Kinetics of the chiral phase transition in a quark-meson $\boldsymbol{\sigma}$ model*

Hendrik van Hees, Alex Meistrenko, Carsten Greiner

Institut für Theoretische Physik, Johann Wolfgang Goethe-Universität,
Max-von-Laue-Str. 1, D-60438 Frankfurt am Main, Germany

Helmholtz Research Academy Hesse for FAIR, Campus Riedberg,
Max-von-Laue-Str. 12, D-60438 Frankfurt am Main, Germany

Using the two-particle irreducible (2PI) Φ-functional formalism for self-consistent approximations of a linear-σ model for quarks and mesons in and out of equilibrium, the build-up of fluctuations of net-baryon number during the time evolution of an expanding fireball is studied within a kinetic theory for the order parameter (σ field) and quark distribution functions. Initializing the system with purely Gaussian fluctuations a fourth-order cumulant is temporarily built up due to the evolution of the σ-field. This is counterbalanced, however, by the dissipative evolution due to collisions between quarks, anti-quarks, mesons, and the mean field, depending on the speed of the fireball expansion.

1. Introduction

One important motivation for ultra-relativistic heavy-ion experiments, as conducted, e.g., with the large-hadron collider at CERN, the Relativistic Heavy-Ion Collider (RHIC) at BNL, and in the future at the Facility for Antiproton and Ion Research (FAIR) is the understanding of the phase diagram of strongly interacting matter under extreme conditions of temperature and density. For small baryo-chemical potentials, μ_{B}, lattice-QCD calculations [1, 2] show that the transition between a quark-gluon plasma and a hadron-resonance gas as well as the chiral transition is a smooth crossover at a transition temperature $T_{\mathrm{c}} \simeq 155\,\mathrm{MeV}$. Based on effective models like the Nambu-Jona-Lasinio model, quark-meson models with constituent quarks [3, 4, 5, 6], and their Polyakov-loop extended versions [7, 8, 9, 10], at larger

* Presented at XXIX$^{\text{th}}$ International Conference on Ultra-relativistic Nucleus-Nucleus Collisions (Quark Matter 2022).

$\mu_{\rm B}$ one expects a $1^{\rm st}$-order transition line ending in a critical point with a $2^{\rm nd}$-order transition [11, 12, 13, 14]. The main challenge is that this phase structure must be reconstructed from the observables, which reflect the state of this medium at the end of the fireball evolution (thermal freeze-out), which lasts only for a very short time at the order of some $10\,{\rm fm}/c \simeq 10^{-23}\,{\rm s}$. A challenging theoretical question therefore is, whether "grand-canonical" higher-order cumulants of the net-baryon density can develop and survive the rapid time evolution of the finite-size fireball, expected to occur when the medium is undergoing a $1^{\rm st}$- or especially a $2^{\rm nd}$-order phase transition and whether corresponding quantitative signatures of a possible critical point can be observed.

In this contribution we study this, employing a set of coupled equations for the quarks, anti-quarks, and mesons as well as the order parameter, σ, of the chiral symmetry within a linear quark-meson σ model, derived from the two-particle irreducible functional (Φ functional) formalism [15].

2. The kinetic equations

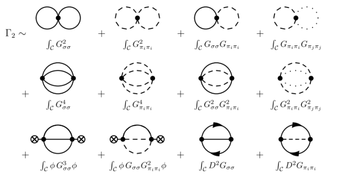
We start from an O(4) linear-σ model for σ-mesons, pions, and u- and d-quarks,

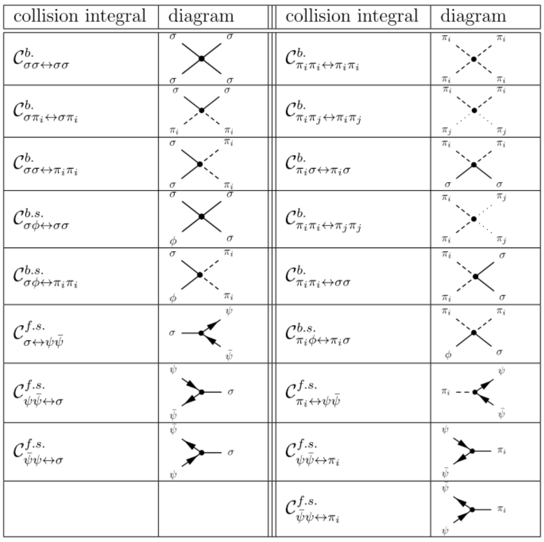
$$\begin{aligned}\mathcal{L} = &\sum_{i=1} \bar{\psi}_i \Big[\mathrm{i}\partial\!\!\!/ - g\left(\sigma + \mathrm{i}\gamma_5 \vec{\pi}\cdot\vec{\tau}\right)\Big]\psi_i \\ &+ \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 + f_\pi m_\pi^2 \sigma + U_0\,,\end{aligned} \tag{1}$$

where $\lambda = 20$, $f_\pi = 93\,\mathrm{MeV}$, $m_\pi = 138\,\mathrm{MeV}$, $\nu^2 = f_\pi^2 - m_\pi^2/\lambda$, and $U_0 = m_\pi^4/(4\lambda) - f_{\pi^2} m_{\pi^2}$ are chosen to lead to the right pion phenomenology in the vacuum. The quark-meson-coupling constant g is varied in the range between 2-5, leading to cross-over as well as $1^{\rm st}$- and $2^{\rm nd}$-order chiral phase transitions at finite T and $\mu_{\rm B}$.

For the kinetic equations to describe both the equilibrium state as well as the off-equilibrium kinetic evolution of this model we use the 2PI Φ-derivable approximation, defined in terms of the corresponding Feynman diagrams in Fig. 1 (left panel). Solving the corresponding self-consistent equations for the propagators and the mean σ-field in thermal equilibrium indeed leads to a phase diagram with a cross-over transition at lower $\mu_{\rm B}$ and a first-order transition line ending in a critical point at $(T, \mu_{\rm B}) = (108, 157)\,\mathrm{MeV}$ (for a quark-meson coupling, $g = 3.3$).

For the derivation of coupled kinetic equations of motion for the mean σ-field and the generalized Boltzmann equations for the quark- and meson-phase-space-distribution functions the diagrams are evaluated within the





collision integral	diagram	collision integral	diagram
$\mathcal{C}^{b.}_{\sigma\sigma\leftrightarrow\sigma\sigma}$		$\mathcal{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_i\pi_i}$	
$\mathcal{C}^{b.}_{\sigma\pi_i\leftrightarrow\sigma\pi_i}$		$\mathcal{C}^{b.}_{\pi_i\pi_j\leftrightarrow\pi_i\pi_j}$	
$\mathcal{C}^{b.}_{\sigma\sigma\leftrightarrow\pi_i\pi_i}$		$\mathcal{C}^{b.}_{\pi_i\sigma\leftrightarrow\pi_i\sigma}$	
$\mathcal{C}^{b.s.}_{\sigma\phi\leftrightarrow\sigma\sigma}$		$\mathcal{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_j\pi_j}$	
$\mathcal{C}^{b.s.}_{\sigma\phi\leftrightarrow\pi_i\pi_i}$		$\mathcal{C}^{b.}_{\pi_i\pi_i\leftrightarrow\sigma\sigma}$	
$\mathcal{C}^{f.s.}_{\sigma\leftrightarrow\psi\bar{\psi}}$		$\mathcal{C}^{b.s.}_{\pi_i\phi\leftrightarrow\pi_i\sigma}$	
$\mathcal{C}^{f.s.}_{\psi\bar{\psi}\leftrightarrow\sigma}$		$\mathcal{C}^{f.s.}_{\pi_i\leftrightarrow\psi\bar{\psi}}$	
$\mathcal{C}^{f.s.}_{\bar{\psi}\psi\leftrightarrow\sigma}$		$\mathcal{C}^{f.s.}_{\psi\bar{\psi}\leftrightarrow\pi_i}$	
		$\mathcal{C}^{f.s.}_{\bar{\psi}\psi\leftrightarrow\pi_i}$	

Fig. 1. Left: Included 2PI part of the effective action with 1st line: Hartree diagrams, 2nd line: basketball diagrams, 3rd line: sunset diagrams, where solid lines stand for the σ propagator, dashed and pointed lines for the pion propagators and solid lines with arrows for the fermion propagator. The circle with a cross represents a σ mean-field. Right: The scattering processes in the collision integrals of the kinetic equation. A full σ-line indicates a mean-field contribution, while a full ϕ-line describes a scattering process involving a σ meson.

Schwinger-Keldysh real-time formalism, leading to corresponding Kadanoff-Baym equations. Then a first-order gradient-expansion approximation to the Wigner transforms of the Green's functions as well as an "on-shell approximation" with self-consistent dispersion relations has been applied. This results in a non-Markovian dissipative equation for the mean field, σ, and a Boltzmann equation with a collision integral including the scattering processes depicted in Fig. 1 (right panel).

3. Simulation of a heavy-ion collision

To simulate the formation of higher-order cumulants of net-baryon-density fluctuations in momentum bins, we describe the fireball of strongly interacting quark-meson matter by an expanding homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker metric, $\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)(\mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2)$. This only leads to a modification in the drift terms of the mean-field and kinetic equations. For the mean field the "Hubble expansion" adds an additional dissipation term $3H\partial_t\sigma$ with the "Hubble constant" $H = \dot{a}/a$, as well as an additional term of the form $-Hp\partial_p f(t,p)$ in the drift terms for the particle phase-space distribution. Note that due to the assumed spatial homogeneity and isotropy the f's only depend on t and $p = |\vec{p}|$.

To initialize the fireball a spherically symmetric bubble of radius $R_0 = 5$ fm is considered, which then is expanding according to the above defined FLRW expansion with $a = vt$. The medium within this bubble is initialized in thermal equilibrium with a temperature T_0 and baryon chemical potential $\mu_{\mathrm{B}0}$. Then the initial net-quark number is Monte-Carlo sampled corresponding to a Gaussian distribution with the mean determined by the thermal initial state and a standard deviation of $\sigma_{q,\mathrm{net}} = \langle N_{q,\mathrm{net}} \rangle/10$. To mimick the expected fluctuations in a heavy-ion collision within a given "centrality bin" we keep the parameters R_0 and T_0 fixed and adjust μ_q such that the fireball contains the net-quark number $N_{q,\mathrm{net}}$ specified by the Monte-Carlo sampling.

With this initial conditions the coupled mean-field and kinetic integro-differential equations of motion are solved numerically on a momentum grid. It has been checked that the total net-quark number is conserved within a few precent numerical accuracy.

In Fig. 2 we show the results for the cumulant ratio, $R_{4,2} = \kappa_4/\kappa_2$, for initial conditions adjusted such that the system undergoes cross-over, second-order, and first-order transition, respectively. The fluctuations are plotted in different momentum intervals and for different expansion velocities, v, as a function of vt. I turns out that the most pronounced fluctuations occur at the critical time scales $\tau_{m_\sigma,\mathrm{min}}$ (dynamical minimum of the σ mass) and $\tau_{\sigma \to q\bar{q}}$ ($q\bar{q}$-pair production from σ decay). The fluctuations become largest for the smallest expansion velocity of $v = 0.05c$, corresponding to a quasi-adiabatic expansion, where the system stays for the longest time close to the critical region. However, in relativistic heavy-ion collisions this intermediate build-up of fluctuations related with the critical region of the phase diagram cannot be observed but only those surviving until the thermal freeze-out, which corresponds in our model to $vt \geq 6$ fm and a fireball radius of $R \geq 11$ fm.

In the most interesting case, $\mu_q = 160$ MeV, where the system evolves close to the critical point of a 2^{nd}-order phase transition the largest cumulant ratio in the final state is observed for intermediate expansion velocities $v = 0.2$-$0.4c$, while for the case when the system goes through a 1^{st}-order phase transition the final fluctuations are rather insensitive to the expansion velocity (for the most interesting momentum range $p = 200$-600 meV). This allows in principle to distinguish between different types of the phase transition and indicates the expected longer relaxation times ("critical slowing down") around a critical point in the phase diagram, i.e., the system needs longer to equilibrate and thus the fluctuations survive until the thermal

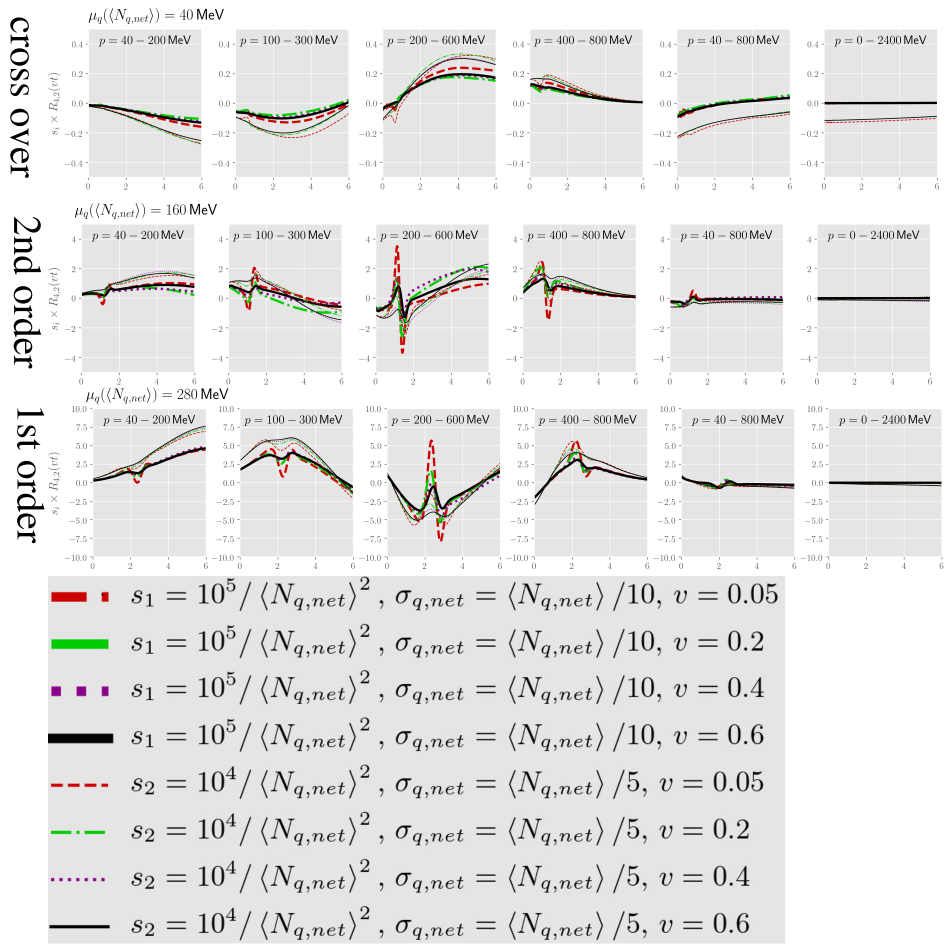


Fig. 2. Results for the rescaled cumulant ratio $R_{4,2}$ for different initial conditions where the fireball evolves through a cross-over, second-order, or first-order transition.

freeze-out. The absolute magnitude of the cumulant ratio increases with an increasing net-baryon number (note the scaling factors $s_1, s_2 \sim 1/\langle N_{q,\mathrm{net}}\rangle^2$ in the plots of Fig. 2).

4. Conclusions

Although the fluctuations of net-baryon numbers in an expanding finite system are less pronounced compared to the expectations from a equilibrated infinite strongly interaction matter, our simulations suggest that a significant deviation from the crossover behavior is observable through higher-order cumulant ratios in different momentum bins, providing a positive candidate for an experimental signature of the chiral phase transition and a possible critical region in the phase diagram of strongly interacting matter.

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