Exercise sheet IV

May 15 [correction: May 22]

Problem 1 [ϕ^4 *theory, perturbation theory*]

Given the action of the Euclidean ϕ^4-theory,

$$S_E[\phi] = \int \mathrm{d}^4 x \left[\frac{1}{2}\partial_\mu\phi\partial_\mu\phi + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4\right] \quad ,$$

in the Lecture you were given the relation

$$\begin{aligned} Z[J] &:= \frac{1}{Z(0)}\int \mathcal{D}\phi \exp\left(-S_E + (J,\phi)\right) \\ &= \frac{Z_0(0)}{Z(0)} \exp\left(-S_I\left[\frac{\delta}{\delta J}\right]\right) \exp\left(\frac{1}{2}(J,\Delta_F J)\right)\Big|_{J=0} \quad , \end{aligned}$$

where

$$\exp\left(-S_I\left[\frac{\delta}{\delta J}\right]\right) \exp\left(\frac{1}{2}(J,\Delta_F J)\right) = \exp\left(\frac{1}{2}(J,\Delta_F J)\right)\left\{1 + \lambda W_1[J] + \lambda^2 W_2[J] + \mathcal{O}(\lambda^3)\right\} \quad .$$

(i) We derived $W_1[J]$ and parts of $W_2[J]$ during the Lecture; now compute the full expression for $W_2[J]$, where

$$W_2[J] = \frac{1}{2}\Big(\frac{1}{4!}\Big)^2 \exp\left(-\frac{1}{2}(J,\Delta_F J)\right)\left\{\int \mathrm{d}^4 x \Big[\frac{\delta}{\delta J}\Big]^4\right\}^2 \exp\left(\frac{1}{2}(J,\Delta_F J)\right) \quad .$$

(ii) Translate the expression obtained above into a diagrammatic language.

(iii) Use the resulting $W_2[J]$ from *(i)* to derive $G_2(y_1,y_2)$ up to order $\mathcal{O}(\lambda^2)$.