Exercise sheet V

May 22 [correction: June 5]

Problem 1 [*Path integral formalism and perturbation theory in quantum mechanics*]

Consider a one dimensional quantum mechanical oscillator, action

$$S[x] \quad = \quad \int dt \left(\frac{m}{2}\dot{x}^2 - \frac{m\omega^2}{2}x^2 - \frac{\lambda}{4!}x^4\right).$$

(a) Derive within the path integral formalism of quantum mechanics the essential equations of perturbation theory in λ for this oscillator.
(Proceed in close analogy to what has been presented in the lecture for ϕ^4 theory.)

(b) Apply the equations you have derived in (a) to calculate the VEV of x^2, i.e. $\langle\Omega|x^2|\Omega\rangle$ up to first order in λ.

(c) Cross check your result by using techniques and methods you learned in your standard lectures on quantum mechanics (canonical formalism, creation and annihilation operators, ordinary quantum mechanical perturbation theory).

Problem 2 [*Relations between connected and disconnected n-point functions*]

Consider an interacting, translationally invariant, scalar quantum field theory with generating functionals $Z[J]$ and $W[J] = \log Z$. The connected and disconnected n-point functions are denoted by $G_n^c(x_1, \ldots, x_n)$ and $G_n(x_1, \ldots, x_n)$, respectively. Show the following relations:

$$\begin{aligned}
G_1(x_1) \quad &= \quad G_1^c(x_1) \\
G_2(x_1, x_2) \quad &= \quad G_2^c(x_1, x_2) + (G_1)^2 \\
G_3(x_1, x_2, x_3) \quad &= \quad G_3^c(x_1, x_2, x_3) - 2(G_1)^3 \\
&\quad + G_2(x_1, x_2)G_1(x_3) + G_2(x_2, x_3)G_1(x_1) + G_2(x_3, x_1)G_1(x_2).
\end{aligned}$$