Numerische Methoden der Physik

SoSe 2021 – Prof. Marc Wagner

Lasse Müller: lmueller@itp.uni-frankfurt.de
Laurin Pannullo: pannullo@itp.uni-frankfurt.de

Exercise sheet 3

To be discussed on 04.05.2021 and 05.05.2021

Exercise 1 [*Pendulum motion*] (2+2+9+2+3+2=20 pts.)

Consider the so-called *simple* pendulum (mass m and length l), for which the only external considered force is the gravitational one, $\vec{F} = -mg\vec{e}_z$. The angle ϕ describes the angular displacement of the pendulum from the neutral position $\phi = 0$. Suppose to have an initial displacement $\phi(t = 0) = \phi_0$.

(i) Derive the equation of motion of the pendulum.

(ii) Solve analytically the equation of motion assuming $\phi_0 \ll 1$.

(iii) Implement the 4^{th} order Runge-Kutta method discussed in the lecture, in order to solve the equation of motion obtained in task (i). How can you solve a second order ordinary differential equation with Runge-Kutta methods? What would you do if you had to solve a higher order ordinary differential equation?

(iv) Test your code in the small-angle regime. Set ϕ_0 properly and compare the numerical solution to the previously calculated analytical one.

(v) Use your program to solve the equation of motion for initial conditions $\phi_0 \in \left\{\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}$. Calculate the motion $\phi(t)$ for at least a complete period of the pendulum. Compare again your analytical solution with the numerical one. Can you explain the discrepancy?

(vi) Set $\phi = \frac{\pi}{2}$ and think of a way to calculate the period of the pendulum. Provide an error estimate for your result.