ADVANCED QUANTUM MECHANICS
SS 2019 – PROF. DR. MARC WAGNER

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Exercise sheet 8

To be handed in 13.06.19 before the lecture.
To be discussed in the week of 17.06.19.
06.06.19

Exercise 1 [*Perturbative relativistic corrections*] (7+4=11 pts.)

(a) Calculate the leading order relativistic correction of the ground state energy of the 1-dimensional harmonic oszillator (mass m, frequency ω). To do this, expand the relativistic energy-momentum relation $E = \sqrt{m^2 + \mathbf{p}^2}$ for small momenta and use the correspondence principle to write down the corresponding Schrödinger equation. Explain intuitively, why the energy correction is negative.

(b) Repeat part (a) for the energy eigenstates $|nlm\rangle$ of the hydrogen atom with potential $V(r) = -Ze^2/r$. For this the expectation values

$$\langle nlm| r^{-1} |nlm\rangle = Z/an^2 \ , \ \langle nlm| r^{-2} |nlm\rangle = Z^2/(l+1/2)a^2n^3 \qquad (1)$$

might be useful (a derivation can be found e.g. in F. Schwabl, "Quantenmechanik (QM I)", Springer, section 12.3).

Exercise 2 [*Klein-Gordon equation and continuity equation*] (3+1=4 pts.)

In the lecture it has been shown that solutions of the Klein-Gordon equation fulfill a continuity equation $\partial_\mu j^\mu = 0$.

(a) Calculate the density ρ and the current $\mathbf{j}$, where $j^\mu = (\rho, \mathbf{j})$, for both a solution with positive energy and one with negative energy.

(b) Using your results from (a), discuss a possible interpretation of ρ as probability density or electric charge density.

Exercise 3 [*Dirac equation in 1+1 dimensions*] (5 pts.)

Consider the Dirac equation for a spacetime with 1 temporal and only 1 spatial dimension. Find a representation of the γ-matrices with minimal matrix dimension. Is the representation unique? Find the solutions of the Dirac equation in this 2-dimensional spacetime. Discuss differences between the solutions in 2 and 4 spacetime dimensions, in particular concerning spin.