Self-Consistent Conserving Approximations for Gauge Theories at Finite Temperature?

Hendrik van Hees

Jörn Knoll

Contents

- Φ-derivable approximation schemes
- Vector particles $\leftrightarrow$ gauge theories
- Gauge symmetries and Φ-functional
- A gauge invariant scheme for π and ρ
- Results
- Perspectives

Motivation

How to treat particles with finite mass width?

- How to find a self-consistent approximation which
  - ☞ respects conervation laws
  - ☞ is thermodynamically consistent
  - ☞ can be treated numerically
  - ☞ povides non-equilibrium equations beyond quasi-particle description?

The answer

- Φ-derivable schemes (Luttinger, Ward, Kadanoff, Baym)

The Φ-Functional

- Introduce local and bilocal auxiliary sources
- Generating functional

$$Z[J,K] = N \int \mathrm{D}\phi \exp\left[\mathrm{i}S[\phi] + \mathrm{i}\left\{J_1\phi_1\right\}_1 + \left\{\frac{\mathrm{i}}{2} K_{12}\phi_1\phi_2\right\}_{12}\right]$$

- Generating functional for connected diagrams

$$Z[J,K] = \exp(\mathrm{i}W[J,K])$$

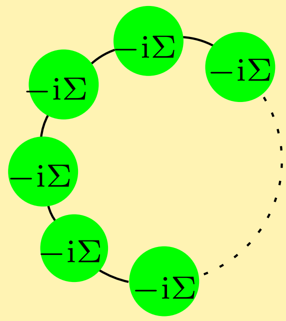
- The mean field and the connected Green's function

$$\varphi_1 = \frac{\delta W}{\delta J_1}, \quad G_{12} = -\frac{\delta^2 W}{\delta J_1 \delta J_2} \quad \Rightarrow \quad \frac{\delta W}{\delta K_{12}} = \frac{1}{2}[\varphi_1\varphi_2 + \mathrm{i}G_{12}]$$

- Legendre transformation for φ and G:

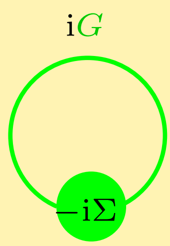
$$\Gamma[\varphi, G] = W[J,K] - \{\varphi_1 J_1\}_1 - \frac{1}{2}\{(\varphi_1\varphi_2 + \mathrm{i}G_{12})K_{12}\}_{12}$$

$\mathrm{i}\Gamma[\varphi, G] = \mathrm{i}S_0[\varphi] + \sum_{n=1}^{\infty} \frac{1}{n}$

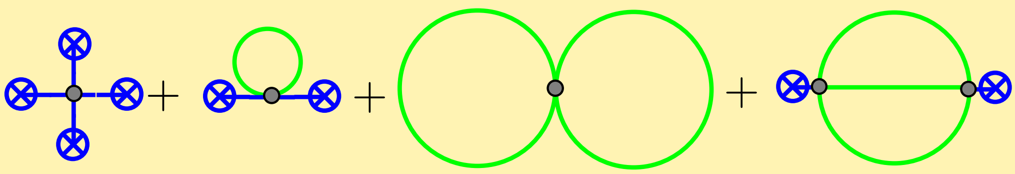


n insertions of $-\mathrm{i}\Sigma$

$-$



$+$



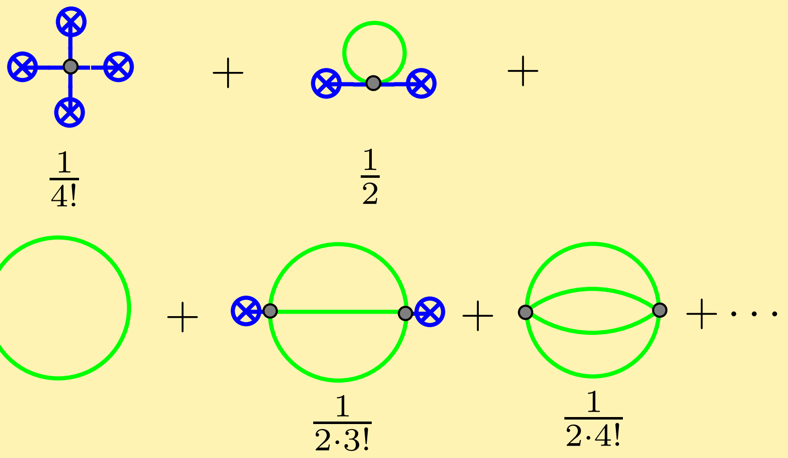
$\rightarrow \Phi[\varphi, G]$

Equations of Motion

- Physical solution defined by vanishing auxiliary sources

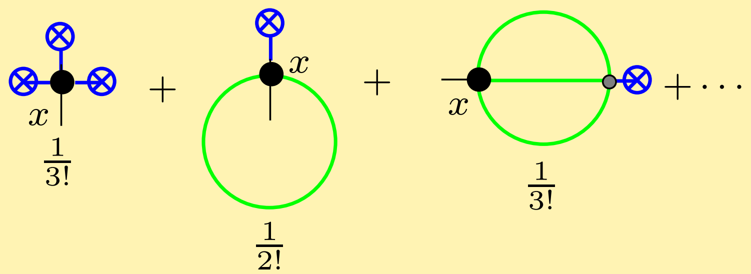
- Φ-functional (2PI closed diagrams)

$i\Phi[\varphi, G] =$



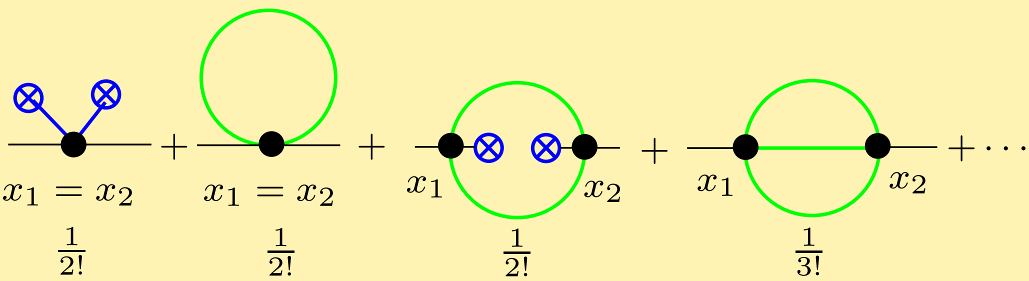
- Equation of motion for the mean field φ

$i(\Box + m^2)\varphi(x) =$



- For the "full" propagator $G \Rightarrow G = G_0 + G_0 \circ \Sigma \circ G$

$-i\Sigma_{12} =$



- Closed set of equations of motion for φ and G

Properties of the Formalism

- Provides natural scheme for truncation of the Schwinger-Dyson hierarchy
- Truncation of Φ at a certain loop order

  ☞ respects conservation laws for expectation values of energy, momentum, angular momentum, ...
  Noether charges from linearly realized global symmetries

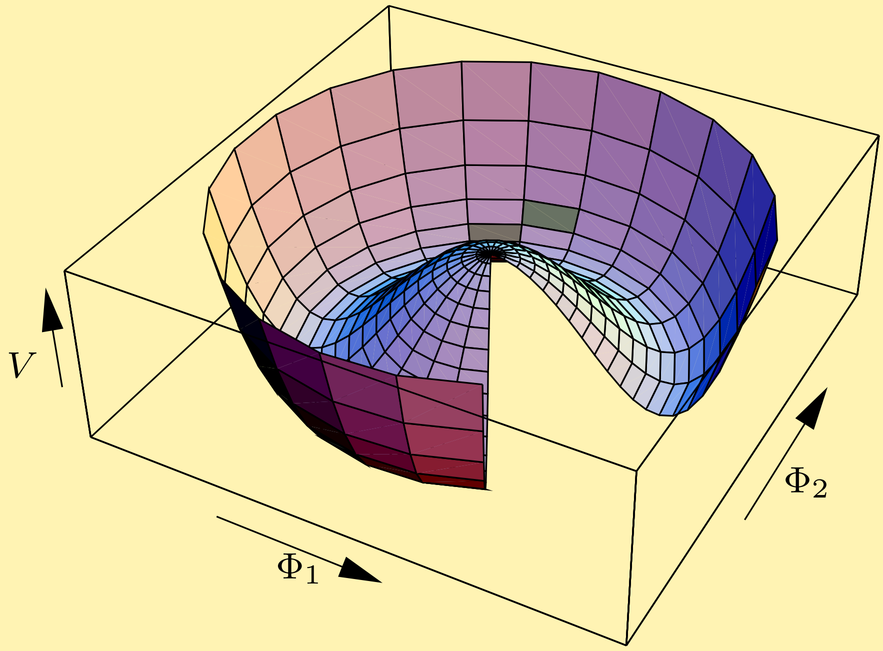
- Thermodynamically consistent
- It is the only self-consistent scheme with these properties

Gauge Model for ρ and π

- ρ-mesons $\Rightarrow$ Higgs-Kibble mechanism
- Φ: $SU(2)$-duplett

$$\mathscr{L} = -\frac{1}{2}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \frac{1}{2}(D_\mu\Phi)^\dagger D^\mu\Phi - V(\Phi)$$

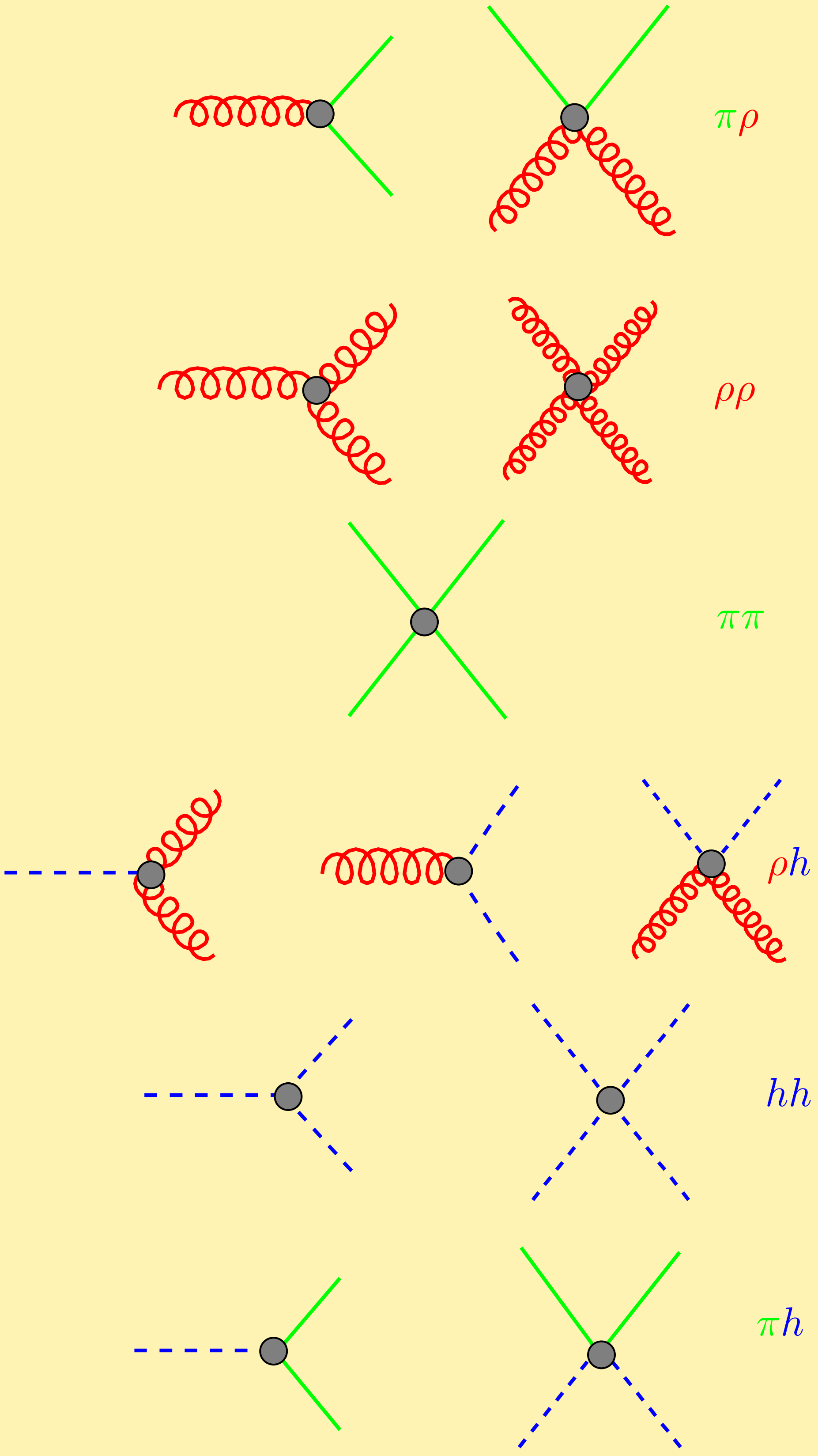
$$V(\Phi) = -\frac{\mu^2}{2}\Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2$$



- Add pions to the model:

$$\mathscr{L}_\pi = \frac{1}{2}(D_\mu\vec{\pi})(D_\mu\vec{\pi}) - \frac{\lambda_2}{8}(\vec{\pi}^2)^2 - \frac{\lambda_3}{4}\vec{\pi}^2\Phi^\dagger\Phi$$

Interactions in Physical Gauge



Quantization

Gauge fixing

- "Physical gauge"
  - ☞ ρ-fields massive $m_\rho^2 = g^2\mu^2/(4\lambda)$
  - ☞ Three ϕ-degrees of freedom $\Rightarrow$ absorbed to ρ-mesons
  - ☞ One ϕ-degree $\Rightarrow$ massive Higgs-particle
  - ☞ only phys. d.o.f. $\Rightarrow$ manifestly unitary
- R_ξ-gauges ('t Hooft) $\Rightarrow$ renormalizable
  - ☞ Unphysical d.o.f. $\Leftrightarrow$ Faddeev-Popov ghosts

Quantized theory

☞ Defines physical states

☞ Only physical states propagate $\Leftrightarrow$ dynamical consistency

☞ Physical quantities
eg. S-matrix, thermodynamical quantities
independent of the gauge fixing

☞ Theory renormalizable (R_ξ-gauge) and
physically consistent (unitary gauge)

Symmetries and Gauge Theories

- Global linear symmetries of the action (eq. of motion) ⇒ conserved quantities (energy, momentum, angular momentum, charges and currents)
- Local gauge theories for vector particles (ρ-mesons)

☞ defines couplings (QED, QCD, QFD, VMD, ...)

☞ Only physical states are interacting in quantum theory

☞ Ensures Lorentz invariance, unitarity and renormalizability of the S-matrix

- Quantized theory symmetric at any loop order

- Φ-functional: need also approximations

☞ Current correlators used in internal lines

$$\Sigma_{\text{int}}(1,2) = \langle j(1)j(2)\rangle_{\text{int}} = \mathrm{i}\frac{\delta\Phi[\varphi,G]}{\delta G(1,2)}$$

describe decay of states (and not where they are going!) different from "external lines" defined by

$$\Sigma_{\text{ext}}(1,2) = \langle j(1)j(2)\rangle_{\text{ext}} = \frac{\delta^2\Phi[\varphi,\mathscr{G}[\varphi]]}{\delta\varphi(1)\delta\varphi(2)}$$

fulfilling the Ward identity ⇒ take into account exactly the part of rescattering corresponding to processes in G

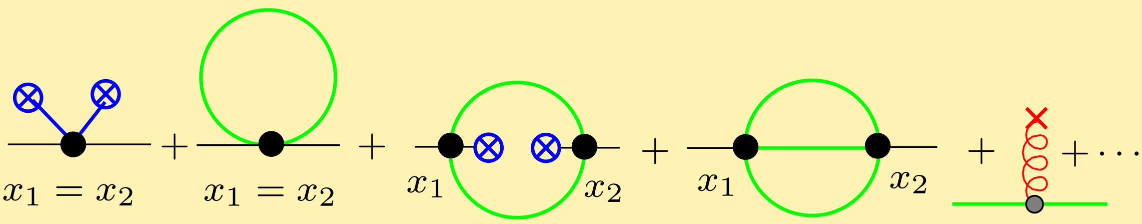
- Self-consistent formalism is not gauge invariant

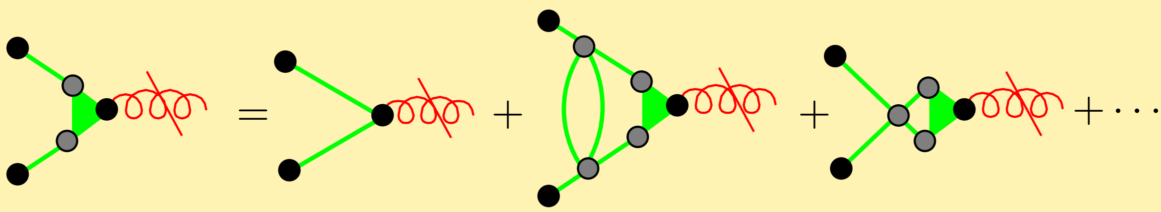
Gauge Invariant Formalism

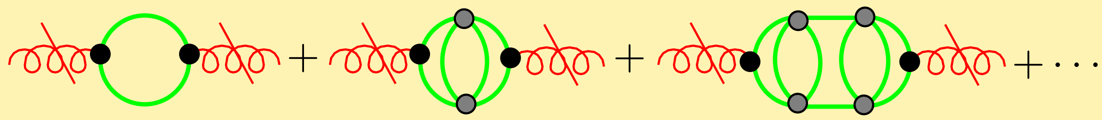
- A way out: Treat gauge field only at mean field level
- Φ-formalism works well for linearly realized global symmetries
- Couple external vector field to conserved current

- Derivative of G:

$$G^{-1} \circ G = \mathbb{1} \qquad \Rightarrow \qquad \frac{\delta G}{\delta \rho^\mu} = -G \circ \frac{\delta \Sigma}{\delta \rho^\mu} \circ G$$

$-\mathrm{i}\Sigma_{12} =$ 

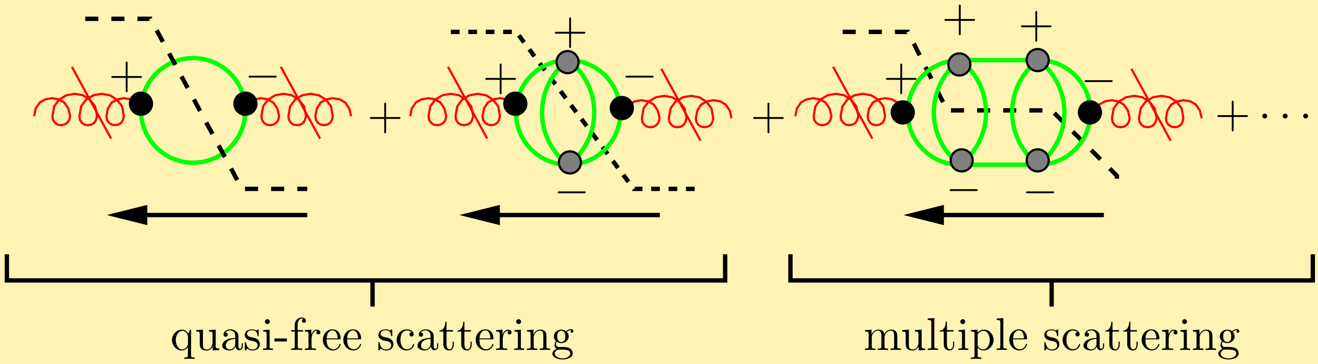
$\frac{\delta G[\rho]}{\delta \rho^\mu} =$ 

$\Pi_{\mu\nu} =$ 

Classical Transport Analogue

- Real processes in the classical limit ⇔ cutting rules in real time diagram formalism

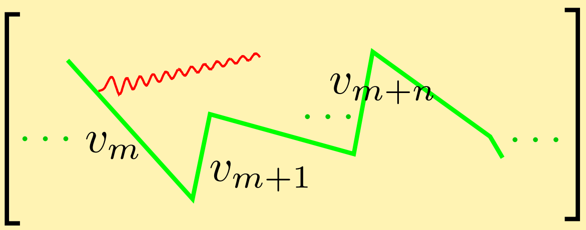
$$\Pi^{+-}_{\mu\nu} = \langle j_\mu j_\nu \rangle =$$



- Classical picture: Maxwell equations

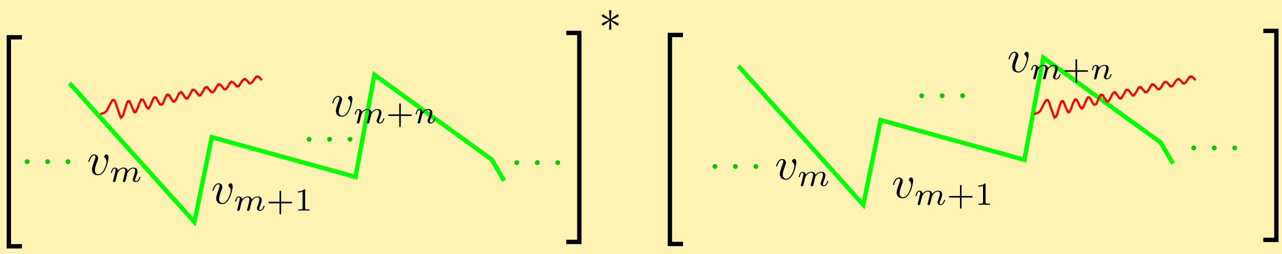
$$\partial_\nu \gamma^{\nu\mu} = j^\mu = e \int \mathrm{d}^3\vec{v}\, v^\mu n(t, \vec{x}, \vec{v})$$

$j^\mu = e \sum_m$



- Langevin process ⇔ FP–equation ⇔ HTL approximation

$$\langle v_m v_{m+n} \rangle \exp(-\Gamma\tau) \frac{(\Gamma\tau)^n}{n!} =$$



Summary and perspectives

- ✔ Finite width of particles ⇒
  important feature compared to quasi-particle approach
- ✔ Φ-functional
  self-consistent conserving approximations
  thermodynamically consistent
- ✔ Numerical treatment possible (including renormalization)

!!! Problems with self-consistent treatment of vector particles

- Φ-functional with internal vector lines ⇒
  violates gauge invariance
- ✔ Strong π-in-medium width effects on ρ-properties
- ✔ Gauge invariant description ⇒ requires multiple scattering
- ✔ Numerical treatment of vertex summation is possible
  (work in progress)

??? Is there a feasable self-consistent treatment including internal vector-lines?

- ✔ Applicable to transport processes