Exercise sheet III

October 29 [solution: November 7]

Problem 1 [*Gamma matrices*] Check the anticommutation relations for the Dirac gamma matrices:

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}. \tag{1}$$

Problem 2 [*Dirac and Klein-Gordon*] Show that the Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \tag{2}$$

implies the Klein-Gordon equation:

$$(\partial^\mu \partial_\mu + m^2)\psi = 0. \tag{3}$$

Problem 3 [*Continuity equation (Dirac)*] Use the Dirac equation and its adjoint to derive a continuity equation. Show that the four-current is given by

$$j^\mu = \bar{\psi}\gamma^\mu\psi \,. \tag{4}$$

Is $j^0 \equiv \rho$ positive? Prove it.

Problem 4 [*Solutions of the Dirac equation*] Using the spinors u_1, u_2, v_1 and v_2 from the lecture, write the full solution of the Dirac equation (including the appropriate exponential) for:
a) particle with spin $s_z = +1/2$ moving along the z-axis,
b) particle with spin $s_z = -1/2$ moving along the z-axis,
c) antiparticle with spin $s_z = +1/2$ moving along the z-axis,
d) antiparticle with spin $s_z = -1/2$ moving along the z-axis.
Is it possible to write a solution for particles/antiparticles with definite s_x and s_y?

Problem 5 [*Spin sums*] Calculate the so-called spin sums:

$$\sum_{s=1,2} u_s(p)\bar{u}_s(p), \tag{5}$$

$$\sum_{s=1,2} v_s(p)\bar{v}_s(p). \tag{6}$$