Exercise sheet V

November 16 [correction: November 23]

Problem 1 [*Proca equation*] If we add a mass term to the Maxwell Lagrangian, we obtain the *Proca* Lagrangian,

$$\mathscr{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu \, .$$

(i) Show that gauge invariance is not present anymore.

(ii) Derive the equations of motion.

(iii) Apply the four divergence to the equations of motion obtained in (ii). How do you interpret the result?

(iv) Use the result of (iii) to simplify/rephrase the equations of motion obtained in (ii).

(v) Derive the general solution of this theory. Hint: use the results obtained in (ii) and (iii).

(vi) Argue that this theory describes massive spin-1 particles by showing that the $\mathbf{p} = 0$ contributions of the solution in (v) transform as a spin-1 (angular momentum-1) state at rest (as you know from quantum mechanics).

Problem 2 [*Majorana fermions*] Neutral massive spin-1/2 particles are described by the Dirac equation with an additional constraint, the so-called Majorana condition: $\psi_c = -i\gamma^2\psi^* = \psi$.

(i) Starting from the Dirac equation, proceed as for the Weyl equation, but keep the mass term. Namely, after splitting the spinor

$$\psi = \begin{pmatrix} \phi \\ \eta \end{pmatrix} ,$$

make the following change of variables

$$\phi_R = \phi + \eta \, , \quad \phi_L = \phi - \eta \, ,$$

and write the equations of motion for ϕ_R and ϕ_L.

(ii) Use the Majorana condition on the mass term to decouple the ϕ_R and ϕ_L components.

(iii) Show that the equation for ϕ_R is identical to the one for ϕ_L.