Exercise sheet XI

January 25 [correction: February 1]

Problem 1 [*Massless Fermion Scattering in Yukawa Theory*]
Consider the Yukawa theory Lagrangian density:

$$\mathcal{L}_{\text{Yukawa}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Klein-Gordon}} - g\,\bar{\psi}\,\psi\,\phi \; ,$$

where g is a dimensionless coupling constant, ϕ is a real scalar field and $\psi, \bar{\psi}$ are Dirac fields. This is a simplified model of Quantum Electrodynamics and was originally invented by Yukawa to describe nucleons ($\rightarrow \psi$) and pions ($\rightarrow \phi$). In modern particle theory, the Standard Model contains Yukawa interaction terms which couple the scalar Higgs field to quarks and leptons (most of the free parameters in the Standard Model are Yukawa coupling constants).
Let us focus on the two-particle scattering,

$$n(p) + n(p') \rightarrow n(k) + n(k') \; ,$$

where the symbol n indicates a ***massless*** fermion.

(i) Draw all $\mathcal{O}(g^2)$ Feynman diagrams that describe this process.

(ii) Write the corresponding scattering amplitude $\imath\mathcal{M}$, defined as

$$\langle \mathbf{k}, \mathbf{k}' |\, \imath T \,| \mathbf{p}, \mathbf{p}' \rangle = (2\pi)^4\, \delta^{(4)}(p + p' - k - k') \cdot \imath\mathcal{M}\big[(p, p') \rightarrow (k, k')\big] \; .$$

Remember that it is crucial to go all over fermions lines in the opposite direction of their arrows. Which is the right sign between the two terms of $\imath\mathcal{M}$? Why?

(iii) Using standard trace technology [*Gammatics*] and recalling that the only mass involved in the process is m_ϕ, compute the spin averaged square amplitude $\overline{|\mathcal{M}|^2}$ (i.e. the amplitude summed over final states and averaged out of inital states).

(iv) Express $\overline{|\mathcal{M}|^2}$ as a function of t and u Mandelstam variables,

$$t = (p - k)^2 = (p' - k')^2 \qquad\qquad u = (p - k')^2 = (k - p')^2 \; .$$

(v) Consider the center of mass frame and express the four-momenta p, p', k and k' as functions of $s = (p + p')^2 = (k + k')^2$ and θ (scattering angle). Rewrite $\overline{|\mathcal{M}|^2}$ in terms of s and θ.

(vi) Calculate both the differential and the total cross section in the center of mass frame. To make your expressions simpler, define

$$B \equiv \frac{2m_\phi^2}{s} + 1 \; .$$

(vii) Reduce the result (iv) to the case $m_\phi = 0$ and calculate both the differential cross section and the total cross section in the center of mass frame.

(viii) Check that the limit $m_\phi \rightarrow 0$ of total cross section obtained in (vi) leads to the same result obtained in (vii).