ADVANCED QUANTUM MECHANICS
SS 2019 – PROF. DR. MARC WAGNER

Organization: Room GSC 0|21
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Exercise sheet 5

To be handed in 23.05.19 before the lecture. To be discussed in the week of 27.05.19.

16.05.19

Exercise 1 [*Probability density, continuity equation*] (2+2+1+5=10 pts.)

(a) Show that, for a solution of the Schrödinger equation $\psi(\mathbf{r},t)$, the probability density $\rho(\mathbf{r},t) = \psi(\mathbf{r},t)(\psi(\mathbf{r},t))^*$ fulfills a continuity equation. Derive an expression for the corresponding current $\mathbf{j}(\mathbf{r},t)$.

(b) Using the continuity equation found in (a), show that the probability is conserved (i.e. constant in time).

(c) Show that, for a solution of the stationary Schrödinger equation $\psi(\mathbf{r})$ and an arbitrary volume V

$$\oint_{\partial V} \mathrm{d}\mathbf{A}\ \mathbf{j}(\mathbf{r}) = 0, \tag{1}$$

where $\oint_{\partial V} \mathrm{d}\mathbf{A}$ denotes an integral over the volume V enclosing surface.

(d) Using the results from (a) and (c), show that the function $S_l(E)$, which was introduced in the lecture, fulfills the equation

$$|S_l(E)| = 1. \tag{2}$$

Exercise 2 [*Angular momentum eigenfunctions*] (5 pts.)

Expand the plane wave e^{+ikz} in terms of eigenfunctions of the free Schrödinger equation, which are also eigenfunctions of angular momentum, i.e. show the relation

$$e^{+ikz} = \sum_{l=0}^{\infty} i^l (2l+1) j_l(kr) P_l(\cos\vartheta), \tag{3}$$

which was used in the lecture.

Exercise 3 [*Particle in a spherically symmetric potential*] (4+1=5 pts.)

Consider a particle with mass m in the potential

$$V(r) = \begin{cases} 0 & \text{if } R_1 < r < R_2 \\ \infty & \text{otherwise} \end{cases}. \tag{4}$$

(a) Determine all the energy eigenstates (i.e. wave functions and energy eigenvalues) with vanishing angular momentum. You do not need to normalize the wave functions.

(b) What are additional difficulties when trying to determine the energy eigenstates with non-vanishing angular momentum analytically?