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b-baryon masses from lattice QCD: quantum numbers and creation operators

Bachelorarbeit

eingereicht am
21. August 2013

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Abstract

I compute the masses of the Λ_b and Ω_b-baryon using Wilson twisted mass lattice QCD with $N_f = 2$ flavors of sea quarks. I will consider light quarks corresponding to a pion mass of $m_\pi = 336$ MeV. I will discuss a creation operator which will generate the quantum numbers of the particle of interest when applied to the vacuum state. In contrast to previous, similar works the inversions leading to the light quark propagators are done by using the point source method. I compare the statistical errors with those results obtained in these previous works where timeslice sources were used for the inversions.

Zusammenfassung

In der vorliegenden Arbeit berechne ich die Massen des Λ_b und Ω_b-Baryons mittels Wilson twisted mass Gitter-QCD mit $N_f = 2$ Seequark-Flavors. Die leichten Quarks werden mit Massen korrespondierend zu einer Pion Masse von $m_\pi = 336$ MeV implementiert. Ich werde einen Erzeugungsoperator vorschlagen und zeigen, dass dieser angewandt auf den Vakuumzustand die gewünschten Quantenzahlen des Teilchens erzeugt. Im Gegensatz zu früheren Arbeiten werden die Invertierungen, die zu den Propagatoren der leichten Quarks führen, mit Hilfe von Punktquellen durchgeführt. Ich vergleiche die statistischen Fehler mit jenen aus den genannten früheren Arebeiten, in denen für die Invertionen timeslice Quellen verwendet wurden.

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1 Introduction

How to compute the mass of a particle seems to be a rather simple question. Naively one would think to simply sum up the ingredients of the particle which would be in case of baryons the valence quark masses. But this simple assumption fails even in the simplest cases, e.g. the proton. While the proton has a mass of about 1 GeV the sum of the valence quarks (up, up, down) only reaches out to about 15 MeV which is about 1000 times smaller. It is already known for a long time that determining the mass of a baryon is much more complicated due to the fact that it is mainly generated by the interaction of the quarks and gluons within the baryon. This makes the question on how to compute the mass of a particle quite fascinating. The correct way to describe such interaction is given by **Q**uantum **C**hromo **D**ynamics (*QCD*) which describes interaction due to the strong force.

In the following work I will compute bottom baryon masses from lattice QCD using Monte Carlo methods. A bottom baryon is a bound state consisting of one bottom and two light quarks. I will consider up, down and strange quarks as these light degrees of freedom. I choose Wilson twisted mass fermions which automatically improve numerical computations by $O(a)$, with a being the lattice spacing, and a static approach, i.e. I will set the mass of the bottom quark to infinity, because it is hardly feasible to perform computation with a dynamical bottom quark.
The main goal of this thesis is to investigate whether the statistical errors which arise while computing baryon masses using timeslice sources will be reduced by implementing the point source method. It was shown in [11] that the error for meson masses could be reduced drastically in this way. In fact a bottom baryon also consists of two light quarks. Thus the idea was to reduce the error in exact the same way, i.e. using point sources rather than timeslice sources. In case of the point source method the errors occur only because of the gauge fields whereas using timeslice sources there also will be an error due to statistical noise. I will compare my results to those presented in [10, 3], where the same lattice setup has been made.

This work was done in close collaboration with the work done by another bachelor student. Some important theoretical aspects can be found in more detail in his thesis [1].

In the following I will use the euclidean formulation of (lattice) QCD. Hence the γ-matrices fulfill the following commutator relations:

$$\{\gamma^E_\mu, \gamma^E_\nu\} = 2\delta_{\mu,\nu}$$
$$\{\gamma^E_5, \gamma^E_\mu\} = 0 \ \forall\mu$$

which implies that $\forall\mu: \ (\gamma^E_\mu)^2 = 1$, as well as $(\gamma^E_5)^2 = 1$.
I will omit 'E' indices on γ-matrices to simplify the notation.

2 QCD Basics

2.1 Continuum QCD

A theory like QCD can be defined by the Lagrangian or the action of a system. The equation of motions follow from the *principle of least action.* The QCD Lagrangian is given by:

$$\mathcal{L}_{QCD}\left[\psi,\bar{\psi},A_\mu\right]=\mathcal{L}_F\left[\psi,\bar{\psi},A_\mu\right]+\mathcal{L}_G\left[A_\mu\right] \tag{2.1}$$

$\mathcal{L}_F$ refers to the gauge invariant fermionic Lagrangian, which describes the dynamics of the quarks. As can be seen in (2.2) and (2.3), $\mathcal{L}_F$ depends on the gauge fields as well as the fermionic fields.

$$\mathcal{L}_F\left[\psi,\bar{\psi},A_\mu\right]=\sum_f\bar{\psi}_f\left(i\gamma_\mu D_\mu+m_f\right)\psi_f \tag{2.2}$$

$$D_\mu=\left(\partial_\mu-igA_\mu\right) \tag{2.3}$$

Note that the gluon field A_μ is of the form $A_\mu=A_\mu^a\frac{\lambda^a}{2}$ where λ^a are the generators of the $SU(3)$ Lie-Algebra, i.e. the eight ($a\in\{1...8\}$) Gell-Mann matrices.
The gauge field Lagrangian is given by:

$$\mathcal{L}_G\left[A_\mu\right]=\frac{1}{4}F_{\mu\nu}^aF_{\mu\nu}^a=\frac{1}{2}tr\left(F_{\mu\nu}F_{\mu\nu}\right) \tag{2.4}$$

In this case $F^{\mu\nu}$ is the *gluon field strength tensor* defined as:

$$F_{\mu\nu}=F_{\mu\nu}^a\frac{\lambda^a}{2} \tag{2.5}$$

$$F_{\mu\nu}^a=\partial_\mu A_\nu^a-\partial_\nu A_\mu^a+gf^{abc}A_\mu^bA_\nu^c \tag{2.6}$$

In order to perform numerical calculations it is of advantage to quantize QCD with the path integral formalism. In this approach the observables are vacuum expectation values (VEV) which consist of time ordered products of suitable operators. Using the Euclidean formalism VEVs can be calculated by

$$\langle\Omega|T\{\mathcal{O}_1(x_1)...\mathcal{O}_n(x_n)\}|\Omega\rangle=\frac{1}{Z}\int D\psi D\bar{\psi}DA_\mu\mathcal{O}_1(x_1)...\mathcal{O}_n(x_n)e^{-S_E[\psi,\bar{\psi},A_\mu]} \tag{2.7}$$

where $x_j=x(\tau_j)$, $Z=\int D\psi D\bar{\psi}DA_\mu e^{-S_E[\psi,\bar{\psi},A_\mu]}$ and $S_E\left[\psi,\bar{\psi},A_\mu\right]$ is the Euclidean action of the system.
Note that the integration $D\psi D\bar{\psi}DA$ stands for all possible fermionic field configurations, i.e.

$$D\psi D\bar{\psi}DA:=\prod_{x,\alpha}\prod_{y,\beta}\prod_{z,\mu}D\psi_\alpha(x)D\bar{\psi}_\beta(y)DA_\mu(z)$$

where ψ and $\bar{\psi}$ are Grassmann variables.

2.2 Lattice QCD

Since QCD is up to now not solvable analytically I have to rely on numerical methods and therefore will use the approach of lattice QCD for my investigation. When going on the lattice the quark fields $\psi(x)$, $\bar{\psi}(x)$ are defined on discrete spacetime points, i.e. $\psi^c_{S,f}(n)$, $\bar{\psi}^c_{S,f}(n)$, with c being the color, S being the spin, f being the flavor index and n the lattice site [10]. The gauge fields are now represented by so-called *links* which correspond in the continuum to

$$U_\mu(n) \;\equiv\; U(n, n+a\mu) = \exp\left(ig \int_n^{n+a\mu} dz_\mu \; A_\mu(z)\right) \tag{2.8}$$

A possible gluonic action expressed by link variables is given in [6]:

$$S_G\left[U\right] = \sum_P \frac{2}{g^2} \mathrm{Tr}\left[1 - \frac{1}{2}\left(U_{\mu\nu}(n) + U^\dagger_{\mu\nu}(n)\right)\right] \tag{2.9}$$

where the summation is over all possible plaquettes P and $U_{\mu\nu}(n)$ refers to a product of link variables forming a closed loop on the lattice (cf. [6]).
When trying naively a discretized version of the fermion action from (2.2), one will be confronted with the so-called *fermion doubling problem.* One way to avoid this is to use a variant of the below defined *Wilson fermionic action* (cf. (2.10)), i.e. using so-called Wilson twisted mass fermions described in subsection 2.3.

$$S_F^{(W)}\left[\psi, \bar{\psi}, U\right] = a^4 \sum_n \bar{\psi}(n)(D_W + m)\psi(n) \tag{2.10}$$

$$D_W = \frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \frac{ar}{2}\nabla_\mu\nabla^*_\mu \tag{2.11}$$

In the above expression ∇_μ and ∇^*_μ are the forward and backward covariant derivatives on the lattices [10, 1]. Note that the Wilson term will vanish in the continuum limit.

More details on lattice QCD can be found in [1].

2.3 Wilson twisted mass fermions

To get rid of $O(a)$ lattice discretization errors (cf. [8]) in numerical calculations I will work with so-called *wilson twisted mass fermions.* In order to do so I will use the following action:

$$S_F\left[\chi, \bar{\chi}, U\right] = a^4 \sum_n \bar{\chi}(D_W + m + i\mu\gamma_5\tau_3)\chi \tag{2.12}$$

with $\chi = (\chi_u, \chi_d)$, μ the so-called *twisted mass* and τ_3 being the Pauli matrix in flavor space.
I will refer to $\{\chi, \bar{\chi}\}$ as the *twisted mass* basis which can be obtained from the *physical basis* $\{\psi, \bar{\psi}\}$ by the twist rotation

$$\psi = exp\left(i\frac{\omega}{2}\gamma_5\tau_3\right)\chi, \qquad \bar{\psi} = \bar{\chi}\; exp\left(i\frac{\omega}{2}\gamma_5\tau_3\right) \tag{2.13}$$

3 Quantum Numbers and Creation Operators

In order to excite a b-baryon, I need an operator producing suitable quantum numbers when applied to the vacuum. In the following I will choose an ansatz from the literature motivated by phenomenological considerations and verify that indeed this operator produces the desired quantum numbers like parity, spin and isospin:

$$\mathcal{O}(\vec{r}) = \epsilon^{abc} Q^a(\vec{r}) \left((\psi_1^b)^T(\vec{r}) \mathcal{C}\Gamma(\psi_2^c(\vec{r})) \right) \tag{3.1}$$

with $\mathcal{C} = \gamma_0\gamma_2$ standing for the charge conjugation matrix and Γ representing a certain combination of γ-matrices which has to be chosen appropriately for the particle being investigated. While Q stands for the heavy quark field, $\psi_{1/2}$ name the light quark fields in the physical basis.

3.1 Gauge invariance

To describe a physical state (3.1) has to be gauge invariant. In QCD a spinor transforms under gauge transformations $G(\vec{r}) \in SU(3)$ in the following way:

$$\psi^a(\vec{r}) \to \psi^{a\prime}(\vec{r}) = G^{ab}(\vec{r})\psi^b(\vec{r}) \tag{3.2}$$

Thus the chosen operator transforms like

$$\begin{aligned}
\mathcal{O}(\vec{r}) \to \mathcal{O}'(\vec{r}) &= \epsilon^{abc} G^{ad}(\vec{r}) Q^d(\vec{r}) \left(G^{be}(\vec{r}) (\psi_1^e)^T(\vec{r}) \mathcal{C}\Gamma G^{cf}(\vec{r}) \psi_2^f(\vec{r}) \right) \\
&= \epsilon^{abc} G^{ad}(\vec{r}) G^{be}(\vec{r}) G^{cf}(\vec{r}) Q^d(\vec{r}) \left((\psi_1^e)^T(\vec{r}) \mathcal{C}\Gamma \psi_2^f(\vec{r}) \right) \\
&= \epsilon^{def} \det(G) Q^d(\vec{r}) \left((\psi_1^e)^T(\vec{r}) \mathcal{C}\Gamma \psi_2^f(\vec{r}) \right) \\
&= \mathcal{O}(\vec{r})
\end{aligned} \tag{3.3}$$

and is therefore invariant under gauge transformations. Note that $(\psi)^T$ is being transposed in the spin basis. Therefore $(\psi)^T$ transforms according to (3.2).
In the third step I used the following identity

$$\epsilon^{abc} G^{ad} G^{be} G^{cf} = \det(G)\epsilon^{def}$$

followed by $\det G = 1$ since $G \in SU(3)$.

3.2 Parity

A light quark field transforms under parity in the following way:

$$P\psi(\vec{r}) = \gamma_0 \psi(-\vec{r}) \tag{3.4}$$

while a static quark field does not transform at all:

$$PQ(\vec{r}) = Q(\vec{r}) \tag{3.5}$$

Therefore $\mathcal{O}$ transforms under a parity transformation like

$$\begin{aligned}\mathcal{O} \to \mathcal{O}' &= \epsilon^{abc} Q^a \left((\psi_1^b)^T \gamma_0 \mathcal{C} \Gamma \gamma_0 \psi_2^c \right) \\ &= \epsilon^{abc} Q^a \left((\psi_1^b)^T \gamma_0 \gamma_0 \gamma_2 \Gamma \gamma_0 \psi_2^c \right) \\ &= \epsilon^{abc} Q^a \left((\psi_1^b)^T \mathcal{C} (-\gamma_0 \Gamma \gamma_0) \psi_2^c \right) \end{aligned} \tag{3.6}$$

Note that the γ-matrices (in euclidean representation) suffice the algebra: $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$.
Since according to (3.5) the static quark is invariant under parity transformation the parity of the exited b-baryon is given by

$$P = + \quad \text{for} \quad \Gamma = -\gamma_0 \Gamma \gamma_0 \tag{3.7}$$

$$P = - \quad \text{for} \quad \Gamma = +\gamma_0 \Gamma \gamma_0 \tag{3.8}$$

It is important to mention that the twisted mass formalism breaks the parity-symmetry of the action. However a specific combination of parity and isospin, i.e.

$$\begin{aligned} P^{(tm)} \chi &:= \gamma_0 \tau_1 \chi \\ P^{(tm)} \bar{\chi} &:= \bar{\chi} \gamma_0 \tau_1 \end{aligned} \tag{3.9}$$

is still a symmetry:

$$\begin{aligned} S_F[\chi, \bar{\chi}, U] \to S'_F[\chi, \bar{\chi}, U] &= a^4 \sum_n \bar{\chi} \tau_1 \gamma_0 (D_W + m + i\mu\gamma_5\tau_3) \gamma_0 \tau_1 \chi \\ &= a^4 \sum_n \bar{\chi} (D_W + m + i\mu\tau_1\gamma_0\gamma_5\gamma_0\tau_3\tau_1) \chi \\ &= a^4 \sum_n \bar{\chi} (D_W + m + i\mu\tau_1(-\gamma_5)\tau_3\tau_1) \chi \\ &= a^4 \sum_n \bar{\chi} (D_W + m + i\mu\gamma_5) \chi \qquad (3.10) \\ &= S_F[\chi, \bar{\chi}, U] \qquad (3.11) \end{aligned}$$

where $(\tau^a)^2 = 1$ and $\tau^a \tau^b = i\epsilon^{abc}\tau^c + \delta^{ab}$ was used.
In consequence a mixing of states with different parity in the correlator occurs. Nevertheless this is not a problem since I will investigate only the lightest baryon for a given light flavor combination. A slight mixing is unavoidable but in the effective mass the heavier states with opposite parity are exponentially suppressed.

3.3 Spin

In this thesis I consider operators where the quark *orbital momentum* L is zero. This means that the *total momentum* J is equal to the *spin* S. Since the spin of the static quark has no influence on the mass of the baryon it is appropriate

to label the states by the spin j of the light quarks.
In quantum field theory the spin is given by the *spinor representation* of the Lorentz transformations $S(\Lambda)$ generated by the spin operator $\vec{\mathcal{S}}$ which can often be found in the literature to be written as (cf. [7])

$$\vec{\mathcal{S}} = \frac{1}{2}\vec{\Sigma} \tag{3.12}$$

Here $\vec{\Sigma}$ stands for a three component object, where each component Σ_j is itself a 4x4 matrix. In the literature Σ_j can be found to be (cf. [4, 7]):

$$\Sigma_j = -i\gamma_1\gamma_2\gamma_3\gamma_j \tag{3.13}$$

Now a rotation by an angle α is given by:

$$e^{-(i\alpha^a \mathcal{S}_a)} = e^{-\left(i\frac{\alpha^a}{2}\Sigma_a\right)} = e^{-\left(\frac{\alpha^a}{2}\gamma_1\gamma_2\gamma_3\gamma_a\right)} = e^{\left(\frac{\alpha^a}{2}\gamma_0\gamma_5\gamma_a\right)} \tag{3.14}$$

where $(\gamma_0)^2 = 1$ and $\gamma_5 = -\gamma_0\gamma_1\gamma_2\gamma_3$ was used in the last step.

Therefore a spinor transforms like

$$\psi \to \psi' = e^{\left(\frac{\alpha^a}{2}\gamma_0\gamma_5\gamma_a\right)}\psi \tag{3.15}$$

$$(\psi)^T\mathcal{C} \to (\psi^T)'\mathcal{C} = (e^{\left(\frac{\alpha^a}{2}\gamma_0\gamma_5\gamma_a\right)}\psi)^T\mathcal{C} = (\psi)^T\mathcal{C}e^{-\left(\frac{\alpha^a}{2}\gamma_0\gamma_5\gamma_a\right)} \tag{3.16}$$

This means that the spin of the light quarks depend on the expression

$$e^{-\left(\frac{\alpha^a}{2}\gamma_0\gamma_5\gamma_a\right)}\Gamma e^{\left(\frac{\alpha^a}{2}\gamma_0\gamma_5\gamma_a\right)} \tag{3.17}$$

A general rotation, e.g. around the z-axis, by an infinitesimal small angle α of some state $\mathcal{O}|\Omega\rangle$ can be written as

$$\begin{aligned} R_z(\alpha)\left(\mathcal{O}|\Omega\rangle\right) &= \exp\left(-i\alpha_z\mathcal{J}_z\right)\left(\mathcal{O}|\Omega\rangle\right) \\ &= \left(1 - i\alpha_z\mathcal{J}_z\right)\left(\mathcal{O}|\Omega\rangle\right) \\ &= \mathcal{O}|\Omega\rangle - i\alpha\mathcal{J}_z\mathcal{O}|\Omega\rangle \end{aligned} \tag{3.18}$$

On the other hand I could transform the operator $\mathcal{O}$ first and than afterwards apply it to the vacuum. This again should yield the same rotated state. Doing the algebra one finds that for infinitesimal small angles the operator $\mathcal{O}$ transforms like:

$$\begin{aligned} \mathcal{O}|\Omega\rangle \to \mathcal{O}'|\Omega\rangle &= (1 - \frac{\alpha^z}{2}J_z)\mathcal{O}(1 + \frac{\alpha^z}{2}J_z)|\Omega\rangle \\ &= \mathcal{O}|\Omega\rangle - i\alpha\tilde{\mathcal{O}}|\Omega\rangle + O(\alpha^2) \end{aligned} \tag{3.19}$$

As mentioned before (3.19) and (3.18) should be the same state. Thus a comparison yields the action of the momentum operator $\mathcal{J}_z$ on the state $\mathcal{O}|\Omega\rangle$, as is shown in (3.20).

$$\mathcal{J}_z\mathcal{O}|\Omega\rangle = \tilde{\mathcal{O}}|\Omega\rangle \tag{3.20}$$

The spin quantum number j is defined by $\mathcal{J}^2\mathcal{O}|\Omega\rangle = \left(\mathcal{J}_x^2 + \mathcal{J}_y^2 + \mathcal{J}_z^2\right)\mathcal{O}|\Omega\rangle = j(j+1)\mathcal{O}|\Omega\rangle$. Following the steps above, acting the momentum operator twice for each space component on the state and summing up will therefore yield the spin. With this procedure I will compute the spin of the light quark bilinear simply by choosing $\mathcal{O} = (\psi^b_{(1)})^T \mathcal{C}\Gamma(\psi^c_{(2)})$ and following the steps described above.

It should be said here that there is a more elegant way to compute the spin of some state $\bar{\psi}\Gamma\psi|\Omega\rangle$. The spin is defined by the transformation behavior of the creation operator, e.g. of $\mathcal{O} = \bar{\psi}\Gamma\psi$. It can be shown that if the γ-matrices combination Γ of the bilinear state is invariant under such a transformation it describes a spin-0 particle. On the other hand if it transforms like a vector, e.g. $\Gamma = \gamma^\mu \to \gamma^{\mu\prime} = S^{-1}(\Lambda)\gamma^\mu S(\Lambda) = \Lambda^\mu{}_\nu\gamma^\nu$ the state corresponds to a spin-1 particle. This is stated in the *Wigner Eckart theorem*.
Note that the rotational symmetry is broken on the lattice and therefore the above consideration are only rigorous valid in the continuum.

3.4 Isospin

The two light quarks, e.g. one up and one down quark, can form an isospin doublet. This doublet has a well defined isospin of 0 or 1 corresponding to the behavior under isospin transformation. It is known from quantum mechanics that an antisymmetric combination of two spin-1/2, or in this case isospin-1/2 particles form a (iso)spin-0 state while a symmetric combination shows a (iso)spin-1 state. Since the operator remains unchanged or changes its sign under exchange of the light quarks, i.e.

$$\begin{aligned} I=1: &\qquad \mathcal{O}(u,d) \underset{u\leftrightarrow d}{\to} \mathcal{O}(d,u) \\ I=0: &\qquad \mathcal{O}(u,d) \underset{u\leftrightarrow d}{\to} -\mathcal{O}(d,u) \end{aligned} \tag{3.21}$$

explicit implementation of the (anti-)symmetrization of (3.1) is not necessary in this case and will therefore be neglected.

4 Quantum Numbers of the Λ_b- and Ω_b-Baryon

In the following I will introduce the quantum numbers and creation operators for the Λ_b- and Ω_b-baryon. I will refer to u, d, s as the physical and χ_u, χ_d, χ_s as the twisted quark fields. While I will investigate all quantum numbers in the physical basis, the twisted quark fields are used in numerical calculations. The states are labeled by isospin I, spin of the light quarks j and parity P. A summary of the quantum numbers of the Λ_b- and Ω_b-baryon can be found in table 1.

4.1 The Λ_b-baryon

The Λ_b-baryon consist of one bottom, one up and one down quark, i.e. $\Lambda_b = b(ud - du)$, and has the following quantum numbers (cf. [3]):

$$\Lambda_b : \qquad I(j^P) = 0(0^+) \tag{4.1}$$

The mass difference to the B meson was determined to be $\Delta m(\Lambda_b, B) = m_{\Lambda_b} - m_B = 461$ MeV [10].
I will show that the choice of $\Gamma = \gamma_5$ in the creation operator from the ansatz (3.1) will produce the desired quantum numbers when the operator is applied to the vacuum.
In the following calculations of quantum numbers I therefore assume $\Gamma = \gamma_5$ and thus the operator for the Λ_b-baryon to be

$$\mathcal{O}_{\Lambda_b} = \epsilon^{abc} b^a \left((u^b)^T \gamma^0 \gamma^2 \gamma^5 d^c\right) \tag{4.2}$$

Note that in fact the creation operator (4.2) with the light quark combination ud is the same as with $-du$ and therefore proportional to the difference $ud - du$ in $b(ud - du) = \Lambda_b$.

4.1.1 Parity

According to (3.7) the parity in the physical basis is determined by

$$-\gamma_0 \Gamma \gamma_0 = -\gamma_0 \gamma_5 \gamma_0 = +\gamma_5 \tag{4.3}$$

With the help of (3.6) it is straight forward to see that the parity is $P = +$.

4.1.2 Spin

Following the steps described in subsection 3.3 an infinitesimal rotation around one arbitrary axis k with $k \in \{1, 2, 3\}$ yields (for simplicity I will only consider the necessary part in (4.2))

$$\begin{aligned}(u^b)^T \gamma_0 \gamma_2 \gamma_5 d^c |\Omega\rangle &\to (u^b)^T \gamma_0 \gamma_2 \left(1 - \frac{\alpha}{2} \gamma_0 \gamma_5 \gamma_k\right) \gamma_5 \left(1 + \frac{\alpha}{2} \gamma_0 \gamma_5 \gamma_k\right) d^c |\Omega\rangle \\ &= (u^b)^T \gamma_0 \gamma_2 \left(1 - \frac{\alpha}{2} \gamma_0 \gamma_5 \gamma_k\right) \left(1 + \frac{\alpha}{2} \gamma_0 \gamma_5 \gamma_k\right) \gamma_5 d^c |\Omega\rangle \\ &= (u^b)^T \gamma_0 \gamma_2 \gamma_5 d^c |\Omega\rangle + O(\alpha^2)\end{aligned} \tag{4.4}$$

The comparison in (3.20) leads to the fact that $\mathcal{J}_k\left((u^b)^T\gamma_0\gamma_2\gamma_5 d^c|\Omega\rangle\right) = 0$ and therefore $\mathcal{J}_k^2\left((u^b)^T\gamma_0\gamma_2\gamma_5 d^c|\Omega\rangle\right) = 0$ which corresponds to a momentum of the light quark doublet of $j = 0$. Since the heavy quark carries a spin of 1/2 the total spin of the particle described by (4.2) is $J = \frac{1}{2}$.

4.1.3 Isospin

According to subsection 3.4 flavor exchange yields the isospin. This means, that by using (3.21) the isospin can be determined to be $I = 0$, as is shown below.

$$\begin{aligned}
\epsilon^{abc}b^a\left((u^b)^T\gamma^0\gamma^2\gamma^5 d^c\right) &= \epsilon^{abc}b^a\left((u^b)^T_A\gamma^0_{AB}\gamma^2_{BC}\gamma^5_{CD}d^c_D\right) \\
&\to \epsilon^{acb}b^a\left((d^c)^T_D\gamma^5_{DC}\gamma^2_{CB}\gamma^0_{BA}u^b_A\right) \\
&= \epsilon^{abc}b^a\left((d^b)^T\gamma^5\gamma^2\gamma^0 u^c\right) \\
&= -\epsilon^{abc}b^a\left((d^b)^T\gamma^0\gamma^2\gamma^5 u^c\right) \qquad (4.5)
\end{aligned}$$

4.2 The Ω_b-baryon

The Ω_b-baryon is made out of one bottom and two strange quarks. It has the following quantum numbers (cf. [3]):

$$\Omega_b : \qquad I(j^P) = 0(1^+) \qquad (4.6)$$

I will make a similar ansatz for the Ω_b creation operator as I did in subsection 4.1 for the Λ_b creation operator (cf. (4.2)), i.e. I will choose γ_j as the suitable Γ in (3.1).

$$\mathcal{O}_{\Omega_b} = \epsilon^{abc}b^a\left((u^b)^T\gamma^0\gamma^2\gamma^j d^c\right) \qquad (4.7)$$

In the following I will show as before that this ansatz applied to the vacuum produces the desired quantum numbers (4.6).

4.2.1 Parity

Looking at (3.7) the parity of the chosen creation operator (4.7) is given by the expression

$$-\gamma_0\Gamma\gamma_0 = -\gamma_0\gamma_j\gamma_0 = +\gamma_j \qquad (4.8)$$

With this the parity of the Ω_b particle described by (4.7) is $P = +$, as can be seen from (3.6).

4.2.2 Spin

Let us assume a certain γ_k-matrix in (4.7), e.g. $k = 3$. Now performing an infinitesimal rotation, e.g. around the x-axis, as stated in subsection 3.3 leads to:

$$\begin{aligned}(u^b)^T\gamma_0\gamma_2\gamma_3 d^c|\Omega\rangle &\to (u^b)^T\gamma_0\gamma_2\left(1-\frac{\alpha}{2}\gamma_0\gamma_5\gamma_1\right)\gamma_3\left(1+\frac{\alpha}{2}\gamma_0\gamma_5\gamma_1\right)d^c|\Omega\rangle\\ &= (u^b)^T\gamma_0\gamma_2\left(1-\frac{\alpha}{2}\gamma_0\gamma_5\gamma_1\right)\left(1-\frac{\alpha}{2}\gamma_0\gamma_5\gamma_1\right)\gamma_3 d^c|\Omega\rangle\\ &= (u^b)^T\gamma_0\gamma_2\left(1-\alpha\gamma_0\gamma_5\gamma_1\right)\gamma_3 d^c|\Omega\rangle + O(\alpha^2)\\ &= (u^b)^T\gamma_0\gamma_2\left(1-\alpha\gamma_0(-\gamma_0\gamma_1\gamma_2\gamma_3)\gamma_1\right)\gamma_3 d^c|\Omega\rangle + O(\alpha^2)\\ &= (u^b)^T\gamma_0\gamma_2\gamma_3 d^c|\Omega\rangle + \alpha(u^b)^T\gamma_0\gamma_2\gamma_2 d^c|\Omega\rangle + O(\alpha^2)\end{aligned} \tag{4.9}$$

where I kept only the interesting part of (4.7) for simplicity. But according to the comparison (3.20) this means that

$$\mathcal{J}_1((u^b)^T\gamma_0\gamma_2\gamma_3 d^c|\Omega\rangle) = -i((u^b)^T\gamma_0\gamma_2\gamma_2 d^c|\Omega\rangle) \tag{4.10}$$

In order to know how the initial state $(u^b)^T\gamma_0\gamma_2\gamma_3 d^c|\Omega\rangle$ transforms under $\mathcal{J}_1^2$ one has to apply $\mathcal{J}_1$ again, this time acting on the new state $(u^b)^T\gamma_0\gamma_2\gamma_2 d^c|\Omega\rangle$. Using the exact same algebra as before I obtain the following result:

$$\mathcal{J}_1((u^b)^T\gamma_0\gamma_2\gamma_2 d^c|\Omega\rangle) = +i((u^b)^T\gamma_0\gamma_2\gamma_3 d^c|\Omega\rangle) \tag{4.11}$$

Hence

$$\begin{aligned}\mathcal{J}_1^2((u^b)^T\gamma_0\gamma_2\gamma_3 d^c|\Omega\rangle) &= \mathcal{J}_1\left(\mathcal{J}_1((u^b)^T\gamma_0\gamma_2\gamma_3 d^c|\Omega\rangle)\right)\\ &= \mathcal{J}_1\left(-i((u^b)^T\gamma_0\gamma_2\gamma_2 d^c|\Omega\rangle)\right)\\ &= -i\left(+i((u^b)^T\gamma_0\gamma_2\gamma_3 d^c|\Omega\rangle)\right)\\ &= +((u^b)^T\gamma_0\gamma_2\gamma_3 d^c|\Omega\rangle)\end{aligned} \tag{4.12}$$

The rotation around the y- and z-axis as well as the rotation of the operator choosing $k \in \{1,2,3\}$ in (4.7) is derived analogously, e.g. one finds that for $k=3$

$$\mathcal{J}_2^2((u^b)^T\gamma_0\gamma_2\gamma_3 d^c|\Omega\rangle) = +((u^b)^T\gamma_0\gamma_2\gamma_3 d^c|\Omega\rangle) \tag{4.13}$$

$$\mathcal{J}_3^2((u^b)^T\gamma_0\gamma_2\gamma_3 d^c|\Omega\rangle) = 0 \tag{4.14}$$

It is easy to see that from the above calculations and according to subsection 3.3 the operator given in (4.7) corresponds to a spin of the light quarks of 1 since $\mathcal{J}^2((u^b)^T\gamma_0\gamma_2\gamma_k d^c|\Omega\rangle) = (\mathcal{J}_1^2+\mathcal{J}_2^2+\mathcal{J}_3^2)((u^b)^T\gamma_0\gamma_2\gamma_k d^c|\Omega\rangle) = 2((u^b)^T\gamma_0\gamma_2\gamma_k d^c|\Omega\rangle)$, with $2 = j(j+1)$ for $j=1$, holds for all $k \in \{1,2,3\}$.

4.2.3 Isospin

Since the Ω_b-baryon consists of two strange quarks and one bottom quark, it has an isospin of $I=0$ and strangeness $S=-2$.

5 Computational Setup

5.1 The two point correlation function

It is shown in [1] that the mass of a b-baryon, interpreted as the difference between the vacuum energy E_Ω and the energy of the ground state E_0 in the sector described by the quantum numbers of the baryon, can be calculated from the the limit at infinite temporal separation of the exponential decay of the two point correlation function.

$$\lim_{t\to\infty} \langle\Omega|\mathcal{O}^\dagger(t)\mathcal{O}(0)|\Omega\rangle = |\langle\Omega|\mathcal{O}|0\rangle|^2 e^{-(E_0-E_\Omega)t} \tag{5.1}$$

It is common to look at the effective mass plateau described by (5.2) in order to determine the mass of the baryon.

$$m_{eff} = \log\left(\frac{\langle\Omega|\mathcal{O}^\dagger(t+1)\mathcal{O}(0)|\Omega\rangle}{\langle\Omega|\mathcal{O}^\dagger(t)\mathcal{O}(0)|\Omega\rangle}\right) \tag{5.2}$$

Hence in order to calculate the mass of the Λ_b-baryon I need an expression for $\langle\Omega|\mathcal{O}^\dagger(t)\mathcal{O}(0)|\Omega\rangle$ which can be computed numerically on the lattice. It is shown in [1] that with the help of (3.1) this correlation function is found to be

$$C(t) = \epsilon^{abc}\epsilon^{def}\langle U^{ad}(t,0)Tr_{spin}\left(\mathcal{C}\Gamma(\Delta^{-1}_{\chi_u})^{cf}\mathcal{C}\Gamma(\Delta^{-1}_{\chi_d})^{be}\right)\rangle \tag{5.3}$$

In this expression $\Delta^{-1}_{\chi_u}$ and $\Delta^{-1}_{\chi_u}$ stand for the propagators of the light quarks in the twisted mass basis. These were calculated performing inversions of the Dirac operator using point sources according to [1]. Furthermore $\langle ... \rangle$ indicates the weighted average over the gauge link configurations $\int dU e^{-S_{eff}[U]}$ where $S_{eff}[U] \propto S_G[U] - \log(\det(\gamma_\mu D_\mu[U]+m))$. Here $D_\mu[U]$ stands for the discretized Dirac operator. $U^{ab} = U^{ab}(t_0, t)$ stands for the Wilson line from one lattice site at some initial time t_0 to the same (spatial) site at some later time t. It represents the heavy quark propagator which can be shown using **H**eavy **Q**uark **E**ffective **T**heory (HQET) [5].

5.2 Smearing techniques

In order to get a good overlap of the trial state $\mathcal{O}|\Omega\rangle$ signal with the ground state $|0\rangle$ I use standard smearing techniques as in [2]. The spatial links were smeared using APE smearing with parameters $N_{\text{APE}} = 40$ and $\alpha_{\text{APE}} = 0.5$. The fermionic fields were smeared according to Gaussian smearing with a smearing level $N_{\text{Gauss}} = 90$ and $\kappa_{\text{Gauss}} = 0.5$. These are the parameters optimized in previous similar computations (cf. [3, 10]). Finally I used HYP2 smearing for the temporal links in order to reduce the self energy of the static quark and therefore to reduce statistical errors.

5.3 Technical parameters

In this analysis I will use $T/a \times (L/a)^3 = 48 \times 24^3$ gauge link configuration produced by the European Twisted Mass Collaboration (ETMC). I used as

mentioned before the Wilson twisted mass action for the fermionic action with twisted masses $\mu_{u/d} = 0.004$ corresponding to a pion mass of $m_\pi = 336$ MeV, $\mu_s = 0.022$ and $\kappa = 0.160856$ which corresponds to a maximal twist. I assumed the sea quarks to be up and down quarks which as well corresponds to $\mu_{sea} = 0.004$. I chose the tree-level Symanzik gauge action (cf. [9]) with a $\beta = 3.9$ which yields a lattice spacing of $a = 0.079$ fm for the computations of the gauge link configurations.
More details on how to generate gauge link configurations can be found in [9].

6 Numerical Results

As already mentioned I obtained the following results in close collaboration with my colleague. This is why some of the figures shown below can also be found in his thesis (cf. [1]).
The results presented in this section were obtained using 23 to 24 gauge configurations. The states of interest were chosen to be two experimental measured states, i.e. the Λ_b and Ω_b-baryon, as well as one state in the following denoted by bss_γ_0, which was not yet experimentally measured. The quantum numbers are listed in table 1.
Due to the small statistics the comparisons of the errors with the results from [3, 10] are based on estimations and have to be investigated in more detail in future works. Because of the usage of Monte Carlo methods the error goes with $1/\sqrt{N}$.
While in [10] a total amount of $\mathrm{N_W} = 200$ gauge configurations were used as well as two 'types' of strange quarks (to improve the statistics), s^+/s^- (cf. [1]), I used $\mathrm{N} = 23$ gauge configurations for the Λ_b-baryon and $\mathrm{N} = 24$ gauge configurations for the other two states Ω_b and bss_γ_0. I also used only one 'type' of strange quark, i.e. either s^+ or s^-. Therefore the ratio R of the relative errors from this work and [10] should have the following values in order for both methods, point source and timeslice method, to be equally good:

1. Λ_b: 23 gauge configurations used; $R_0 = \sqrt{\frac{N_W}{N}} = \sqrt{\frac{200}{23}} \approx 2.95$
2. Ω_b: 24 gauge configurations used; $R_0 = \sqrt{\frac{N_W}{N}} = \sqrt{\frac{400}{24}} \approx 4.08$
3. bss_γ_0: 24 gauge configurations used; $R_0 = \sqrt{2\frac{N_W}{N}} = \sqrt{2 \cdot \frac{400}{24}} \approx 5.77$

Note that in contrast to the other two states the state bss_γ_0 was computed only in positive time direction which makes the ratio of the errors greater by a factor of $\sqrt{2}$.

To be better the errors should be smaller than that.

state	Γ	light quarks	P	I_z	j
Λ_b	γ_5	$ud - du$	$+$	0	0
Ω_b	γ_j	ss	$+$	0	1
bss_γ_0	γ_0	ss	$-$	0	0

Table 1: states of interest in this work; quantum numbers according to [3]

Figure 1 shows the correlation functions for the three investigated states. As was shown in [10] the imaginary part of the correlator is zero. It can be seen that this is the case for each state, i.e. the imaginary part vanishes within the error. This is a valuable cross check of the numerical computation. In the following I will therefore set the imaginary part to zero.

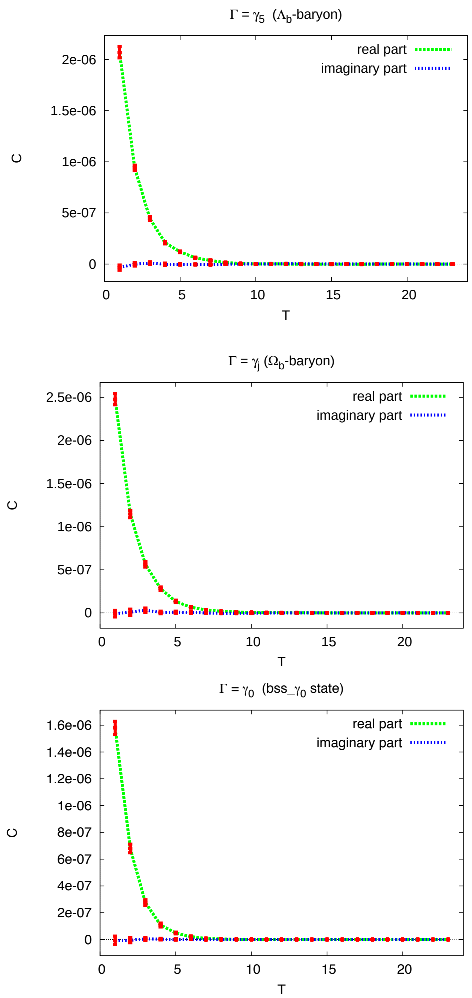

Figure 1: real and imaginary part of the two point correlation function for Λ_b, Ω_b and bss_γ_0

The corresponding effective masses can be found in figure 2. According to (5.1) the mass can be determined only in the limit of infinite times. Therefore I plotted the effective masses for the Λ_b as well as the Ω_b-baryon twice: one time with a smaller and one time with a greater temporal separation. When comparing these masses with the masses m^* obtained in [3] it can be seen that there is a qualitative agreement (cf. table 2). Note that both masses derived in this work are bigger than the ones obtained in previous work. Nevertheless the statistical accuracy is by far not sufficient enough to identify effective mass plateaus and needs to be improved for a more precise comparison.

state	$a \cdot m^*$	$a \cdot m$	$m^* - m_B[MeV]$	$m - m_B[MeV]$
Λ_b	0.5863 ± 0.0085	0.6069 ± 0.0415	461(24)	512(103)
Ω_b	0.7482 ± 0.0034	0.7999 ± 0.0244	865(8)	994(60)

Table 2: comparison of the effective masses obtained in this work (m) and in [3, 10] (m^*)

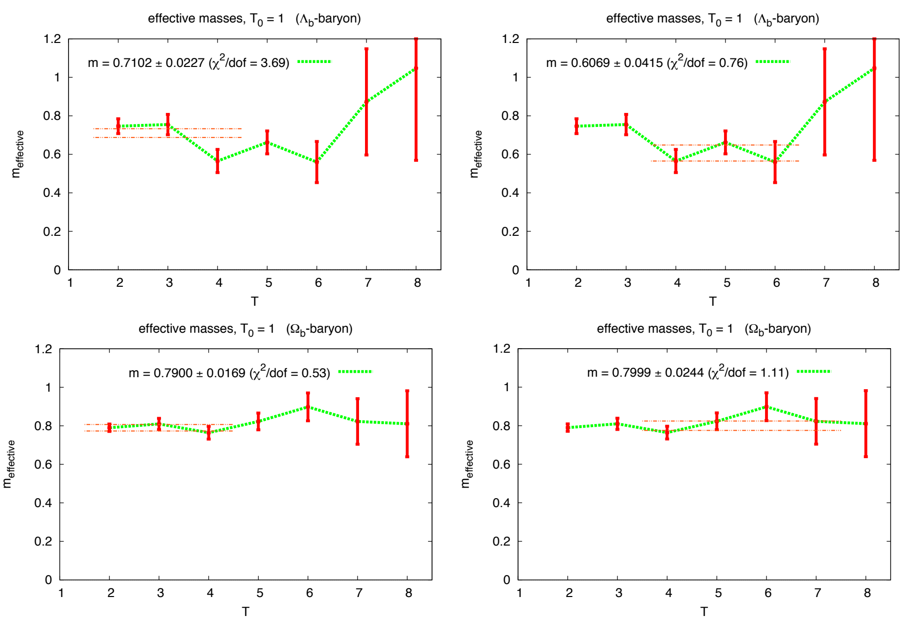

Figure 2: effective mass plot for Λ_b and Ω_b with mass fit for smaller (left) and greater (right) temporal separation

An other cross check for the numerical results is the comparison of the corresponding correlation functions between this work (which were normed with respect to the value of the correlator at $T = 1$) and [10]. This is given in figure 3. It can be seen that except for the bss_γ_0 state the correlation functions match within the errors.

Figure 3 also shows the ratio of the errors obtained in this work and in [10]. At a first glance it seems that in case of the Λ_b and bss_γ_0 the point source method is as good as the timeslice source method. Surprisingly this seems not to be true for the Ω_b anymore. Naively one would think it would be just the other way around: The error in the point source method occurs only due to the gauge

link fluctuations which are stronger suppressed in the timeslice method (note that there is additional statistical noise in case of timeslice sources) . Since these fluctuations are stronger suppressed by heavier quarks the error of the Ω_b should be smaller than the error of the Λ_b and hence the ratio R should be better in case of the Ω_b.
That the ratio in case of the Λ_b is better than for the Ω_b might be due to the different Γ structure of the creation operators and has to be investigated in more detail in future works.

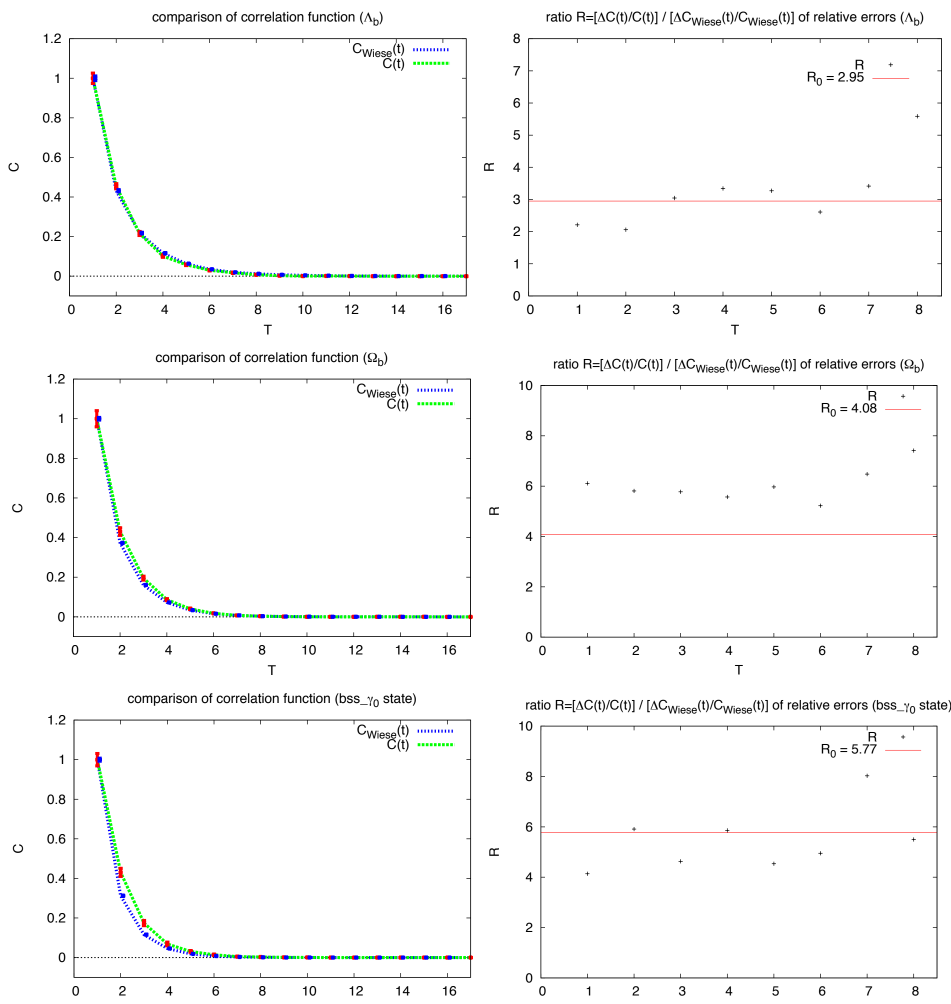

Figure 3: **left**: comparison of the correlation function derived in this work and in [10]; **right**: ratio of relative errors R for Λ_b and Ω_b-baryon as well as the bss_γ_0 state. The red line indicates the ratio R_0 at which point source and timeslice source method are equally good.

7 Summary and Outlook

Surly the ratio of the errors have to be investigated in further analysis, i.e. using more gauge configurations than was done in this work. Nevertheless the results presented above are surprising in the sense that one would naively expect the error of the Ω_b to be better than the error of the Λ_b and not the other way around. Since using the point source method the error only occur due to the gauge link configuration and should be smaller the heavier the state.
Note that the statistics in this work were far to poor to allow a good determination of the particle mass. For more precise physical statements one could consider, next to the use of far more gauge configurations, more particle creation operators which produce the same quantum numbers and therefore the same state. Another possibility is to consider also a correlation matrix $C_{\Gamma_1,\Gamma_2}(t)$ as was done in [10]. However the determination of the mass was not the main goal of this thesis.
In contrast to the calculation of mesons where the error could be improved drastically (cf. [11]) an improvement of the errors using the point source method rather than the timeslice method in case of baryons could not be found. This might be due to the fluctuations of the static quark, which is not present in the meson.

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Selbstständigkeitserklärung

Hiermit erkläre ich, dass ich die vorliegende Arbeit selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel verwendet habe.

Frankfurt am Main, den 21. August 2013

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