Exercise sheet IV

November 9 [correction: November 16]

Problem 1 [*3-vector bilinear*] Show that

$$\bar{\psi}_1 \gamma^0 \gamma^i \gamma^5 \psi_2$$

transforms as a 3-vector (e.g. x^i) under rotations.

Problem 2 [*Chiral projectors*] The projectors on the left- and right-components of Dirac spinors are given by $P_L = \frac{1-\gamma_5}{2}$ and $P_R = \frac{1+\gamma_5}{2}$, respectively. Use the properties of γ_5 to prove the following projector identities:

$$P_L P_R = P_R P_L = 0\,, \quad P_R P_R = P_R\,, \quad P_L P_L = P_L\,.$$

Verify that, given a spinorial solution to the massless Dirac equation, the spinors $P_{L,R} u_s(p)$ are eigenstates of the helicity operator

$$\mathfrak{h} = \frac{1}{2|\mathbf{p}|} \begin{pmatrix} \sigma \cdot \mathbf{p} & 0 \\ 0 & \sigma \cdot \mathbf{p} \end{pmatrix}$$

with eigenvalues $\pm\frac{1}{2}$. Does this hold also when $m \neq 0$?

Problem 3 [*spin and spinors*] Let us consider the following two spinors

$$u_1 = N \begin{pmatrix} 1 \\ 0 \\ \frac{p_3}{E+m} \\ \frac{p_1+ip_2}{E+m} \end{pmatrix}, \quad \text{and } u_2 = N \begin{pmatrix} 0 \\ 1 \\ \frac{p_1-ip_2}{E+m} \\ -\frac{p_3}{E+m} \end{pmatrix}.$$

It is easy to see that they are eigenstates of S_3 with eigenvalues $\pm\frac{1}{2}$, if $\mathbf{p} = (0, 0, p_3)$.

a) Construct the eigenstates of S_1 and S_2 as linear combinations of u_1 and u_2, and the appropiate choice of (p_1, p_2, p_3).

b) Alternatively, to obtain those eigenstates, you can perform a rotation of $\pi/2$ about the y and x axis of the spinors u_1 and u_2. Show that the result is the same as in a), up to an irrelevant phase factor.

Note: Be careful with the sign of the rotation.