Exercise sheet IX

December 21 [correction: January 18]

Problem 1 [*Feynman propagator*] Show that the Feynman propagator for fermions is given by

$$S_F(x-y) \;=\; \int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{i(\not{p}+m)}{p^2-m^2+i\epsilon}e^{-ip(x-y)} \;=\; \langle 0|\, T(\psi(x)\bar{\psi}(y))\,|0\rangle \;,$$

with

$$\langle 0|\, T(\psi(x)\bar{\psi}(y))\,|0\rangle \;\equiv\; \begin{cases} +\psi(x)\bar{\psi}(y) & x^0 > y^0 \\ -\bar{\psi}(y)\psi(x) & x^0 < y^0 \end{cases}$$

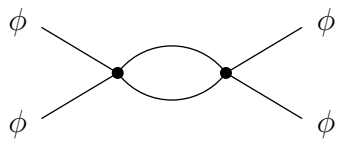
Problem 2 [*Wick theorem*] Compute the following object

$$T\left(\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\right) \;,$$

in the ϕ^4 theory, *without* using the Wick theorem.
Note: The idea of the exercise is to explicitely check the Wick theorem in a particular case.

Problem 3 [*One-loop amplitude in ϕ^4 theory*]

(i) Use the Feynman rules of the ϕ^4 theory to write down an expression for the amplitude corresponding to the following diagram



(ii) Show that a naïve computation leads to a divergent result.
Note: These apparent inconsistencies in quantum field theory are really pointing to the necessity of renormalization, which will be discussed in `QFT II`.