{\mu\nu} - \Theta_{\text{T}}^{\mu\nu} = u^\mu u^\nu - \frac{q^\mu q^\nu}{M^2} + \frac{q_\perp^\mu q_\perp^\nu}{q_\perp^2}.$$

- with $q_\perp^\mu = q^\mu - u^\mu(u \cdot q)$

Differential dilepton production rate



- in **rest-frame of dilepton**

$$\frac{dN_{\ell\ell}}{d^4x d^4q d^2\Omega_-} = L_{\mu\nu} \frac{\alpha_{\text{em}}^2}{32\pi^4} \frac{1}{M^4} \sqrt{1 - \frac{4m_\ell^2}{M^2}} \rho^{\mu\nu}(q) f_{\text{B}}(u \cdot q)$$

- m : lepton mass
- in-medium spectral function of **electromagnetic current-current correlation function**

$$\rho^{\mu\nu} = -2 \text{Im} \Pi_{\text{ret}}^{\mu\nu} = \rho_{\text{T}}(M, u \cdot q) \Theta_{\text{T}}^{\mu\nu} + \rho_{\text{L}}(M, u \cdot q) \Theta_{\text{L}}^{\mu\nu}$$

- “lepton tensor”

$$L_{\mu\nu} = 2 \left[M^2 \Theta_{\mu\nu} - 2(p_\mu^- q_\nu + p_\nu^- q_\mu) + 4p_\mu^- p_\nu^- \right]$$

Polarization parameters in HX' frame

- “heat-bath-helicity frame” HX' frame defined for dileptons from a **thermalized medium**
- **polarization axis**: $\parallel \vec{q}' = Q' \vec{e}'_3$ in **rest-frame of heat bath**, $u' = (1, 0, 0, 0)$
- **boost** to rest-frame of dilepton: $q = (M, 0, 0, 0)$, $u = \gamma(1, 0, 0, -\beta)$, $\beta = Q'/E'_{\gamma^*} = Q'/\sqrt{M^2 + Q'^2}$,
 $p_- = (\sqrt{m_\ell^2 + P^2}, P \sin \theta \cos \phi, P \sin \theta \sin \phi, P \cos \theta)$

$$\begin{aligned} \frac{dN_{\ell\ell}}{d^4x d^4q d^2\Omega_-} &= L_{\mu\nu} \frac{\alpha_{\text{em}}^2}{32\pi^4} \frac{1}{M^4} \sqrt{1 - \frac{4m_\ell^2}{M^2}} \rho^{\mu\nu}(q) f_{\text{B}}(u \cdot q) \\ &= \mathcal{N} A (1 + \lambda'_\theta \cos^2 \theta). \end{aligned}$$

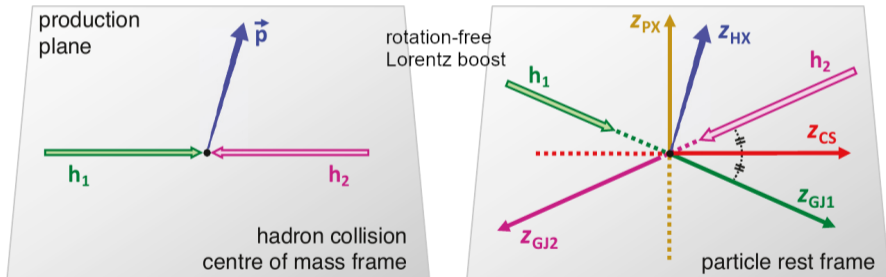
- with

$$\begin{aligned} \mathcal{N} &= \frac{\alpha_{\text{em}}^2}{32\pi^4} \frac{1}{M^4} \sqrt{1 - \frac{4m_\ell^2}{M^2}} f_{\text{B}}(u \cdot q), \\ A &= 2M^2(\rho_{\text{L}} + \rho_{\text{T}}) + 8m_\ell^2 \rho_{\text{T}}, \quad B = A \lambda'_\theta = 2(M^2 - 4m_\ell^2)(\rho_{\text{T}} - \rho_{\text{L}}), \\ \lambda'_\theta &= \frac{B}{A} = \frac{2(M^2 - 4m_\ell^2)(\rho_{\text{T}} - \rho_{\text{L}})}{2M^2(\rho_{\text{L}} + \rho_{\text{T}}) + 8m_\ell^2 \rho_{\text{T}}} \underset{m \ll M}{\cong} \frac{\rho_{\text{T}} - \rho_{\text{L}}}{\rho_{\text{T}} + \rho_{\text{L}}} \end{aligned}$$

- check: integrate over $d^2\Omega_- \Rightarrow$ **McLerran-Toimela formula!**

Polarization parameters in arbitrary frame

- polarization axis determined in **center-momentum frame of hadron collision**
- different choices HX (helicity frame), CS (Collins-Soper frame), GJ (Gottfried-Jackson frame), PX (perpendicular helicity frame)...



[FL22]

Polarization parameters in arbitrary frame

- theory: start with $\vec{q}' = Q' \vec{e}'_3$ (now in **cm frame of collision!**); then boost to **rest-frame of dilepton**
- rotate \vec{u} to basis of chosen **polarization frame** using **Euler angles** ω, ξ, ψ [Faccioli:2022peq]

$$\vec{u} = \gamma \hat{R}_3(\omega) \hat{R}_2(\xi) \hat{R}_3(\psi) (0, 0, -\beta)^T = \beta \gamma (\sin \xi \cos \omega, -\sin \xi \sin \omega, -\cos \xi)^T.$$

- use general expression for differential production rate

$$\frac{dR_{ll}}{d^4q d^2\Omega_-} = \mathcal{N} A (1 + \Lambda) \left(1 + \lambda_\theta \cos^2 \theta + \lambda_\phi \sin^2 \theta \cos 2\phi + \lambda_{\theta\phi} \sin 2\theta \cos \phi \right. \\ \left. + \lambda_\phi^\perp \sin^2 \theta \sin 2\phi + \lambda_{\theta\phi}^\perp \sin 2\theta \sin \phi \right),$$

- with

$$\Lambda = \frac{1}{2} \lambda'_\theta \sin^2 \xi, \quad \lambda_\theta = \frac{\lambda'_\theta}{1 + \Lambda} \left(1 - \frac{3}{2} \sin^2 \xi \right), \\ \lambda_\phi = \frac{1}{2} \frac{\lambda'_\theta}{1 + \Lambda} \cos(2\omega) \sin^2 \xi, \quad \lambda_\phi^\perp = -\frac{1}{2} \frac{\lambda'_\theta}{1 + \Lambda} \sin(2\omega) \sin^2 \xi, \\ \lambda_{\theta\phi} = -\frac{1}{2} \frac{\lambda'_\theta}{1 + \Lambda} \cos \omega \sin(2\xi), \quad \lambda_{\theta\phi}^\perp = \frac{1}{2} \frac{\lambda'_\theta}{1 + \Lambda} \sin \omega \sin(2\xi).$$

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