

Kinetics of the chiral phase transition in a quark-meson σ model

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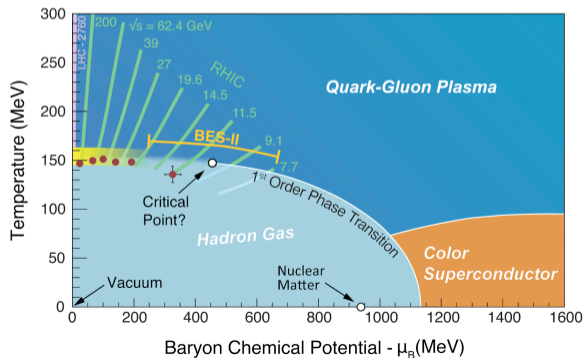
- 1 Chiral Symmetry and the QCD Phase Diagram
- 2 Quark-meson linear- σ model
- 3 Equilibrium properties
- 4 Off-equilibrium evolution: mean-field and transport equations with memory
- 5 Expanding fireball model
- 6 Fluctuations of conserved net-quark number
- 7 Conclusions

Chiral Symmetry and the QCD Phase Diagram

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\psi, A_\mu} + \mathcal{L}_G := \sum_{i \in \{u, d, s, c, b, t\}} \bar{\psi}_{i,j} \left(i\gamma^\mu (D_\mu)^j_k - m_i \delta_k^j \right) \psi_i^k - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu},$$

$$\mathcal{L}_{\psi, A_\mu} = \sum_{i \in \{u, d\}} \left[\bar{\psi}_{i,R} (i\gamma^\mu D_\mu) \psi_{i,R} + \bar{\psi}_{i,L} (i\gamma^\mu D_\mu) \psi_{i,L} \right] - \sum_{i \in \{u, d\}} m_i \left[\bar{\psi}_{i,R} \psi_{i,L} + \bar{\psi}_{i,L} \psi_{i,R} \right]$$

- light-quark sector (u+d quarks): approximate chiral symmetry $SU(2)_L \times SU(2)_R$



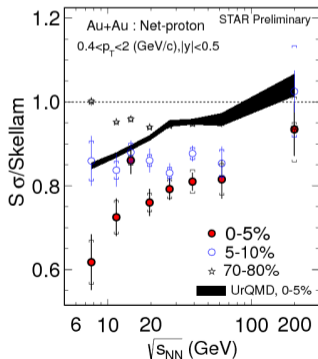
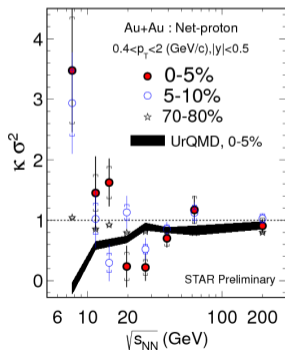
[Fig. from A. A. A. et al. Reaching for the horizon]

Cumulants of net-baryon number fluctuations

Quark-number susceptibilities

$$c_1 = \frac{N_{q,\text{net}}}{VT^3}, \quad c_2 = \frac{1}{VT^3} \left\langle \left(N_{q,\text{net}} - \langle N_{q,\text{net}} \rangle \right)^2 \right\rangle \equiv \frac{1}{VT^3} \sigma_{q,\text{net}}^2$$

$$c_3 = \frac{1}{VT^3} \left\langle \left(N_{q,\text{net}} - \langle N_{q,\text{net}} \rangle \right)^3 \right\rangle, \quad c_4 = \frac{1}{VT^3} \left[\left\langle \left(N_{q,\text{net}} - \langle N_{q,\text{net}} \rangle \right)^4 \right\rangle - 3\sigma_{q,\text{net}}^4 \right],$$



$$\kappa \sigma^2 = c_4 / c_2$$

$$S \sigma = c_3 / c_2$$

[X. Luo (STAR collab.) NPA 956, 75 (2016)]

Quark-Meson linear- σ Model

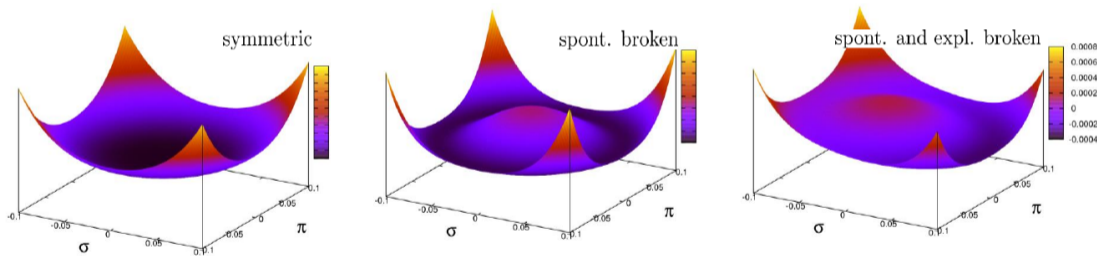
- $SU(2)_L \times SU(2)_R$ linear- σ model
- mesons: $\sigma, \vec{\pi}$, quarks: $\psi = (u, d)$

$$\mathcal{L} = \bar{\psi} \left[i \not{\partial} - g (\sigma + i \gamma_5 \vec{\pi} \cdot \vec{\tau}) \right] \psi + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi}) - \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2 - v^2)^2 + f_\pi m_\pi^2 \sigma + U_0$$

parameter	value	description
λ	20	coupling constant for σ and $\vec{\pi}$
g	2–5	coupling constant between $\sigma, \vec{\pi}$ and ψ
f_π	93 MeV	pion decay constant
m_π	138 MeV	pion mass
v^2	$f_\pi^2 - m_\pi^2 / \lambda$	field shift term
U_0	$m_\pi^4 / (4\lambda) - f_\pi^2 m_\pi^2$	ground state

Quark-Meson linear- σ Model: meson potential

$$\mathcal{L} = \bar{\psi} \left[i \not{\partial} - g (\sigma + i \gamma_5 \vec{\pi} \cdot \vec{\tau}) \right] \psi + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi}) - \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2 - v^2)^2 + f_\pi m_\pi^2 \sigma + U_0$$



Quark-Meson linear- σ Model: 2PI action

$$\mathcal{L} = \bar{\psi} \left[i \not{\partial} - g (\sigma + i \gamma_5 \vec{\pi} \cdot \vec{\tau}) \right] \psi + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi}) - \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2 - v^2)^2 + f_\pi m_\pi^2 \sigma + U_0,$$

$$\Gamma[\sigma, \vec{\pi}, G, D] = S[\sigma, \vec{\pi}] + \frac{i}{2} \text{Tr} \ln G^{-1} + \frac{i}{2} G_0^{-1} G - i \text{Tr} \ln D^{-1} - i \text{Tr} D_0^{-1} D + \Gamma_2[\sigma, \vec{\pi}, G, D]$$

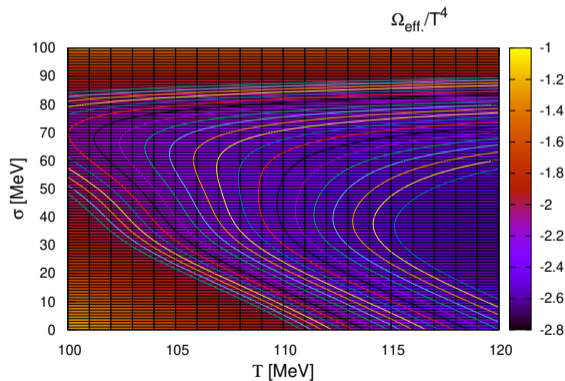
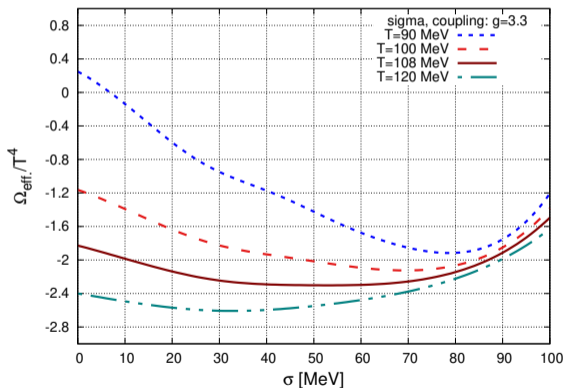
$$\Gamma_2 \sim \int_{\mathcal{C}} G_{\sigma\sigma}^2 + \int_{\mathcal{C}} G_{\pi_i\pi_i}^2 + \int_{\mathcal{C}} G_{\sigma\sigma} G_{\pi_i\pi_i} + \int_{\mathcal{C}} G_{\pi_i\pi_i} G_{\pi_j\pi_j} + \int_{\mathcal{C}} G_{\sigma\sigma}^4 + \int_{\mathcal{C}} G_{\pi_i\pi_i}^4 + \int_{\mathcal{C}} G_{\sigma\sigma}^2 G_{\pi_i\pi_i}^2 + \int_{\mathcal{C}} G_{\pi_i\pi_i}^2 G_{\pi_j\pi_j}^2 + \int_{\mathcal{C}} \phi G_{\sigma\sigma}^3 \phi + \int_{\mathcal{C}} \phi G_{\sigma\sigma} G_{\pi_i\pi_i}^2 \phi + \int_{\mathcal{C}} D^2 G_{\sigma\sigma} + \int_{\mathcal{C}} D^2 G_{\pi_i\pi_i}$$

Equations of motion:

$$\frac{\delta \Gamma}{\delta \sigma} = \frac{\delta \Gamma}{\delta \vec{\pi}} = \frac{\delta \Gamma}{\delta G} = \frac{\delta \Gamma}{\delta D} = 0.$$

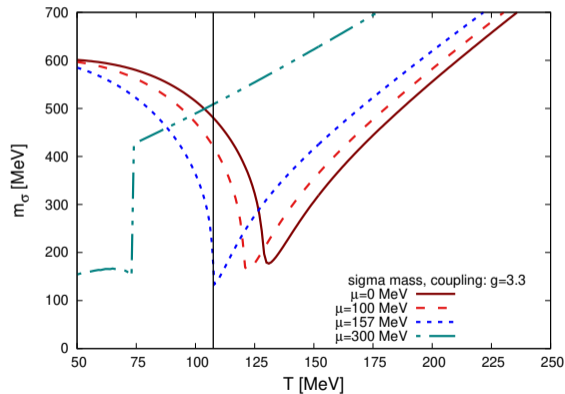
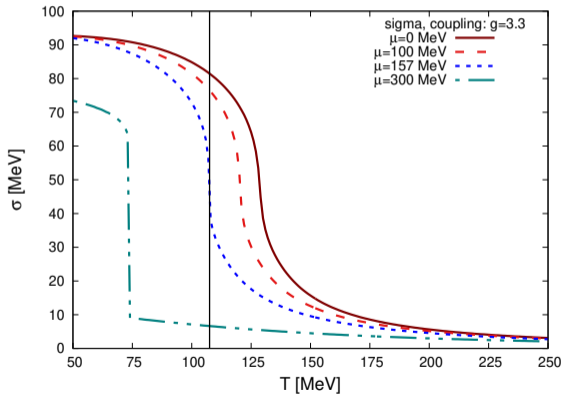
Quark-Meson linear- σ Model: equilibrium effective potential

$$\Omega_{\text{eff}}[\sigma, \vec{\pi}, G, D] = -\frac{1}{\beta V} \text{i}\Gamma[\sigma, \vec{\pi}, G, D]$$



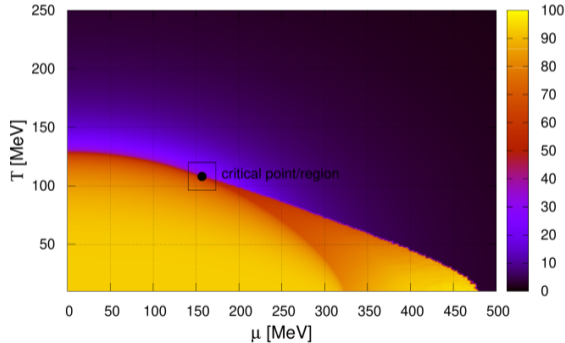
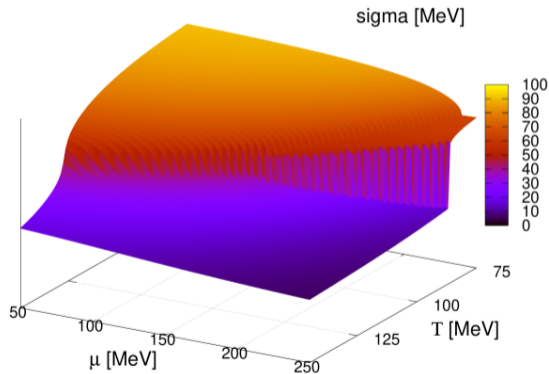
Quark-Meson linear- σ Model: equilibrium order parameter

$$\Omega_{\text{eff}}[\sigma, \vec{\pi}, G, D] = -\frac{1}{\beta V} \text{i}\Gamma[\sigma, \vec{\pi}, G, D], \quad \frac{\partial \Omega_{\text{eff}}}{\partial \sigma} \stackrel{!}{=} 0, \quad M_{\sigma}^2 = \frac{\partial^2 \Omega_{\text{eff}}}{\partial \sigma^2}, \quad M_{\pi}^2 = \frac{\partial^2 \Omega_{\text{eff}}}{\partial \pi_i^2}$$



Quark-Meson linear- σ Model: equilibrium order parameter

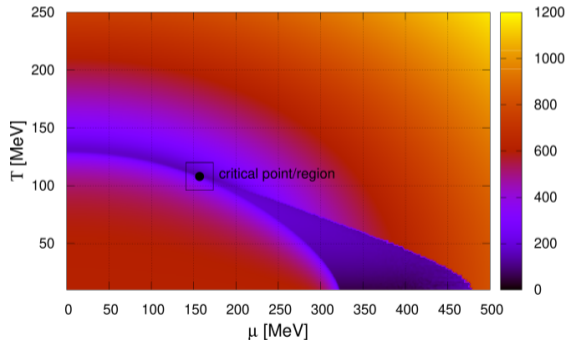
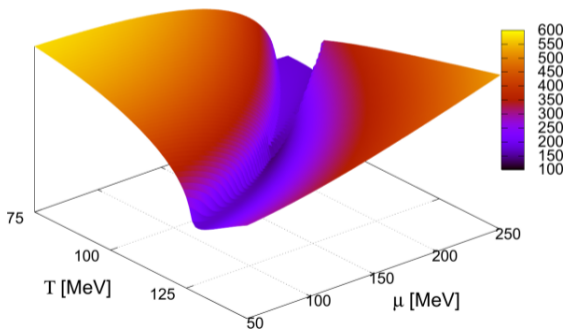
$$\Omega_{\text{eff}}[\sigma, \vec{\pi}, G, D] = -\frac{1}{\beta V} i\Gamma[\sigma, \vec{\pi}, G, D], \quad \frac{\partial \Omega_{\text{eff}}}{\partial \sigma} \stackrel{!}{=} 0, \quad M_{\sigma}^2 = \frac{\partial^2 \Omega_{\text{eff}}}{\partial \sigma^2}, \quad M_{\pi}^2 = \frac{\partial^2 \Omega_{\text{eff}}}{\partial \pi_i^2}$$



Quark-Meson linear- σ Model: effective σ mass

$$\Omega_{\text{eff}}[\sigma, \vec{\pi}, G, D] = -\frac{1}{\beta V} i\Gamma[\sigma, \vec{\pi}, G, D], \quad \frac{\partial \Omega_{\text{eff}}}{\partial \sigma} \stackrel{!}{=} 0, \quad M_{\sigma}^2 = \frac{\partial^2 \Omega_{\text{eff}}}{\partial \sigma^2}, \quad M_{\pi}^2 = \frac{\partial^2 \Omega_{\text{eff}}}{\partial \pi_i^2}$$

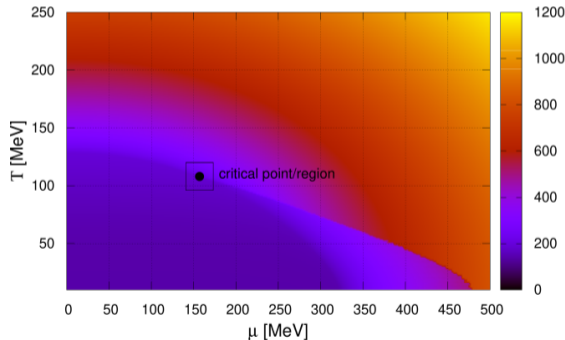
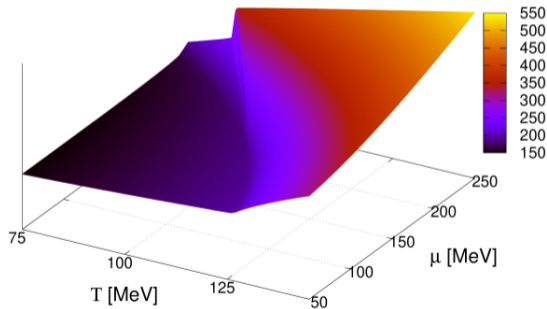
sigma mass [MeV]



Quark-Meson linear- σ Model: effective π mass

$$\Omega_{\text{eff}}[\sigma, \vec{\pi}, G, D] = -\frac{1}{\beta V} i\Gamma[\sigma, \vec{\pi}, G, D], \quad \frac{\partial \Omega_{\text{eff}}}{\partial \sigma} \stackrel{!}{=} 0, \quad M_{\sigma}^2 = \frac{\partial^2 \Omega_{\text{eff}}}{\partial \sigma^2}, \quad M_{\pi}^2 = \frac{\partial^2 \Omega_{\text{eff}}}{\partial \pi_i^2}$$

pion mass [MeV]



Quark-Meson linear- σ Model: off-equilibrium equations

- real-time Keldysh contour \Rightarrow 2PI/Kadanoff Baym \Rightarrow transport equation (spatially homogeneous)
- Mean-field equation

$$\partial_t^2 \phi + D(t) + J(t) = 0, J(t) := \lambda \left(\phi^2 - v^2 + 3G_{\sigma\sigma}^{11} + \sum_i G_{\pi_i\pi_i}^{11} \right) \phi - f_\pi m_\pi^2 + g \langle \bar{\psi} \psi \rangle$$

- transport equations for meson- and quark-phase-space-distribution functions

$$\partial_t f^\sigma(t, \vec{p}_1) = \mathcal{C}_{\sigma\sigma \leftrightarrow \sigma\sigma}^{b.} + \sum_i \mathcal{C}_{\sigma\pi_i \leftrightarrow \sigma\pi_i}^{b.} + \sum_i \mathcal{C}_{\sigma\sigma \leftrightarrow \pi_i\pi_i}^{b.} + \mathcal{C}_{\sigma\phi \leftrightarrow \sigma\sigma}^{b.s.} + \sum_i \mathcal{C}_{\sigma\phi \leftrightarrow \pi_i\pi_i}^{b.s.} + \mathcal{C}_{\sigma \leftrightarrow \psi\bar{\psi}}^{f.s.},$$

$$\begin{aligned} \partial_t f^{\pi_i}(t, \vec{p}_1) &= \mathcal{C}_{\pi_i\pi_i \leftrightarrow \pi_i\pi_i}^{b.} + \sum_{j \neq i} \mathcal{C}_{\pi_i\pi_j \leftrightarrow \pi_i\pi_j}^{b.} + \sum_{j \neq i} \mathcal{C}_{\pi_i\pi_i \leftrightarrow \pi_j\pi_j}^{b.} + \mathcal{C}_{\pi_i\sigma \leftrightarrow \pi_i\sigma}^{b.} + \mathcal{C}_{\pi_i\pi_i \leftrightarrow \sigma\sigma}^{b.} \\ &\quad + \mathcal{C}_{\pi_i\phi \leftrightarrow \pi_i\sigma}^{b.s.} + \mathcal{C}_{\pi_i \leftrightarrow \psi\bar{\psi}}^{f.s.} \end{aligned}$$

$$\partial_t f^\psi(t, \vec{p}_1) = \mathcal{C}_{\psi\bar{\psi} \leftrightarrow \sigma}^{f.s.} + \sum_i \mathcal{C}_{\psi\bar{\psi} \leftrightarrow \pi_i}^{f.s.}$$

$$\partial_t f^{\bar{\psi}}(t, \vec{p}_1) = \mathcal{C}_{\bar{\psi}\psi \leftrightarrow \sigma}^{f.s.} + \sum_i \mathcal{C}_{\bar{\psi}\psi \leftrightarrow \pi_i}^{f.s.},$$

Quark-Meson linear- σ Model: collision terms

collision integral	diagram	collision integral	diagram
$C_{\sigma\sigma\leftrightarrow\sigma\sigma}^b$		$C_{\pi_i\pi_i\leftrightarrow\pi_i\pi_i}^b$	
$C_{\sigma\pi_i\leftrightarrow\sigma\pi_i}^b$		$C_{\pi_i\pi_j\leftrightarrow\pi_i\pi_j}^b$	
$C_{\sigma\sigma\leftrightarrow\pi_i\pi_i}^b$		$C_{\pi_i\sigma\leftrightarrow\pi_i\sigma}^b$	
$C_{\sigma\phi\leftrightarrow\sigma\sigma}^{b.s.}$		$C_{\pi_i\pi_i\leftrightarrow\pi_j\pi_j}^b$	
$C_{\sigma\phi\leftrightarrow\pi_i\pi_i}^{b.s.}$		$C_{\pi_i\pi_i\leftrightarrow\sigma\sigma}^b$	
$C_{\sigma\leftrightarrow\psi\bar{\psi}}^{f.s.}$		$C_{\pi_i\phi\leftrightarrow\pi_i\sigma}^{b.s.}$	
$C_{\phi\bar{\psi}\leftrightarrow\sigma}^{f.s.}$		$C_{\pi_i\leftrightarrow\psi\bar{\psi}}^{f.s.}$	
$C_{\bar{\psi}\psi\leftrightarrow\sigma}^{f.s.}$		$C_{\psi\bar{\psi}\leftrightarrow\pi_i}^{f.s.}$	
		$C_{\bar{\psi}\psi\leftrightarrow\pi_i}^{f.s.}$	

Quark-Meson linear- σ Model: non-Markovian dissipation

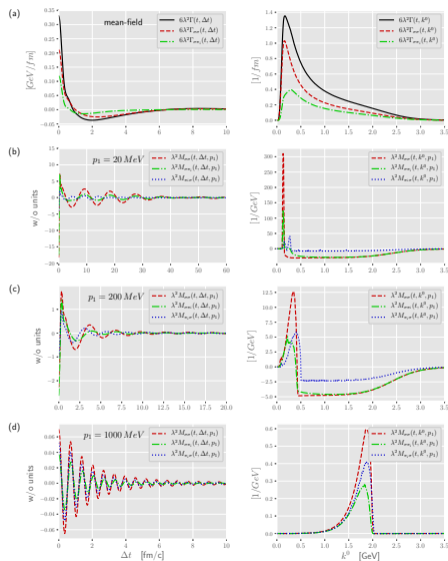
- memory kernel in mean-field dissipation term

$$D(t) = 6\lambda^2 [\Gamma(t, \Delta t = t - t_0) \phi(t_0) - \Gamma(t, \Delta t = 0) \phi(t)] + 6\lambda^2 \int_{t_0}^t dt' \dot{\phi}(t') \Gamma(t, t - t'),$$
$$\Gamma(t, \Delta t) = \int \frac{dk^0}{(2\pi)} e^{-ik^0 \Delta t} \frac{1}{k^0} \left[\tilde{\mathcal{M}}_{\sigma\sigma}(t, k^0) + \frac{1}{3} \sum_i \tilde{\mathcal{M}}_{\sigma\pi_i}(t, k^0) \right],$$

- memory kernels in Boltzmann equation

$$\mathcal{G}_\sigma^{b.s.}(t, \vec{p}_1) := \frac{9}{2} \pi \lambda^2 \phi(t) \int_0^\infty d\Delta t \phi(t - \Delta t) \int \frac{dk^0}{(2\pi)} e^{-ik^0 \Delta t} \mathcal{M}_{\sigma\sigma}(t, k^0, p_1)$$
$$+ \frac{1}{2} \pi \lambda^2 \phi(t) \sum_i \int_0^\infty d\Delta t \phi(t - \Delta t) \int \frac{dk^0}{(2\pi)} e^{-ik^0 \Delta t} \mathcal{M}_{\sigma\pi_i}(t, k^0, p_1),$$
$$\mathcal{G}_{\pi_i}^{b.s.}(t, \vec{p}_1) := \pi \lambda^2 \phi(t) \int_0^\infty d\Delta t \phi(t - \Delta t) \int \frac{dk^0}{(2\pi)} e^{-ik^0 \Delta t} \mathcal{M}_{\pi_i\sigma}(t, k^0, p_1),$$

Quark-Meson linear- σ Model: Memory kernels



Quark-Meson linear- σ Model: expanding-fireball geometry

- Friedmann-Lemaître-Robertson-Walker metric (spatially flat)

$$ds^2 = dt^2 - a^2(t)(dx_1^2 + dx_2^2 + dx_3^2), \quad H = \dot{a}/a$$

- expanding fireball with radius $R(t) = R_0 + v_e t$, $\dot{a}/a = \dot{R}/R$
- mean-field equation

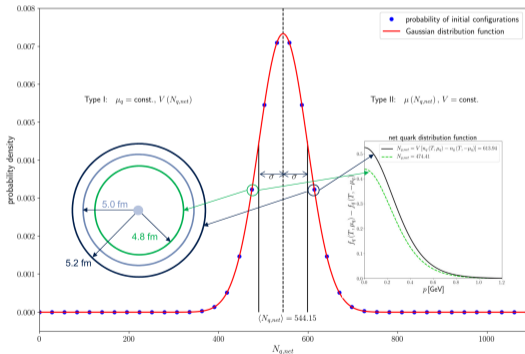
$$\partial_t^2 \phi + 3H \partial_t \phi + D(t) + J(t) = 0$$

- Boltzmann equation

$$\left(\frac{\partial}{\partial t} - Hp \frac{\partial}{\partial p} \right) f = \mathcal{S}$$

Initialization of net-quark numbers

- goal: time-evolution of net-quark number fluctuations
- ensembles with fluctuating initial conditions

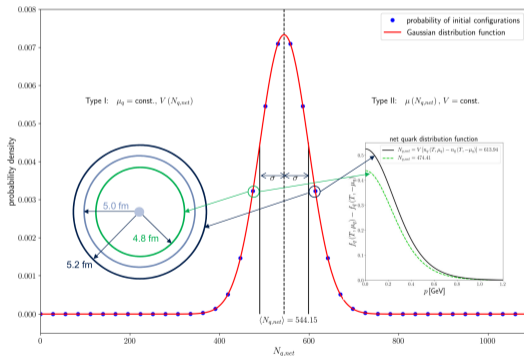


- mean net-quark number

$$\langle N_{q,net} \rangle = \frac{4\pi}{3} R_0^2 \int \frac{d^3 p}{(2\pi)^3} [f_q(T, \mu_q) - f_q(T, -\mu_q)]$$

- standard deviation: $\sigma_{q,net} = \langle N_{q,net} \rangle / 10$
- choose $M = 200-1000$ values for $N_{q,net}$
- initialize type I or type II for each $N_{q,net}$

- goal: time-evolution of net-quark number fluctuations
- ensembles with fluctuating initial conditions

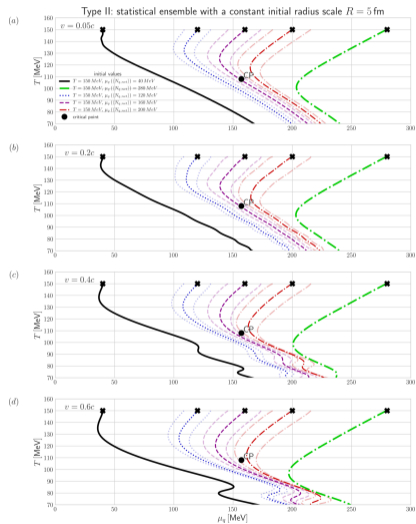
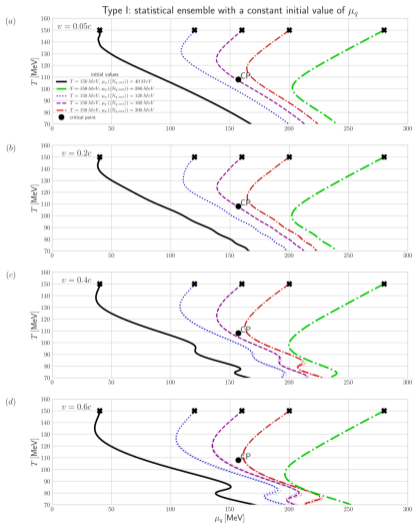


- ensembles with fluctuating initial conditions

$$\langle O \rangle = \frac{p_0 O_0 + p_M O_M}{2} + \sum_{k=1}^{M-1} p_k O_k$$

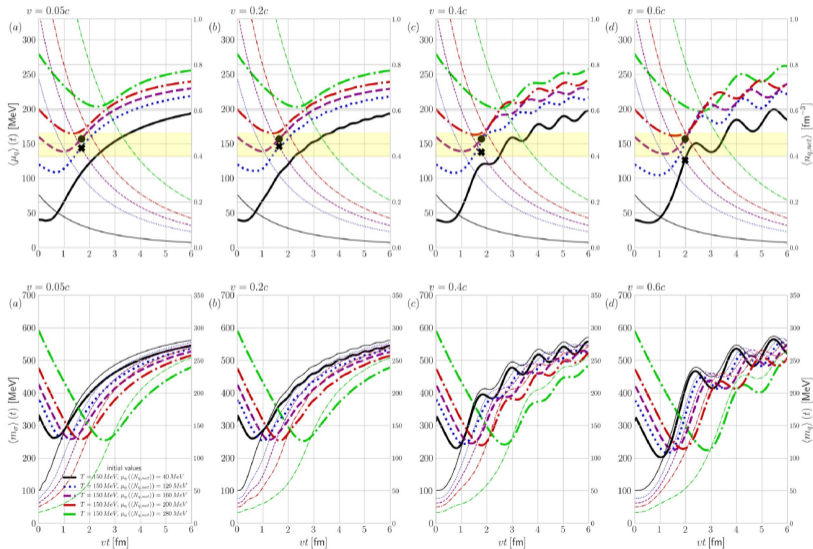
- cumulant ratios $R_{3,1} = c_3/c_1$,
 $R_{4,2} = c_4/c_2 = \kappa\sigma^2$,
 $c_1 = \langle m \rangle$,
 $c_2 = \tilde{m}_2 = \sigma^2$,
 $c_3 = \tilde{m}_3$,
 $c_4 = \tilde{m}_4 - 3\tilde{m}_2^2$

“Trajectories” in phase diagram



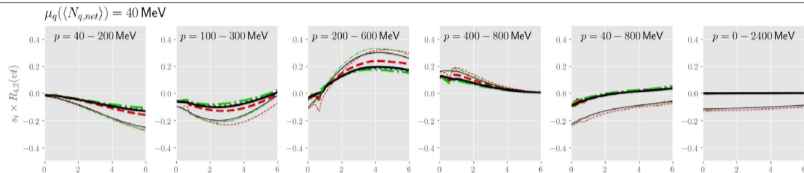
“Trajectories” in phase diagram

Type II: statistical ensemble with a constant initial radius scale $R = 5$ fm

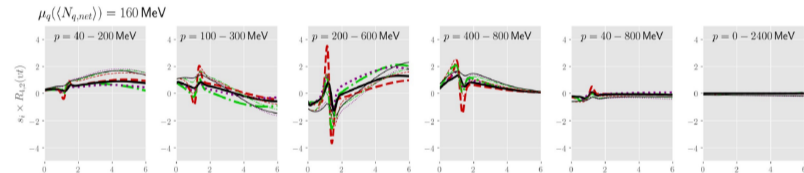


“Trajectories” in phase diagram

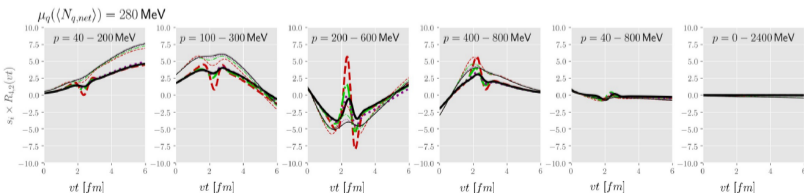
cross over



2nd order



1st order



$T_{\text{ini}} = 160 \text{ MeV}$
 $\langle N \rangle_{q,net} = 123, 544, 1158$

- $s_1 = 10^5 / \langle N_{q,net} \rangle^2, \sigma_{q,net} = \langle N_{q,net} \rangle / 10, v = 0.05$
- $s_1 = 10^6 / \langle N_{q,net} \rangle^2, \sigma_{q,net} = \langle N_{q,net} \rangle / 10, v = 0.2$
- $s_1 = 10^5 / \langle N_{q,net} \rangle^2, \sigma_{q,net} = \langle N_{q,net} \rangle / 10, v = 0.4$
- $s_1 = 10^6 / \langle N_{q,net} \rangle^2, \sigma_{q,net} = \langle N_{q,net} \rangle / 10, v = 0.6$
- $s_2 = 10^4 / \langle N_{q,net} \rangle^2, \sigma_{q,net} = \langle N_{q,net} \rangle / 5, v = 0.05$
- $s_2 = 10^4 / \langle N_{q,net} \rangle^2, \sigma_{q,net} = \langle N_{q,net} \rangle / 5, v = 0.2$
- $s_2 = 10^4 / \langle N_{q,net} \rangle^2, \sigma_{q,net} = \langle N_{q,net} \rangle / 5, v = 0.4$
- $s_2 = 10^4 / \langle N_{q,net} \rangle^2, \sigma_{q,net} = \langle N_{q,net} \rangle / 5, v = 0.6$

- Phase diagram of strongly interacting matter and chiral symmetry
- Quark-meson linear σ model
- 2PI effective action in and off equilibrium
- coupled mean-field and transport equations with memory
 - ensembles with fluctuating initial conditions for net-quark number
 - trajectory through chiral phase transition \Rightarrow non-trivial fluctuations \Rightarrow higher cumulants
 - differences between crossover and 1st or 2nd-order transitions
 - depends on expansion rate and observed momentum bins
 - fluctuations may survive full fireball evolution
- A. Meistrenko, HvH, C. Greiner, *Ann. Phys.* **431**, 168555 (2021); arXiv:2007.09929 [hep-ph]