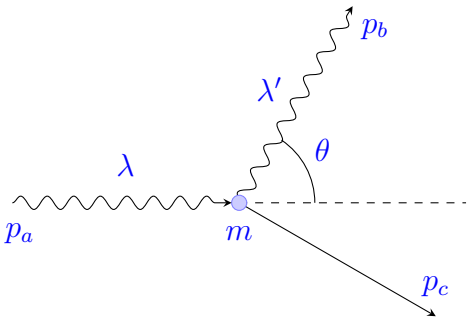
Exercise sheet III

November 2 [correction: November 9]

Problem 1 [*Compton scattering*] A photon of wavelength λ scatters on a particle of mass m at rest. Find the new photon wavelength λ', provided the scattering angle is θ.



Problem 2 [*Pauli equation*] A) Starting from the Schrödinger equation, use *minimal substitution* to show that in the presence of a constant and homogeneous magnetic field, $\mathbf{B}$, a spin zero particle's wave equation can be written as

$$i\frac{\partial}{\partial t}\varphi \;=\; \frac{1}{2m}[\mathbf{p}^2 - q\mathbf{L}\cdot\mathbf{B}]\varphi\,,$$

where q and m are the electric charge and the mass of the particle, respectively.

B) Derive the *Pauli equation*

$$i\frac{\partial}{\partial t}\varphi \;=\; \frac{1}{2m}\left[\mathbf{p}^2 - q(\mathbf{L}+g\mathbf{S})\cdot\mathbf{B}\right]\varphi\,,$$

as the non-relativistic limit of the Dirac equation. What is the value of g?

Note: The above two equations are valid in the *weak field approximation*, where only the first power of the electromagnetic potential is kept. Moreover, consider that the electric field is homogeneous.

Hint (minimal substitution): The principle of minimal coupling says that any derivative, ∂_μ, acting on a charged field (with charge e) has to be replaced by the covariant derivative, $\partial_\mu \to \partial_\mu + ieA_\mu$.

Problem 3 [*Continuity equation (Dirac)*] Use the Dirac equation and its adjoint to derive a continuity equation. Show that the four-current is given by

$$j^\mu \;=\; \bar{\psi}\gamma^\mu\psi\,.$$

Is $j^0 \equiv \rho$ positive? Prove it.