ADVANCED QUANTUM MECHANICS

SS 2019 – PROF. DR. MARC WAGNER

Organization: Room GSC 0|21

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Exercise sheet 3

To be handed in 09.05.19 before the lecture. To be discussed in the week of 13.05.19.

02.05.19

Exercise 1 [*Scattering in a 1D square well potential*] (5+5+5+5=20 pts.)

Consider scattering in the square well potential, which was discussed in the lecture,

$$V(x) = \begin{cases} -V_0 & \text{if } -a \leq x \leq +a \\ 0 & \text{otherwise} \end{cases} . \tag{1}$$

(a) Calculate the solutions of the stationary Schrödinger equation with $E > 0$. In particular, show that

$$B_1 = A_1 S(E) \frac{i}{2} \left(\frac{q}{k} - \frac{k}{q} \right) \sin(2qa), \;\; A_3 = A_1 S(E), \tag{2}$$

with

$$S(E) = \frac{e^{-2ika}}{\cos(2qa) - (i/2)((q/k) + (k/q)) \sin(2qa)} \tag{3}$$

(using the same notation as in the lecture).

(b) Expand $S(E)$ in the vicinity of resonances E_R in terms of the small parameter $E - E_R$ and show that, for a deep square well potential ($V_0 \gg E_R$),

$$S(E) \approx (-1)^n e^{-2ika} \frac{i\Gamma/2}{E - E_R + i\Gamma/2}. \tag{4}$$

You can use $2qa|_{E=E_R} = n\pi$ from the lecture.

(c) Using the results from (a) and the lecture, calculate the wave functions of the bound states, i.e. energies $-V_0 < E < 0$ and imaginary $k = i\sqrt{2m|E|}/\hbar$. Keep in mind that these wave functions must be normalizable. Which (possibly unexpected) behavior does $S(E)$ show close to the energies of the bound states and how can it be explained?

(d) Allowing complex values for E, discuss the existence and positions of the poles of $S(E)$. Make a qualitative plot of the poles in the complex E-plane. Discuss subtleties and problems when expressing e^{-2ika} in terms of E. In your discussion, consider bound states $\text{Re}\{E\} < 0$ and scattering states $\text{Re}\{E\} > 0$ seperately.