Exercise sheet II

`April 27 [correction: May 4]`

Problem 1 [*Observables from v.e.v.'s for the harmonic oscillator*]

Consider the one-dimensional harmonic oscillator and its (Euclidean) generating functional as derived in the Lecture:

$$Z[j] = \exp\left(\frac{1}{4m\omega}\int d\tau_1 \int d\tau_2 j(\tau_1) e^{-\omega|\tau_1-\tau_2|} j(\tau_2)\right) \quad .$$

(i) Calculate the v.e.v. of the mean square deviation (*mittlere quadratische Auslenkung*)

$$\langle\Omega|x^2|\Omega\rangle$$

by means of $Z[j]$.

(ii) Consider now the operator $\mathcal{O} = x^2 - \langle\Omega|x^2|\Omega\rangle$. Compute the two-point function

$$\langle\Omega|\mathcal{O}(\tau)\mathcal{O}(0)|\Omega\rangle$$

and argue that, in the $\tau \to \infty$ limit, it takes the form $Ae^{-B\tau}$.
Show that B is the energy difference $E(\psi) - E(\Omega)$, with $|\psi\rangle$ some higher energy eigenstate. What is the parity of the state $|\psi\rangle$?

[I.e. which excitations of the harmonic oscillator does this operator "feel"? Think in terms of the ordinary quantum-mechanical knowledge of the system's spectrum.]

(iii) Consider the quantity

$$\lim_{\tau\to\infty} \frac{\langle\Omega|x(\tau)x^2(0)x(-\tau)|\Omega\rangle}{\langle\Omega|x(\tau)x(-\tau)|\Omega\rangle} \quad .$$

- What is its meaning, i.e. what kind of observable is it?
- Evaluate it.

(iv) Design an operator $\widetilde{\mathcal{O}}$ to determine the energy difference between the first *negative parity* excitation of the spectrum and $|\Omega\rangle$ by means of a two-point function $\langle\Omega|\widetilde{\mathcal{O}}(\tau)\widetilde{\mathcal{O}}(0)|\Omega\rangle$.