Exercise sheet XII

February 1 [correction: February 8-15]

Problem 1 [*Bhabha Scattering*]
Consider the QED scattering of an electron and a positron. Let the in and out electrons have momenta p and p', respectively, and the initial and final positrons have momenta k and k'.

$$e^-(p) + e^+(k) \to e^-(p') + e^+(k') \ ,$$

Let the angle between the initial and final electrons be θ (in the CM frame).

(i) Draw all $\mathcal{O}(e^2)$ Feynman diagrams that describe this process and write the corresponding scattering amplitude $\imath\mathcal{M}$ (see Exercise Sheet XI for its definition). Be sure that you have the correct relative sign between the diagrams.

(ii) **Optional.** Restrict your attention to the ultrarelativistic case, where you can treat the electron and positron as massless. This means you neglect terms of order $\frac{m_e^2}{E_{cm}^2}$ and for GeV or TeV colliders this term is of order 10^{-8} or 10^{-14}, so you incur no great error. You may drop m_e whenever it occurs additively. Work out the differential cross section in terms of Mandelstam variables.

(iii) Your calculation should end in one of the following equivalent expressions:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left[u^2\left(\frac{1}{s}+\frac{1}{t}\right)^2 + \left(\frac{t}{s}\right)^2 + \left(\frac{s}{t}\right)^2\right] = \frac{\alpha^2}{2s}\left[\frac{s^2+u^2}{t^2} + \frac{2u^2}{st} + \frac{t^2+u^2}{s^2}\right] .$$

Rewrite one of these formulas in terms of $\cos\theta$ and calculate $\frac{d\sigma}{d\cos\theta}$. Try to compute the total cross section. What happens? What feature of the diagrams is responsible for that?

Problem 2 [*3 scalar fields*] Consider the following Lagrangian density

$$\mathscr{L} = \sum_{i=1}^{3}(\partial_\mu\phi_i^\dagger)(\partial^\mu\phi_i) - \mu^2\phi_1^\dagger\phi_1 - \frac{\mu^2}{2}(\phi_2^\dagger\phi_2 + \phi_2^\dagger\phi_3 + \phi_3^\dagger\phi_2 + \phi_3^\dagger\phi_3) ,$$

which describes three *flavours* of complex scalar field, ϕ_1, ϕ_2, ϕ_3. Do they have a definite mass? What are the masses, at tree-level, of the three fields that describes the Lagrangian? Are the flavour eigenstates also mass eigenstates?

Problem 3 [*Superconductivity as a Higgs phenomenon*] Consider the scalar QED theory with Higgs phenomenon, namely

$$\mathscr{L} = (D_\mu \phi^\dagger)(D^\mu \phi) + \frac{\mu^2}{2}\phi^\dagger \phi - \frac{\lambda}{4}(\phi^\dagger \phi)^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

where

$$D_\mu \phi = (\partial_\mu - ieA_\mu)\phi\,, \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu\,.$$

Let us consider the static case, where $\partial^0 \phi = \partial^0 \mathbf{A} = 0$ and $A_0 = 0$.

(i) Show that the equation of motion for $\mathbf{A}$ is of the form

$$\nabla \times \mathbf{B} = \mathbf{J} \quad \text{with}\, \mathbf{J} = ie[\phi^\dagger(\nabla - i\mathbf{A})\phi - (\nabla + i\mathbf{A})\phi^\dagger \phi]\,.$$

(ii) Show that with spontaneous symmetry breaking, in the classical approximation, $\phi = v/\sqrt{2}$, $v = (\mu^2/\lambda)^{1/2}$, the current $\mathbf{J}$ is given by the London equation

$$\mathbf{J} = e^2 v^2 \mathbf{A}\,,$$

thus

$$\nabla^2 \mathbf{B} = e^2 v^2 \mathbf{B}\,.$$

Note: This last equation implies the *Meissner effect.*

(iii) The resistivity ρ for the system is defined by

$$\mathbf{E} = \rho \mathbf{J}\,.$$

Show that, in the case of spontaneous symmetry breaking, $\rho = 0$, and we have superconductivity.