Advanced quantum mechanics

SS 2019 – Prof. Dr. Marc Wagner

Organization: Room GSC 0|21
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Exercise sheet 1

To be handed in 25.04.19 before the lecture. To be discussed in the week of 29.04.19.
18.04.19

Exercise 1 [*Position and momentum space representation*] (3+6+4=13 pts.)

In the lecture, we saw that a representation of the position and momentum operators can be either

- $\hat{p} \equiv -i\hbar\, d/dx,\ \hat{x} = x$ (representation in position space) or
- $\hat{p} = p,\ \hat{x} \equiv +i\hbar\, d/dp$ (representation in momentum space).

(a) Write down the eigenvalue equation for the Hamilton operator with arbitrary potential $V(x)$ (i.e. the Schrödinger equation) both in position space and in momentum space. Why is it more straightforward to use a position space representation for most potentials? Discuss obvious mathematical problems when using the momentum space representation for e.g. a square well potential.

(b) Give eigenvalues and eigenfunctions in both representations for the potential $V(x) = m\omega^2 x^2/2$. Which representation is more appropriate to solve this problem?

(c) Calculate explicitly the eigenvalues and eigenfunctions for a free particle in both representations.

Exercise 2 [*Time evolution*] (7 pts.)

Consider a particle in one spatial dimension in an infinite square well

$$V(x) = \begin{cases} 0 & \text{if } 0 \le x \le L \\ +\infty & \text{else} \end{cases} . \tag{1}$$

At time $t = 0$ the particle is in a state described by the wave function

$$\psi(x) = \frac{1}{\sqrt{2}}\Big(\sin(\pi x/L) + \sin(2\pi x/L)\Big). \tag{2}$$

Determine the probability to find the particle at time t at position x as a function of x and t. Visualize your result using a computer, e.g. by plotting the probability for several values of x as a function of t (and vice versa).