

Dileptons in heavy-ion-collisions

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Electromagnetic probes theory perspective

Electromagnetic probes in heavy-ion collisions

- γ, ℓ^\pm : no strong interactions
- reflect whole “history” of collision:
 - from **pre-equilibrium phase**
 - from thermalized medium
QGP and hot hadron gas
 - from VM decays **after thermal freezeout**

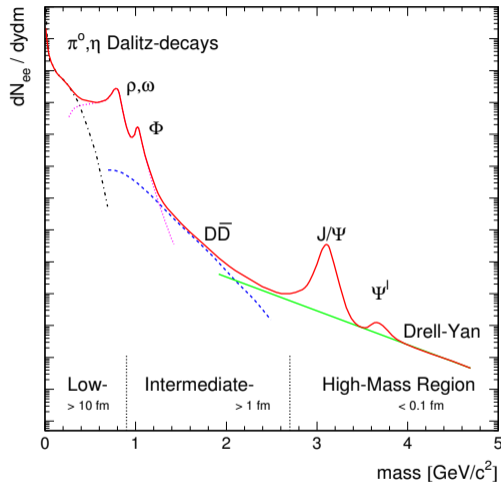
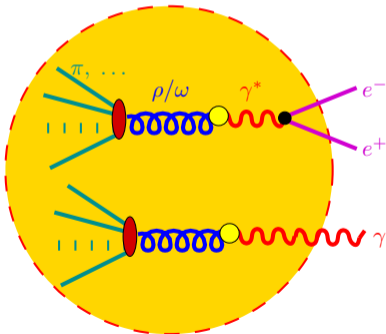


Fig. by A. Drees

Electromagnetic probes from thermal source

- retarded electromagnetic-current-correlation function

$$\Pi_{\text{em},i}^{\mu\nu} = i \int d^4x \exp(iq \cdot x) \Theta(x^0) \langle [j_{\text{em},i}^\mu(x), j_{\text{em},i}^\nu(0)] \rangle$$

- McLerran-Toimela formula [MT85, GK91]

$$q_0 \frac{dN_\gamma}{d^4x d^3\vec{q}} = -\frac{\alpha_{\text{em}}}{2\pi^2} g^{\mu\nu} \text{Im} \Pi_{\mu\nu}^{(\text{ret})}(q, u) \Big|_{q_0=|\vec{q}|} f_B(q \cdot u)$$

$$\frac{dN_{e^+e^-}}{d^4x d^4q} = -g^{\mu\nu} \frac{\alpha^2}{3q^2\pi^3} \text{Im} \Pi_{\mu\nu}^{(\text{ret})}(q, u) \Big|_{q^2=M_{e^+e^-}^2} f_B(q \cdot u)$$

- Lorentz covariant (dependent on four-velocity of fluid cell, u)
- $q \cdot u = E_{\text{cm}}$: Doppler blue shift of q_T spectra!
- to lowest order in α : $4\pi\alpha\Pi_{\mu\nu} \simeq \Sigma_{\mu\nu}^{(\gamma)}$
- vector-meson dominance model:

$$\Sigma_{\mu\nu}^{\gamma} = \text{---} \overset{G_\rho}{\text{---}} \text{---}$$

- $\ell^+\ell^-$ -inv.-mass spectra \Rightarrow in-med. spectral functions of vector mesons (ρ, ω, ϕ)!

Radiation from thermal QGP: $q\bar{q}$ annihilation

- General: **McLerran-Toimela formula**

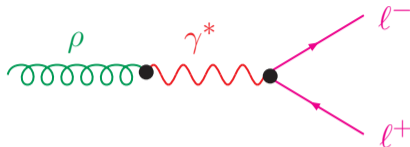
$$\frac{dN_{l+l-}^{(\text{MT})}}{d^4x d^4q} = -\frac{\alpha^2}{3\pi^3} \frac{L(M^2)}{M^2} g_{\mu\nu} \text{Im} \sum_i \Pi_{\text{em},i}^{\mu\nu}(M, \vec{q}) f_B(q \cdot u)$$

- in **QGP** phase: $q\bar{q}$ annihilation
- hard-thermal-loop improved em. current-current correlator

$$-i\Pi_{\text{em},\text{QGP}} = \text{Diagram}$$

Radiation from hadronic sources: ρ , ω , ϕ decays

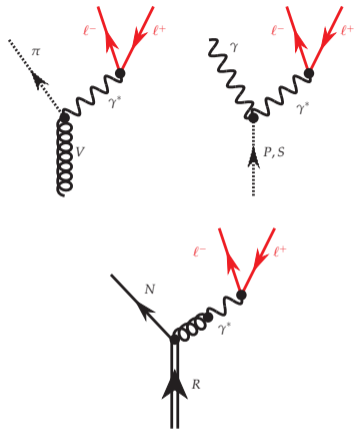
- model assumption: **vector-meson dominance**



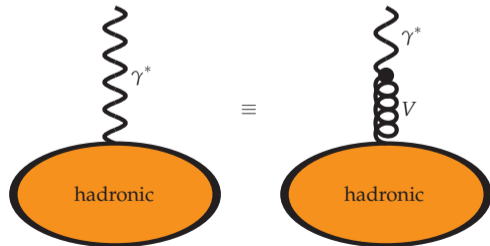
$$\begin{aligned} \frac{dN_{\rho \rightarrow l+l-}^{(\text{MT})}}{d^4x d^4q} &= \frac{M}{q^0} \Gamma_{\rho \rightarrow l+l-}(M) \frac{dN_{\rho}}{d^3\vec{x} d^4q} \\ &= -\frac{\alpha^2}{3\pi^3} \frac{L(M^2)}{M^2} \frac{m_{\rho}^4}{g_{\rho}^2} g_{\mu\nu} \text{Im} D_{\rho}^{\mu\nu}(M, \vec{q}) f_B \left(\frac{q \cdot u - 2\mu_{\pi}(t)}{T(t)} \right) \end{aligned}$$

- special case of McLerran-Toimela (MT) formula
- $M^2 = q^2$: invariant mass, M , of dilepton pair
- $L(M^2) = (1 + 2m_l^2/M^2) \sqrt{1 - 4m_l^2/M^2}$: dilepton phase-space factor
- $D_{\rho}^{\mu\nu}(M, \vec{q})$: (four-transverse part of) in-medium ρ propagator at given $T(t)$, $\mu_{\text{meson/baryon}}(t)$
- analogous for ω and ϕ

Transition form factors: “ ρ mesons” via VMD



- vector mesons have “vacuum spectral shapes”
- propagated as “on-shell particles” of finite lifetime and variable mass
- Dalitz decay: 1 particle \rightarrow 3 particles
- V : $\omega \rightarrow \pi + \gamma^* \rightarrow \pi + l^+ + l^-$
- P, S : $\pi, \eta \rightarrow \gamma + \gamma^* \rightarrow \gamma + l^+ + l^-$
- R : Baryon resonances $\Delta, N^* \rightarrow N + V \rightarrow N + \gamma^* \rightarrow N + l^+ + l^-$
- vector-meson dominance: model for hadron em. trans. FF

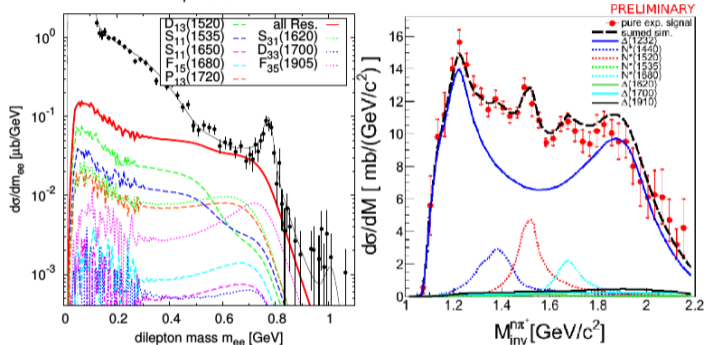


GiBUU: “ ρ meson” in pp

- production through hadron resonances

$$NN \rightarrow NR \rightarrow NN\rho, NN \rightarrow N\Delta \rightarrow NN\pi\rho$$

$$\rho \rightarrow e^+e^-$$



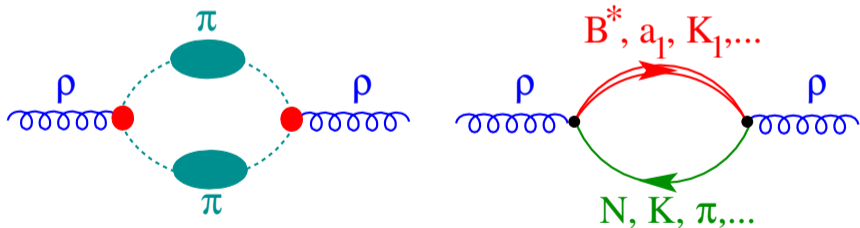
- plots: J. Weil et al [WHM12, ABB⁺14]
- VMD model \Leftrightarrow em. transition form factors of baryon resonances!
- “ ρ ”-line shape “modified” already in elementary hadronic reactions
- due to production mechanism via resonances

Hadronic many-body theory

- hadronic many-body theory (HMBT) for vector mesons

[Ko et al, Chanfray et al, Herrmann et al, Rapp et al, ...]

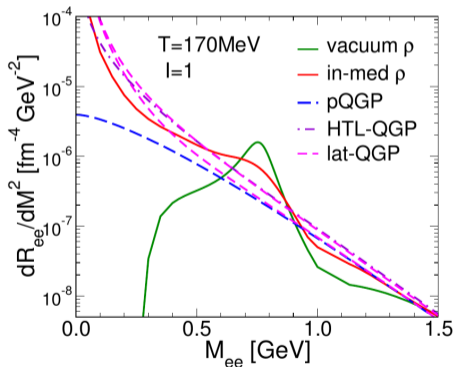
- $\pi\pi$ interactions and **baryonic excitations**
- effective hadronic models, implementing symmetries
- parameters fixed from phenomenology (photon absorption at nucleons and nuclei, $\pi N \rightarrow \rho N$)
- evaluated at **finite temperature and density**
- self-energies \Rightarrow **mass shift and broadening** in the medium



- Baryons** important, even at low **net** baryon density $n_B - n_{\bar{B}}$
- reason: $n_B + n_{\bar{B}}$ relevant (CP inv. of strong interactions)

Dilepton rates: Hadron gas \leftrightarrow QGP

- in-medium **hadron gas** matches with **QGP**
- similar results also for γ rates
- “quark-hadron duality”?



[Rap13]

Bulk-medium evolution

Bulk evolution with transport and coarse graining

- established transport models for **bulk evolution**
 - e.g., **UrQMD**, GiBUU, BAMPS, (p)HSD,...
 - solve **Boltzmann equation** for hadrons and/or partons
- dilemma: need medium-modified **dilepton/photon emission rates**
- usually available only in **equilibrium QFT calculations**
- ways out:
 - **(ideal) hydrodynamics** \Rightarrow local thermal equilibrium \Rightarrow use equilibrium rates
 - transport-hydro hybrid model: treat early stage with transport, then **coarse grain** \Rightarrow switch to hydro \Rightarrow switch back to transport (**Cooper-Frye “particlization”**)
- here: **UrQMD transport** for entire bulk evolution
 \Rightarrow use **coarse graining** in space-time cells \Rightarrow extract $T, \mu_B, \mu_\pi, \dots \Rightarrow$ use equilibrium rates locally

Coarse-grained UrQMD (CGUrQMD)

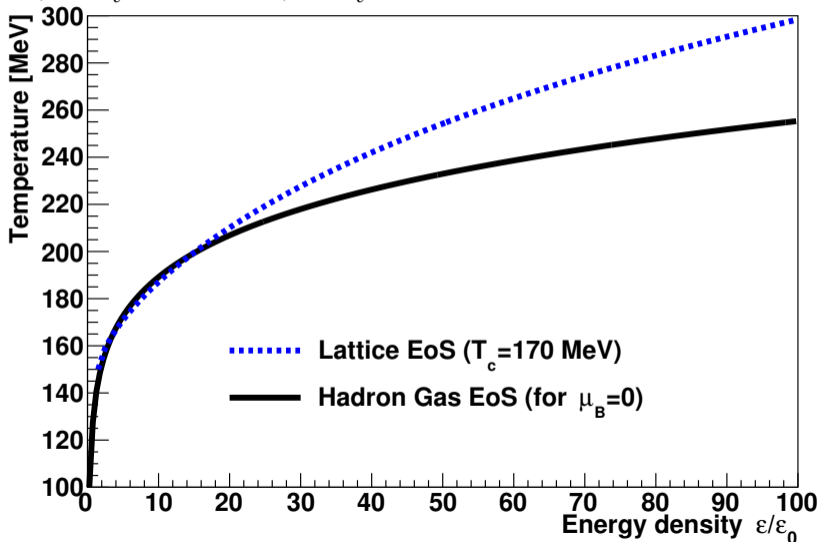
- problem with **medium modifications** of spectral functions/interactions
- only available in equilibrium many-body QFT models
- use “in-medium cross sections” naively: **double counting???**
- way out: map transport to **local-equilibrium fluid**
- use **ensemble of UrQMD** runs with an **equation of state**
- space-time grid with $\Delta t = 0.2 \text{ fm}/c$, $\Delta x = 0.8 \text{ fm}$
- fit **temperature, chemical potentials, flow-velocity field** from anisotropic energy-momentum tensor [FMRS13]

$$T^{\mu\nu} = (\epsilon + P_{\perp})u^{\mu}u^{\nu} - P_{\perp}g^{\mu\nu} - (P_{\parallel} - P_{\perp})V^{\mu}V^{\nu}$$

- thermal rates from **partonic/hadronic QFT become applicable**
- here: **extrapolated lattice QGP** and **Rapp-Wambach HMBT**
- caveat: **consistency between EoS, matter content of QFT model/UrQMD!**

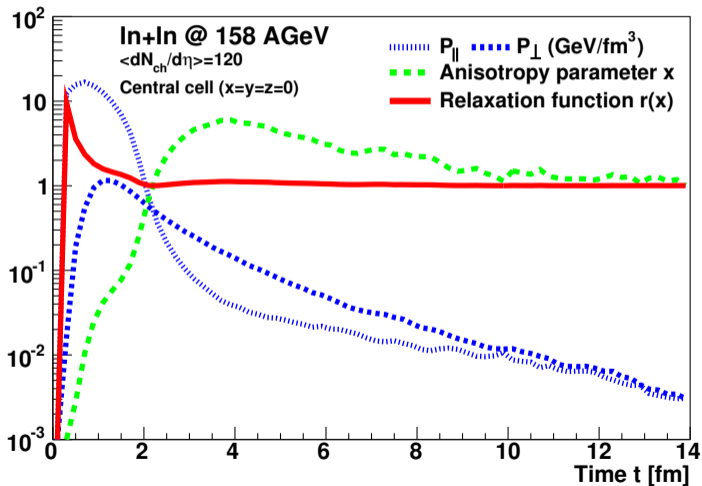
Coarse-grained UrQMD (CGUrQMD)

- $T_c = 170$ MeV; $T > T_c \Rightarrow$ lattice EoS; $T < T_c \Rightarrow$ HRG EoS



Coarse-grained UrQMD (CGUrQMD)

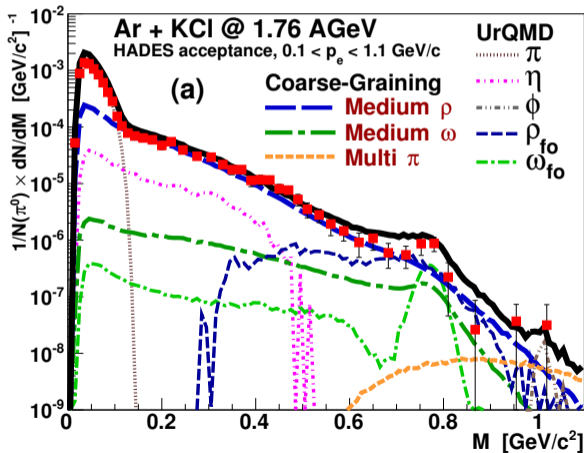
- pressure anisotropy (for In+In @ SPS; NA60)



Dielectrons (SIS/HADES)

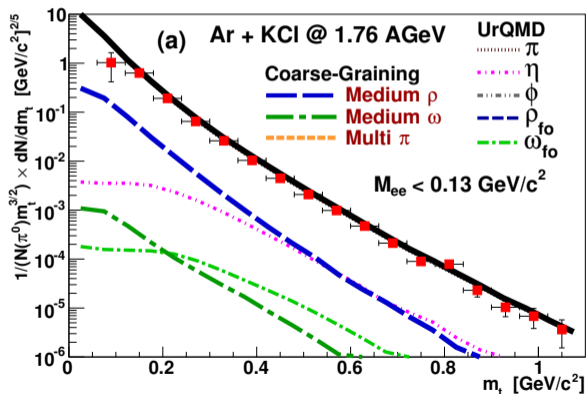
CGUrQMD: Ar+KCl (1.76 AGeV) (SIS/HADES)

- coarse-graining method works at low energies!
- UrQMD-medium evolution + RW-QFT rates



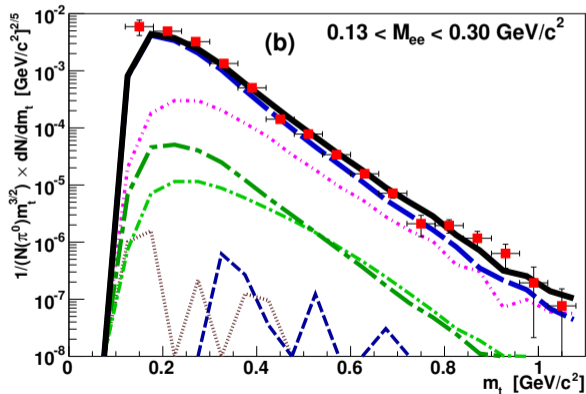
CGUrQMD: Ar+KCl (1.76 AGeV) (SIS/HADES)

- dielectron spectra from Ar + KCl(1.76 AGeV) $\rightarrow e^+e^-$ (SIS/HADES)
- m_t spectra
- $M_{ee} < 0.13$ GeV

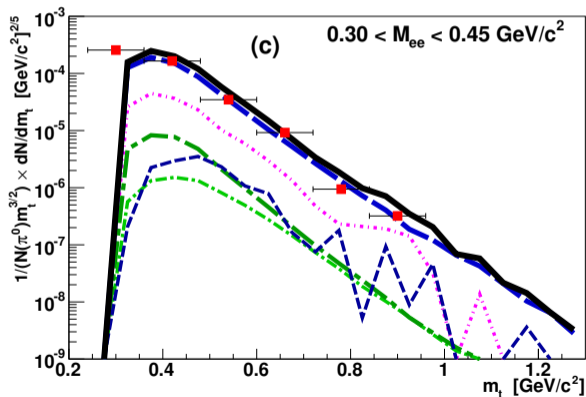


CGUrQMD: Ar+KCl (1.76 AGeV) (SIS/HADES)

- dielectron spectra from Ar + KCl(1.76 AGeV) $\rightarrow e^+e^-$ (SIS/HADES)
- m_t spectra
- $0.13 \text{ GeV} < M_{ee} < 0.30 \text{ GeV}$

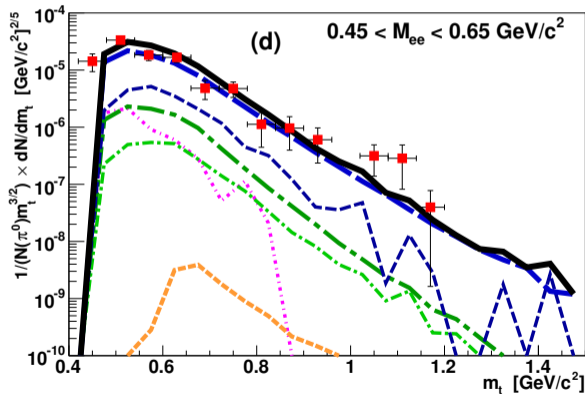


- dielectron spectra from Ar + KCl(1.76 AGeV) $\rightarrow e^+e^-$ (SIS/HADES)
- m_t spectra
- $0.3 \text{ GeV} M_{ee} < 0.45 \text{ GeV}$



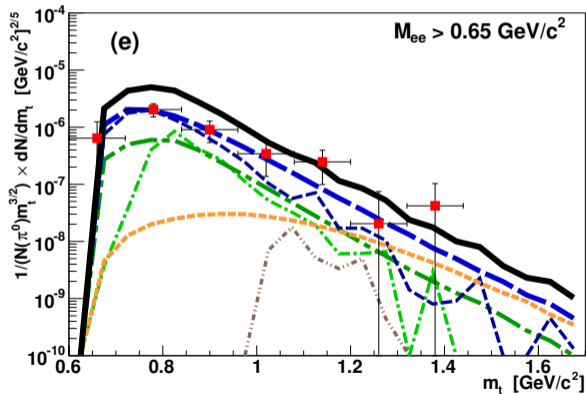
CGUrQMD: Ar+KCl (1.76 AGeV) (SIS/HADES)

- dielectron spectra from Ar + KCl(1.76 AGeV) $\rightarrow e^+e^-$ (SIS/HADES)
- m_t spectra
- $0.45 \text{ GeV} < M_{ee} < 0.65 \text{ GeV}$



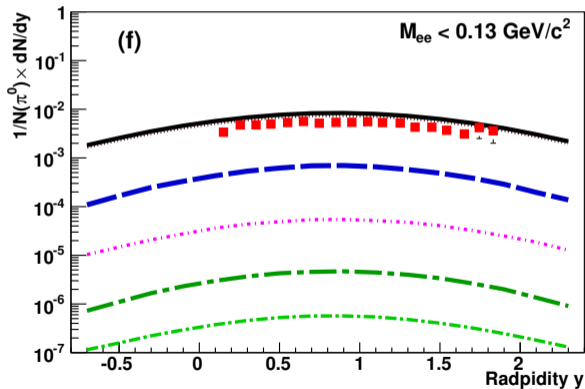
CGUrQMD: Ar+KCl (1.76 AGeV) (SIS/HADES)

- dielectron spectra from Ar + KCl(1.76 AGeV) $\rightarrow e^+e^-$ (SIS/HADES)
- m_t spectra
- $M_{ee} > 0.65$ GeV

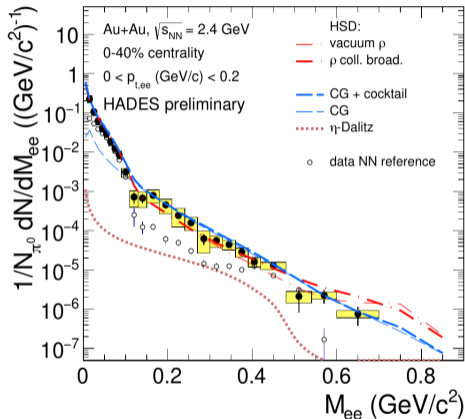


CGUrQMD: Ar+KCl (1.76 AGeV) (SIS/HADES)

- dielectron spectra from Ar + KCl(1.76 AGeV) $\rightarrow e^+e^-$ (SIS/HADES)
- m_t spectra
- rapidity spectrum ($M_{ee} < 0.13$ GeV)



CGUrQMD: Au+Au (1.24 AGeV) (SIS/HADES)



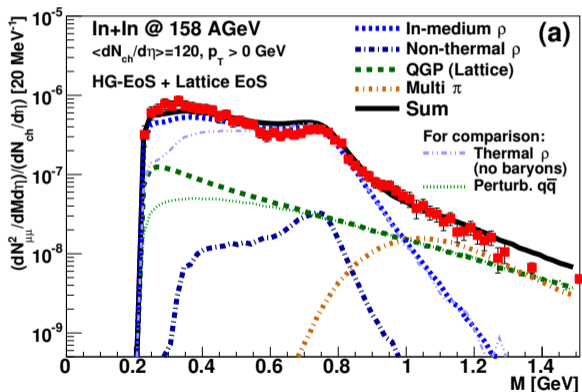
[Gal20]

- excellent agreement between models and data

Dimuons (SPS/NA60)

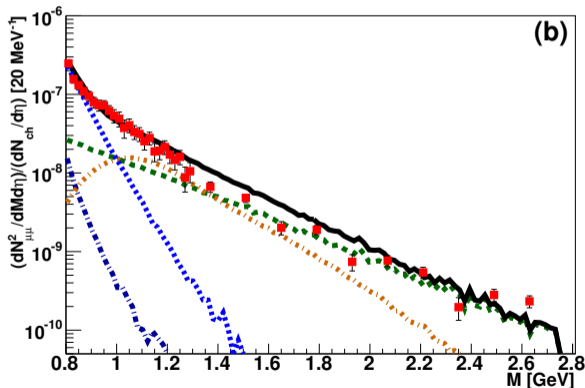
CGUrQMD: In+In (158 AGeV) (SPS/NA60)

- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+\mu^-$ (NA60) [EHWB15]
- min-bias data ($dN_{ch}/dy = 120$)



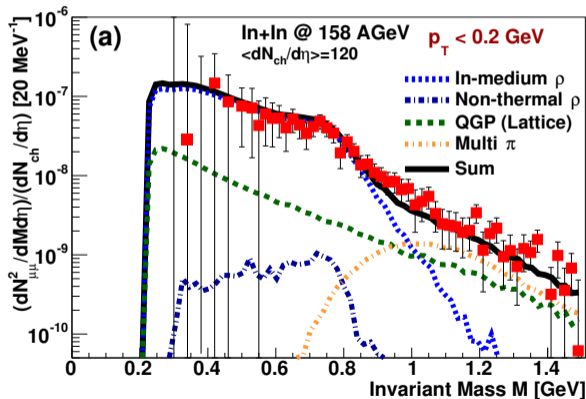
CGUrQMD: In+In (158 AGeV) (SPS/NA60)

- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+ \mu^-$ (NA60) [EHWB15]
- min-bias data ($dN_{\text{ch}}/dy = 120$)
- higher IMR: provides **averaged true temperature** $\langle T \rangle_{1.5 \text{ GeV} \lesssim M \lesssim 2.4 \text{ GeV}} = 205\text{-}230 \text{ MeV}$
- clearly above $T_c \simeq 150\text{-}160 \text{ MeV}$ (no blueshifts in the **invariant-mass** spectra!)



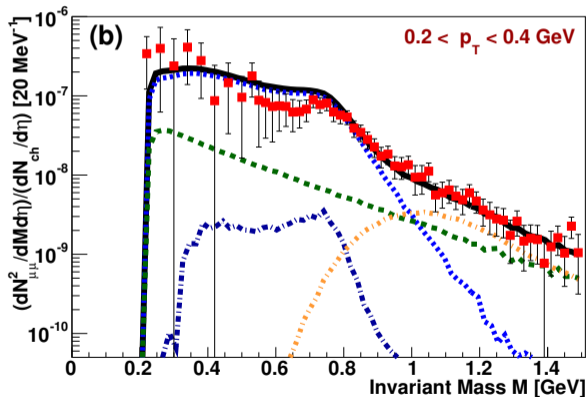
CGUrQMD: In+In (158 AGeV) (SPS/NA60)

- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+ \mu^-$ (NA60) [EHWB15]
- min-bias data ($dN_{\text{ch}}/dy = 120$)
- $p_T < 0.2$ GeV



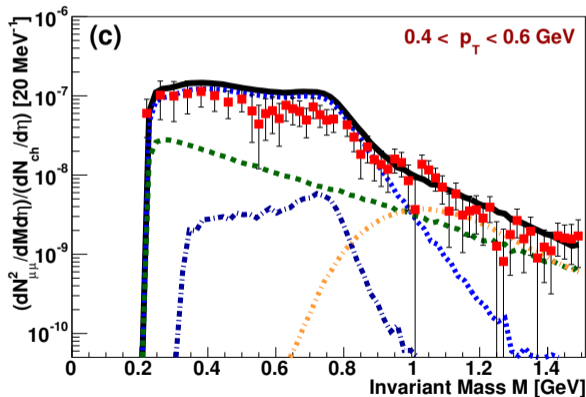
CGUrQMD: In+In (158 AGeV) (SPS/NA60)

- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+ \mu^-$ (NA60) [EHWB15]
- min-bias data ($dN_{\text{ch}}/dy = 120$)
- $0.2 \text{ GeV} < p_T < 0.4 \text{ GeV}$



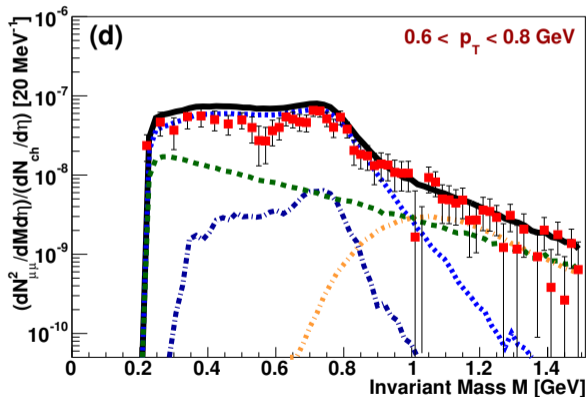
CGUrQMD: In+In (158 AGeV) (SPS/NA60)

- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+ \mu^-$ (NA60) [EHWB15]
- min-bias data ($dN_{\text{ch}}/dy = 120$)
- $0.4 \text{ GeV} < p_T < 0.6 \text{ GeV}$



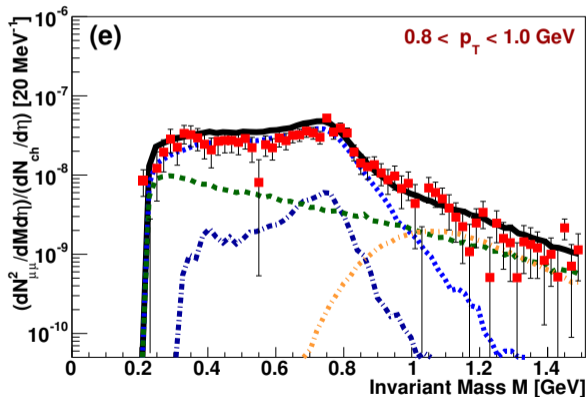
CGUrQMD: In+In (158 AGeV) (SPS/NA60)

- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+ \mu^-$ (NA60) [EHWB15]
- min-bias data ($dN_{\text{ch}}/dy = 120$)
- $0.6 \text{ GeV} < p_T < 0.8 \text{ GeV}$



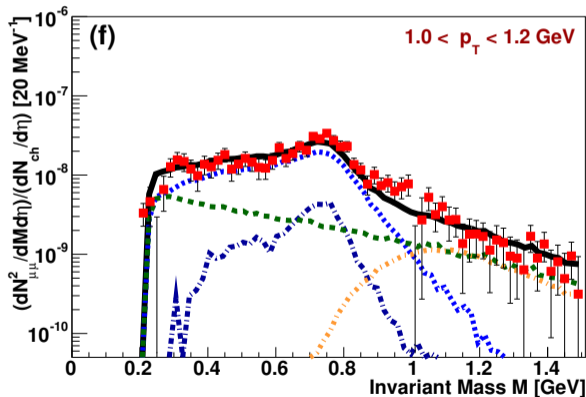
CGUrQMD: In+In (158 AGeV) (SPS/NA60)

- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+ \mu^-$ (NA60) [EHWB15]
- min-bias data ($dN_{\text{ch}}/dy = 120$)
- $0.8 \text{ GeV} < p_T < 1.0 \text{ GeV}$



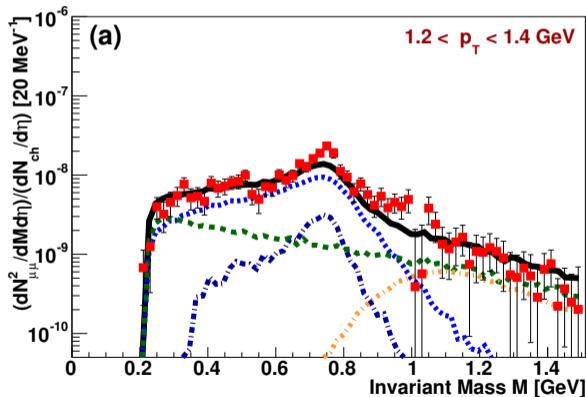
CGUrQMD: In+In (158 AGeV) (SPS/NA60)

- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+ \mu^-$ (NA60) [EHWB15]
- min-bias data ($dN_{\text{ch}}/dy = 120$)
- $1.0 \text{ GeV} < p_T < 1.2 \text{ GeV}$



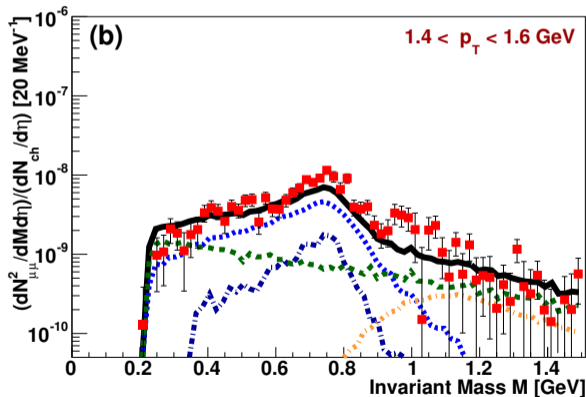
CGUrQMD: In+In (158 AGeV) (SPS/NA60)

- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+ \mu^-$ (NA60) [EHWB15]
- min-bias data ($dN_{\text{ch}}/dy = 120$)
- $1.2 \text{ GeV} < p_T < 1.4 \text{ GeV}$



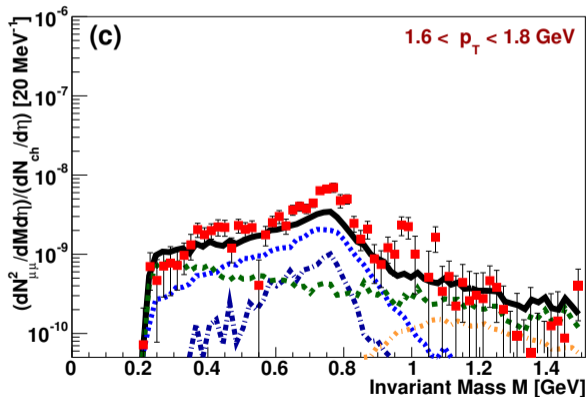
CGUrQMD: In+In (158 AGeV) (SPS/NA60)

- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+ \mu^-$ (NA60) [EHWB15]
- min-bias data ($dN_{\text{ch}}/dy = 120$)
- $1.4 \text{ GeV} < p_T < 1.6 \text{ GeV}$



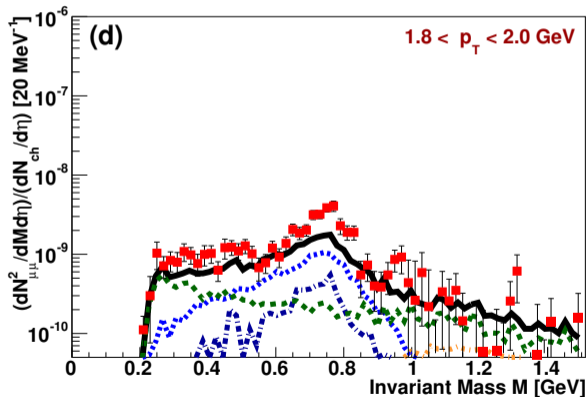
CGUrQMD: In+In (158 AGeV) (SPS/NA60)

- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+ \mu^-$ (NA60) [EHWB15]
- min-bias data ($dN_{\text{ch}}/dy = 120$)
- $1.6 \text{ GeV} < p_T < 1.8 \text{ GeV}$



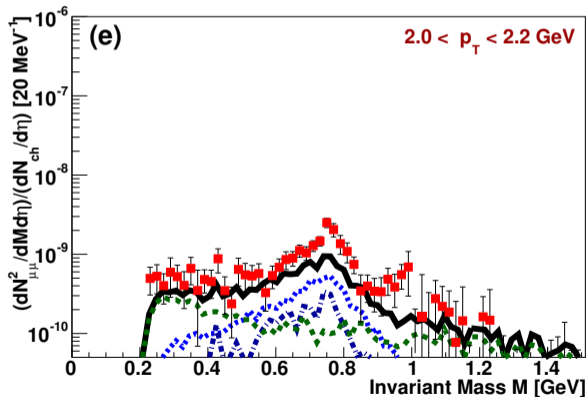
CGUrQMD: In+In (158 AGeV) (SPS/NA60)

- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+ \mu^-$ (NA60) [EHWB15]
- min-bias data ($dN_{\text{ch}}/dy = 120$)
- $1.8 \text{ GeV} < p_T < 2.0 \text{ GeV}$

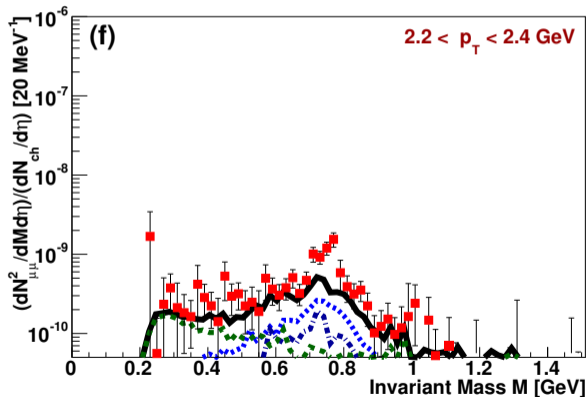


CGUrQMD: In+In (158 AGeV) (SPS/NA60)

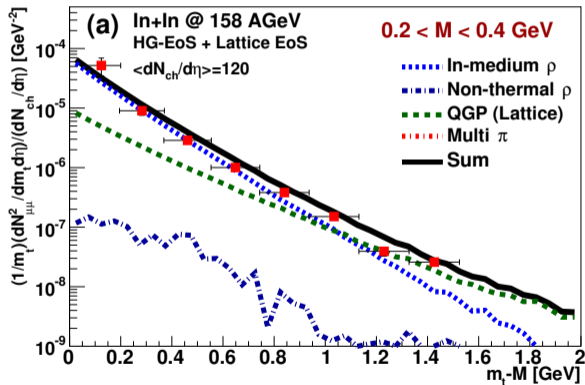
- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+ \mu^-$ (NA60) [EHWB15]
- min-bias data ($dN_{\text{ch}}/dy = 120$)
- $2.0 \text{ GeV} < p_T < 2.2 \text{ GeV}$



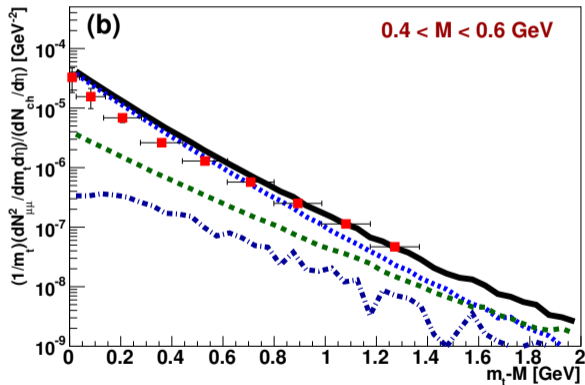
- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+ \mu^-$ (NA60) [EHWB15]
- min-bias data ($dN_{\text{ch}}/dy = 120$)
- $2.2 \text{ GeV} < p_T < 2.4 \text{ GeV}$



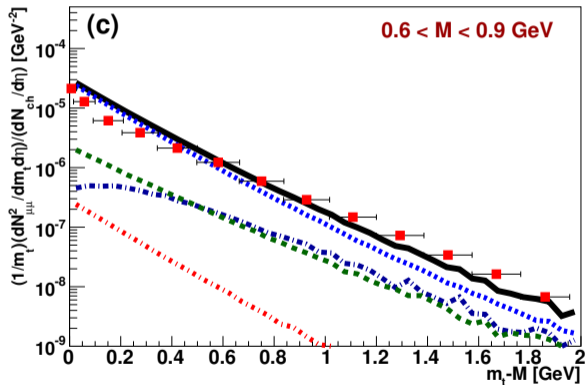
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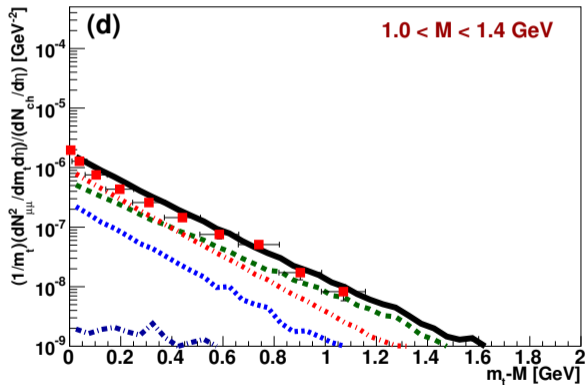
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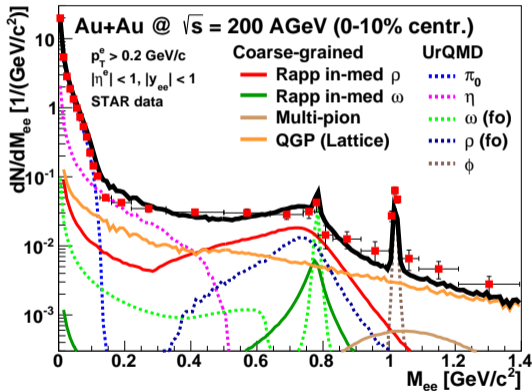
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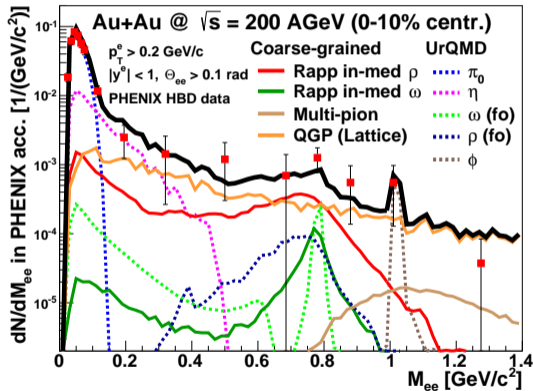
- dimuon spectra from In + In(158 AGeV) $\rightarrow \mu^+\mu^-$ (NA60) [EHWB15]
- min-bias data ($dN_{\text{ch}}/dy = 120$)



Dielectrons at RHIC

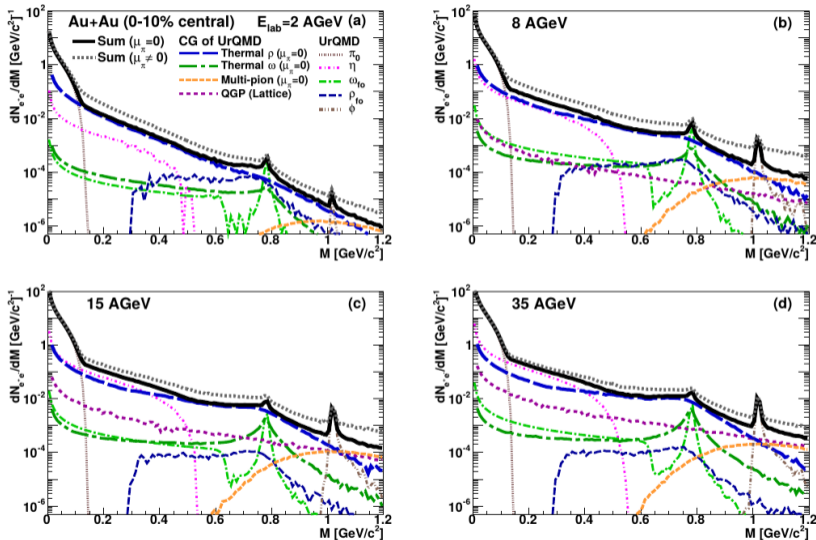


CGUrQMD: Au+Au ($\sqrt{s}_{NN} = 200 \text{ GeV}$) (RHIC/PHENIX)



Dielectrons at RHIC-BES/FAIR/NICA

CGUrQMD: Au+Au ($E_{\text{lab}} = 2\text{-}35 \text{ AGeV}$)

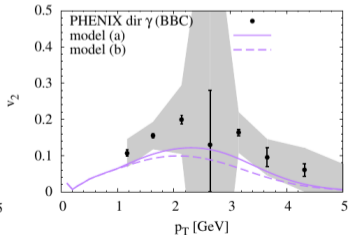
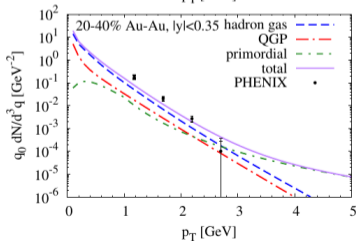
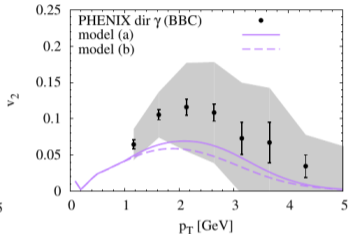
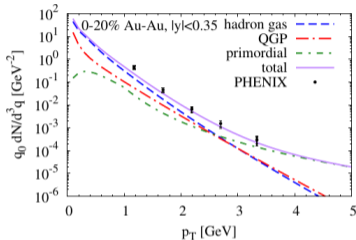


NB: also photon spectra [EHB16b]

Direct photons (RHIC/LHC)

Direct Photons at RHIC

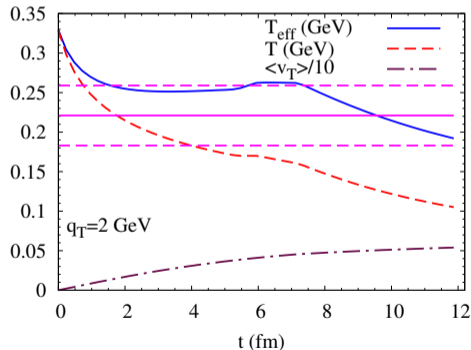
- same model [TRG04] for rates as for dileptons
- photons inherit v_2 from hadronic sources



Effective slopes vs. temperatures

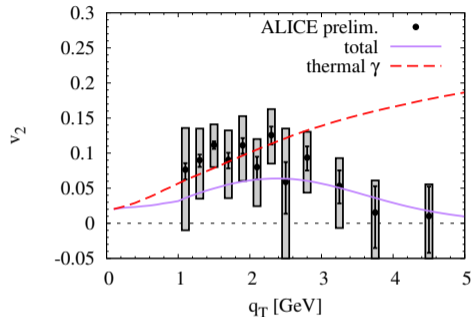
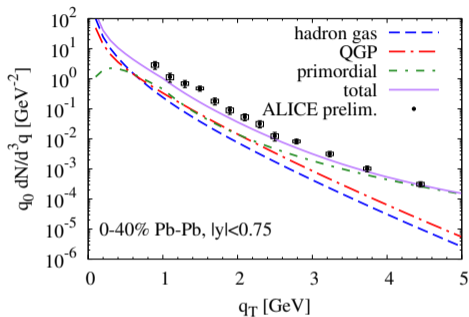
- effective slopes of photon p_T spectra are **NOT temperatures!**
- emission from a **flowing medium** \Rightarrow **Doppler effect**

$$T_{\text{eff}} \simeq \sqrt{\frac{1 + \langle v_T \rangle}{1 - \langle v_T \rangle}} T$$



Direct Photons at the LHC

same model, fireball adapted to hadron data from ALICE [HHR15]



- large direct-photon v_2
- early buildup of v_2 ; here developed already at end of QGP phase
- emission mostly around T_c (dual rates!) \Rightarrow
- \Rightarrow source has already developed radial flow and v_2
- large effective slopes **include blueshift from radial flow!**
- still additional (hadronic?) sources (bremsstrahlung?) missing!?

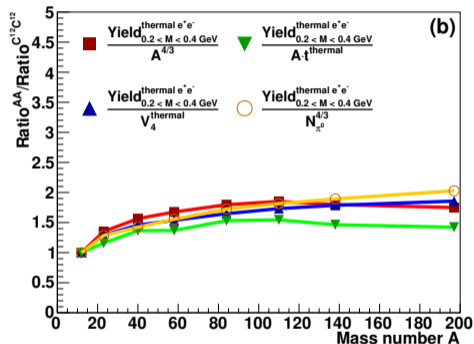
Signatures of the QCD-phase structure?

QCD phase structure from em. probes?

- hadronic observables like p_T spectra: “snapshot” of the stage after **kinetic freezeout**
- particle abundancies: **chemical freezeout**
- em. probes: emitted during the whole medium evolution
life time of the medium \Rightarrow “four-volume of the fireball”
- use CGUrQMD to study **system-size dependence**
- study AA collisions for different A [EHWB15]
- **“excitation functions”**:
systematics of $\ell^+\ell^-$ (and γ) emission vs. beam energy [EHB16b, RH16]
similar study in [GHR⁺16]
- **caveat**: phase transition not really implemented!!!

Scaling behavior of thermal-dilepton yield

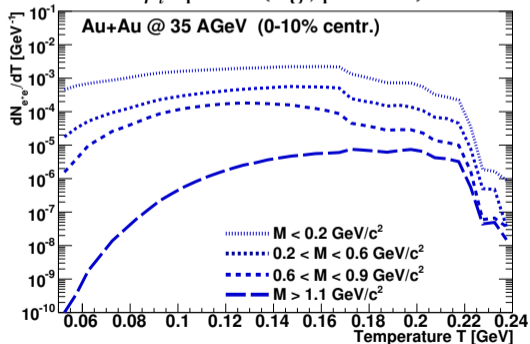
- central collisions from C+C to Au+Au at $E_{\text{kin}} = 1.76$ AGeV



- thermal-dilepton yield roughly $\propto V_{\text{therm}}^{(4)} \propto A^{4/3} \propto A t_{\text{therm}} \propto N_{\pi^0}^{4/3}$
- at low(est) beam energies: lifetime of “medium” $\hat{=}$ time nuclei pass through each other

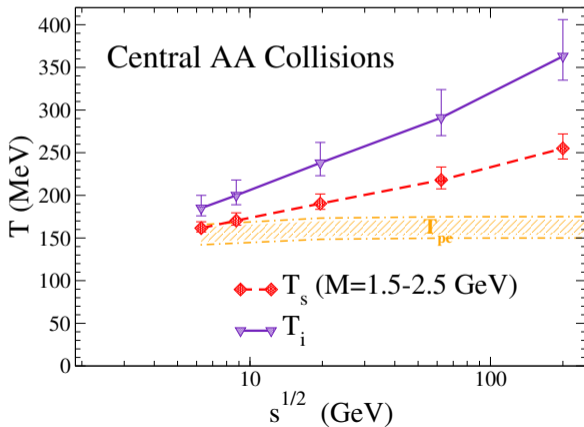
Mass-temperature relation in dilepton emission

- interplay between increasing volume and decreasing temperature of fireball
- in IMR ($T < m_\phi < M_{\ell+\ell^-} < m_{J/\psi}$) biased towards **early hot stages**
- only “background”: correlated $D\bar{D}$ decays, some Drell-Yan
- otherwise emission from **thermal** QGP and hadronic sources
- invariant-mass slope \Leftrightarrow true **invariant** space-time averaged **temperature**
- no blueshift due to radial flow as in p_t spectra (e.g., photons)



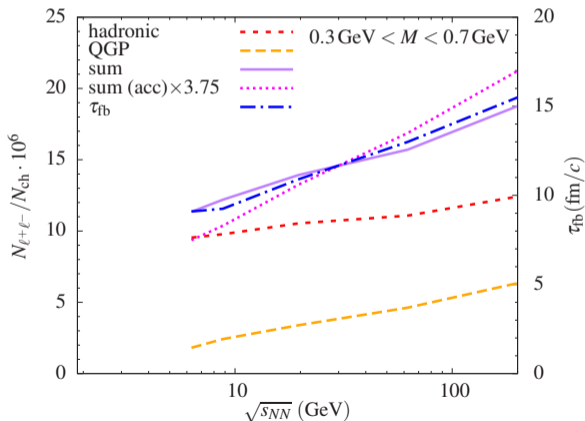
Dilepton systematics in the beam-energy scan

- thermal-fireball model [RH16, EHB16a]
- invariant-mass slope in IMR \Rightarrow true temperature!
- no blue shift from radial flow as in p_T/m_T spectra



Dilepton systematics in the beam-energy scan

- thermal-fireball model [RH16]
- beam-energy scan at RHIC and lower energies at FAIR and
- dilepton yield as **fireball-lifetime clock**



Conclusions and Outlook

• General ideas

- em. probes \Leftrightarrow **in-medium em. current-correlation function**
- dual rates around T_c (compatible with χ **symmetry restoration**)
- **medium modifications of ρ, ω, ϕ**
- importance of **baryon-resonance interactions**

• Application to dileptons in HICs

- **coarse-grained transport** (here: CGUrQMD)
- allows use of **thermal-QFT spectral VM functions**
- applicable also at low collision energies
- allows use of **thermal-QFT models** for dilepton rates
- successful description from **SIS to RHIC energies**
- consistent description of **M and m_T spectra!**
- effective slope of M spectra ($1.5 \text{ GeV} < M < M_{J/\psi}$) **provides $\langle T \rangle$**
- beam-energy scan at RHIC and FAIR \Rightarrow **signature of phase transition?**

• Outlook

- signature of **cross-over vs. 1st order (or even critical endpoint)???**
- challenge: **phase transition in (coarse-grained) transport???**

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