Exercise Sheet I

April 21.

In the lecture we studied a free quantum mechanical particle, and saw that the transition amplitude from a state where the particle is prepared at position x_1 at time t_1, $|x_1, t_1\rangle$, to a state where the particle is at position x_2 at time t_2, $|x_2, t_2\rangle$, is given by the formula

$$\langle x_2, t_2 | x_1, t_1 \rangle = \sqrt{\frac{m}{2\pi i (t_2 - t_1)}} \exp\left(i \frac{m(x_2 - x_1)^2}{2(t_2 - t_1)} \right). \tag{1}$$

In which m is the mass of the particle, and $\hbar = c = 1$.

Task 1. *Transition probabilities for a localised particle*

- What is the probability of finding the particle at position x_2 at time t_2 if we prepared it to be at position x_1 at time t_1?
- Which variables do the probability depend on? Does it make sense?
- Is this probability properly normalised?

To further investigate the situation we replace the initial quantum state by a Gaussian packet of breadth σ, centred around the position x_1:

$$\langle x | \psi \rangle = \frac{1}{\sqrt{\sigma\sqrt{2\pi}}} \exp\left\{ -\frac{1}{4}\left(\frac{x - x_1}{\sigma} \right)^2 \right\} \tag{2}$$

Task 2. *Transition probabilities for a non-localised particle*

- Calculate the scalar product between $|\psi\rangle$ and a momentum state $|p\rangle$, $\langle p|\psi\rangle$.
- Use this result to repeat the calculation of the scalar product between the two states, $\langle x_2, t_2 | \psi, t_1 \rangle$.

Task 3. *Comparisons*

- With the result from the previous task, calculate the probability of finding the particle at position x_2 at time t_2 if it was prepared as a wave packet. How does this compare to the result from question 1?
- Take the limit $\sigma \to 0$ and discuss the result.