Michael Eichberg: eichberg@itp.uni-frankfurt.de
Lasse Müller: lmueller@itp.uni-frankfurt.de

Exercise sheet 11

To be handed in on 10.01.2024 and discussed on 12.01.2024 and 15.01.2024

Exercise 1 [χ^2-*fitting*] (1+10+1+6+2(+5)=20(+5) pts.)

The potential between an infinitely heavy quark and an infinitely heavy antiquark, the so-called *static potential*, as function of the quark separation, can be computed using lattice QCD and Monte Carlo methods. In this framework, both the quark separation $a\,\hat{r} \equiv r$ and the potential $\hat{V}_{Q\bar{Q}} \equiv a\,V_{Q\bar{Q}}$ are expressed in terms of the a priori unknown lattice spacing a, so that only dimensionless quantities appear. As it is common in particle physics, natural units are used ($\hbar = c = 1$) and thus, all quantities can be written in terms of MeV (in particular $[V] = [1/a]$ =MeV and 1 fm^{-1} = 197.3 MeV).

At the web address

`http://th.physik.uni-frankfurt.de/~mwagner/teaching/numerik/V.dat`

you can find the result of a lattice QCD computation of the static potential, which has been done on a $24^3 \times 48$ discretized space-time lattice. The file contains three columns with, respectively, $\hat{r}$, $\hat{V}_{Q\bar{Q}}$ and the statistical error on the static potential $\Delta\hat{V}_{Q\bar{Q}}$. It is known that the parametrization

$$V(r) = V_0 + \frac{\alpha}{r} + \sigma\,r$$

is appropriate to describe the r dependence of the static potential.

(i) Rewrite this parametrization in terms of dimensionless quantities using the lattice spacing a.

(ii) Perform a χ^2-minimising fit to the data and determine $\hat{V}_0$, $\hat{\alpha}$ and $\hat{\sigma}$. Ignore the data for $\hat{r} \leqslant 2$, because it is known that they are affected by sizeable discretization errors. Judge and discuss the quality of your fit by calculating χ^2/d.o.f. (χ^2 per degree of freedom).

(iii) Use the known physical value of the so-called string tension $\sigma = 4.97/\text{fm}^2$ to determine the lattice spacing a of this lattice QCD calculation. [1]

(iv) Perform a polynomial interpolation of the data points for $\hat{r} > 3$ using Lagrange polynomials.

[1] The lattice spacing in lattice QCD calculations is set indirectly by adjusting the so called coupling constant. The relation of the coupling constant and the lattice spacing depends on the particular choice of the discretization and is usually not known. Thus for every lattice QCD calculation the value of a has to be determined by using a suitable observable (the string tension σ).

(v) Compare the obtained potential with the outcome of your previous fit from task (ii) and discuss which approach is more appropriate to parametrize the static potential.

(vi) (Optional) Compute the error of V_0, α and σ using resampling.