Advanced quantum mechanics

SS 2019 – Prof. Dr. Marc Wagner

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Exercise sheet 0

Presence sheet, to be discussed in the week of 22.04.19.
18.04.19

Exercise 1 [*Spin and measurements in quantum mechanics*]

In quantum mechanics, the spin $\mathbf{s} = (s_x, s_y, s_z)$ of a particle is described by a set of three operators fulfilling the angular momentum algebra

$$[\hat{s}_j, \hat{s}_k] = i\hbar\epsilon_{jkl}\hat{s}_l,$$

which can be represented by 2×2 matrices.

(a) Write down a representation of the three spin operators in terms of 2×2 matrices.

(b) Find the eigenvectors and eigenvalues for each of the three spin operators.

(c) Suppose a system is in the eigenstate of $\hat{s}_x$ with eigenvalue $+\hbar/2$. Calculate the expectation value of a measurement of s_y in this system. What are the possible results of the measurement and what are the corresponding probabilities?

(d) For the same system as in (c), calculate the expectation value for a measurement of spin in the direction $(+1, +1, 0)/\sqrt{2}$, i.e. along an axis at a 45° angle w.r.t. the x-axis. What are the possible results of the measurement and what are the corresponding probabilities?

(e) For the same system as in (c), we first measure spin in the direction $(+1, +1, 0)/\sqrt{2}$. After that we measure s_y. Calculate the expectation value of the latter measurement. What are the possible results of the latter measurement and what are the corresponding probabilities?

(f) Same as (e), but instead of a single measurement at a 45° angle, we take $N - 1$ measurements for which the direction of measurement is increased by $90°/N$ for each successive measurement, starting from 0° (along the x-axis) and ending at 90° (along the y-axis). Calculate the expectation value of a measurement of s_y taken after these $N - 1$ measurements in the limit $N \to \infty$. What are the possible results of the measurement and what are the corresponding probabilities?

Write down a representation of the operators and states of the angular momentum algebra given above, such that the eigenvalues of $\hat{s}_x, \hat{s}_y, \hat{s}_z$ are $+\hbar, 0$ and $-\hbar$. Represent the operators using

(g) 3×3 matrices,

(h) suitable combinations of $\mathbf{r}$ and ∇

Which physical systems can be described by these representations?