Advanced quantum mechanics

SS 2019 – Prof. Dr. Marc Wagner

Organization: Room GSC 0|21
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Exercise sheet 2

To be handed in 02.05.19 before the lecture. To be discussed in the week of 06.05.19.
25.04.19

Exercise 1 [*Two-state system*] (1+6+4=11 pts.)

Consider a hilbert space containing the two states $|0\rangle$ and $|1\rangle$, which are eigenstates of the Hamiltonian H_0:

$$H_0 |n\rangle = E_n |n\rangle \, , \; n = 0, 1. \tag{1}$$

At $t = 0$ the system is in the state $|0\rangle$.

(a) Write down the state of the system at arbitrary time t.

(b) Consider an additional time-dependent term such that the Hamiltonian takes the form $H = H_0 + H_1(t)$, where the matrix elements of H_1 are

$$\langle n| H_1(t) |m\rangle = \omega_0 \left(\delta_{n,m-1} e^{+i\omega t} + \delta_{n-1,m} e^{-i\omega t}\right) , \; n, m = 0, 1. \tag{2}$$

Calculate the transition probability from $|0\rangle$ to $|1\rangle$ for time $t \geq 0$. (Hint: When solving the time-dependent Schrödinger equation, you will obtain a system of coupled differential equations. Decouple the equations and use an exponential ansatz.)

(c) Calculate the same transition probability as in (b) up to leading order time-dependent perturbation theory. Discuss the region of validity of the approximation.

Exercise 2 [*Second order time-dependent pert. theory*] (4+5=9 pts.)

(a) In the lecture the transition probability from some initial state $|i\rangle$ to some final state $|f\rangle$ was derived for a time-dependent Hamiltonian of the form $H_0 + \lambda H_1(t)$ up to leading order in time-dependent perturbation theory. Extend the calculation by deriving the equations for the transition probability up to next-to-leading order in λ (i.e. terms $\sim \lambda^3$).

(b) Consider a one dimensional harmonic oszillator which at time $t = -\infty$ is in the ground state $|0\rangle$. The system is perturbed by a weak time-dependent potential

$$V(x, t) = -eEx \exp\left(-\frac{t^2}{\tau^2}\right) . \tag{3}$$

What is the probability to find the system in the second excited state $|2\rangle$ at time $t = +\infty$?