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HIC for FAIR Helmholtz International Center

The Nanbu-Jona-Lasinio model in a Relativistic Quantum Molecular Dynamics

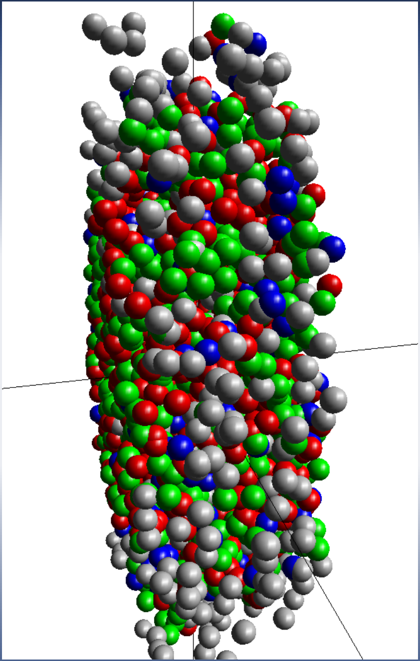
from medium properties to Heavy Ions Collisions

Rudy Marty

June 24^{e} 2012

in collaboration with J. Aichelin

special thanks to E. Bratkovskaya



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Outline

What What kind of transport model ?

How How does the hadronization take place ?

Why Why is a microscopic approach so interesting ?

Outline

What What kind of transport model ?

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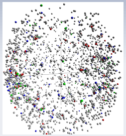
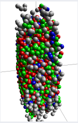
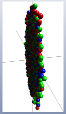
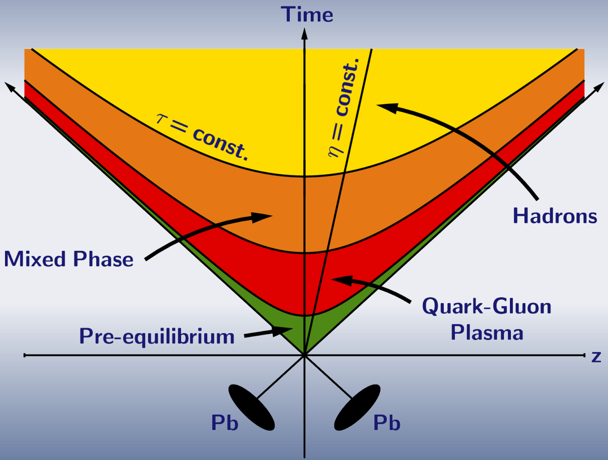
Heavy Ions Collisions









Numerical steps for a simulation

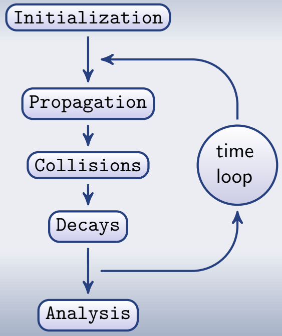
Simulation = computer

$\Downarrow$

Discretization of time steps and processes

$\Downarrow$

Each step must be **carefully** taken into account !



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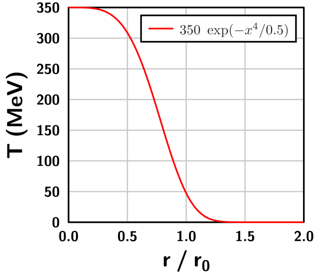
Initial conditions

Simulations similar to RHIC conditions :

Parameters of the simulations

- $A = 208$ nucleons,
- $\sqrt{s_{NN}} = 200\ \mathrm{GeV/N}$,
- b variable,
- $T_0 = 350\ \mathrm{MeV}, \mu_0 = 0\ \mathrm{MeV}$,
- $t = 20\ \mathrm{fm/c}$.



Phase space is **saturated** by partons : no big fluctuations.
The plasma is assumed to reach the **local equilibrium** before the simulations start.

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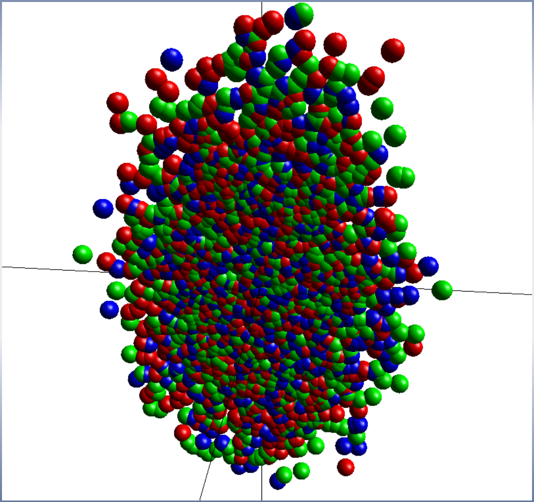
Initial conditions



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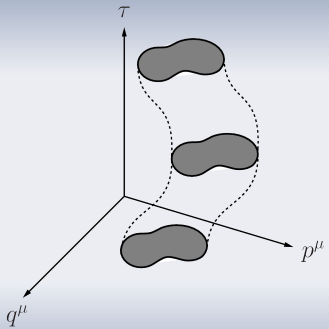
Relativistic Dynamics

Relativistic equations of motion

$$\frac{\partial q_i^\mu}{\partial \tau} = \{q_i^\mu, \mathcal{Z}\}_D = 2\lambda_i p_i^\mu$$

$$\frac{\partial p_i^\mu}{\partial \tau} = \{p_i^\mu, \mathcal{Z}\}_D = \sum_k \lambda_k \frac{\partial V_k}{\partial q_i^\mu} + \langle \text{ coll. } \rangle$$



Relativistic particles in the **Minkowski phase space** (q^μ, p^μ) but classical phase space for dynamics $(\vec{q}, \vec{p})$!

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Relativistic Dynamics

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- τ : invariant time evolution parameter,
- $\{\cdot,\cdot\}_D$: Dirac bracket,
- $\mathcal{Z}$: system with constraints,
- λ : relativistic factor.

Relativistic particles in the **Minkowski phase space** (q^μ, p^μ) but classical phase space for dynamics $(\vec{q}, \vec{p})$!

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Relativistic Dynamics

$8N \rightarrow 6N$ dimensions : $2N$ constraints ϕ_k
to **fix the times and the energies** of the N particles.
For this constrained dynamics we use :

Dirac bracket :

$$\{a, b\}_D = \{a, b\} - \{a, \phi_i\} C_{ij} \{\phi_j, b\}$$

with the matrix of constraint $C_{ij}^{-1} = \{\phi_i, \phi_j\}$.

(**Dirac, Lectures on Quantum Mechanics (1964)**)

Relativistic Dynamics

$\mathcal{Z}$ is a quantity related to the **evolution parameter** τ.
It is related to the **energy conservation** and the **causality** :

$$\mathcal{Z} = \sum_k \lambda_k \phi_k$$

using the relativistic factor λ, which
can be calculated from the Dirac bracket :

$$\lambda_k = C_{k2N}$$

Now : masses ? cross sections ?

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Nambu-Jona-Lasinio model

NJL Lagrangian for $SU(3)$:

$$\mathscr{L}_{NJL} = \mathscr{L}_2 + \mathscr{L}_4 + \mathscr{L}_6$$

$$\mathscr{L}_2 = \bar{q}_f \left(i\partial\!\!\!/ - m_{0f} \right) q_f$$

(kinetic term, break explicitly chiral sym.)

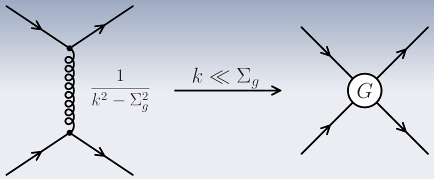
$$\mathscr{L}_4 = G_S \sum_{a=0}^{8} \left[\left(\bar{q}_f \lambda^a q_f \right)^2 + \left(\bar{q}_f i\gamma_5 \lambda^a q_f \right)^2 \right]$$

$$+ G_V \sum_{a=0}^{8} \left[\left(\bar{q}_f \gamma_\mu \lambda^a q_f \right)^2 + \left(\bar{q}_f i\gamma_\mu \gamma_5 \lambda^a q_f \right)^2 \right]$$

(4-fermions term, respect chiral sym.)

$$\mathscr{L}_6 = K \left[\det \bar{q}_f \left(1+\gamma_5\right) q_f + \det \bar{q}_f \left(1-\gamma_5\right) q_f \right]$$

('t Hooft term, $U_A(1)$ anomaly)



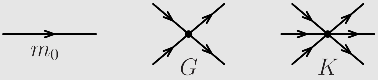
- Same symmetries than QCD ...
- ... but no gluons for confinement,
- A model for q and $\bar{q}$ ...
- ... but which can describe hadrons.

(**Klevansky, Rev. Mod. Phys. 64(1992)**)

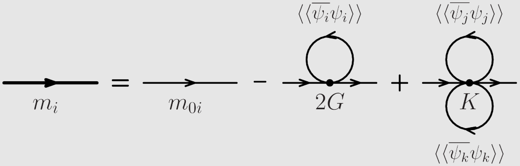
NJL masses for transport

Quark masses :

Starting from **interactions** contained in the Lagrangian :



we can get the effective masses of quarks in a **mean field** :



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NJL masses for transport

Quark masses :

Starting from **interactions** contained in the Lagrangian :

$$m_0 \qquad G(\phi\bar{\phi})^2 \qquad K(\phi\bar{\phi})^3$$

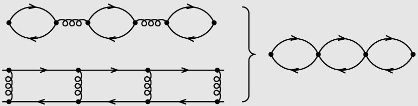
we can get the effective masses of quarks in a **mean field** :

$$M_i = m_{0i} - 2G\langle\langle\bar{\psi}_i\psi_i\rangle\rangle + K\langle\langle\bar{\psi}_j\psi_j\rangle\rangle\langle\langle\bar{\psi}_k\psi_k\rangle\rangle$$

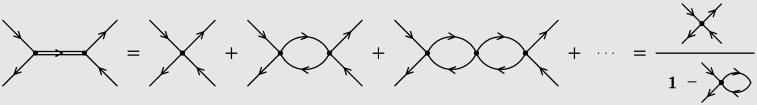
NJL masses for transport

Meson masses :

Starting from the **bound state** description (Random Phase Approximation or Lippmann-Schwinger equation) :



we can extract the meson masses as poles of the **propagator** :



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NJL masses for transport

Meson masses :

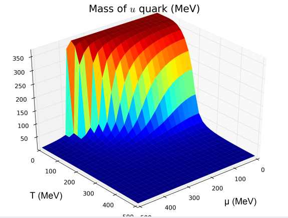
Starting from the **bound state** description (Random Phase Approximation or Lippmann-Schwinger equation) :

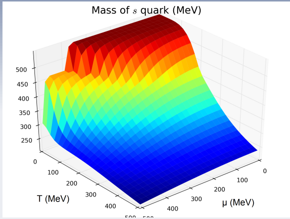
$$\Pi(k^2) = -i\ N_c\ \mathrm{Tr} \int \left(\Gamma S(p + k/2) \Gamma S(p - k/2)\right) \frac{\mathrm{d}^4 p}{(2\pi)^4}$$

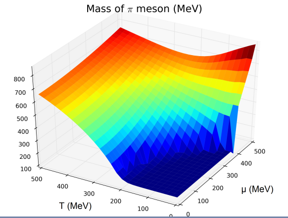
we can extract the meson masses as poles of the **propagator** :

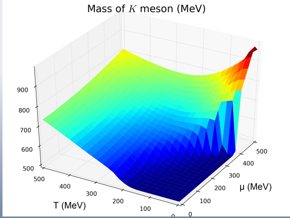
$$\det(1 - G_\pi \Pi(k^2)) = k^2 - m_{\bar{q}q}^2$$

NJL masses for transport



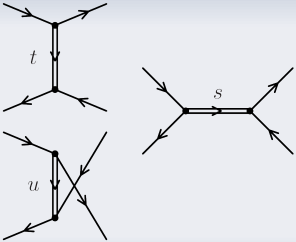




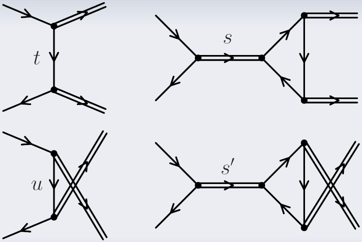


NJL cross sections for transport

The NJL cross sections can be obtained from the NJL Lagrangian and contain the following processes :



$q\ \bar{q} \rightarrow q\ \bar{q}$



$q\ \bar{q} \rightarrow M\ M$

We have an **explosion** of the cross sections close to the **critical temperature** $(m_q + m_{q'} = m_M)$ and close to the **threshold** $(\sqrt{s} = m_M + m_{M'})$.

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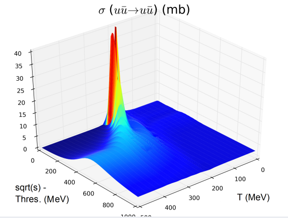
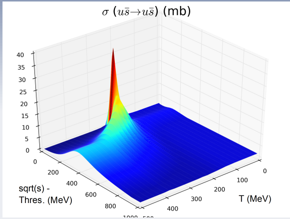
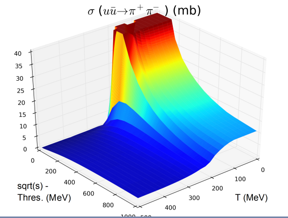
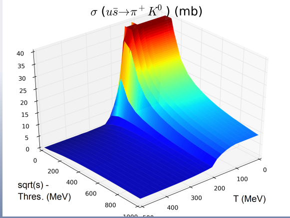
NJL cross sections for transport

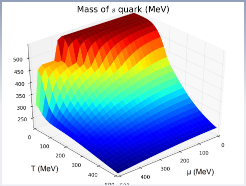


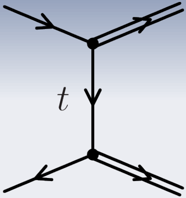






NJL + RQMD = Phase Transition





$$\frac{\mathrm{d}q_i^\mu}{\mathrm{d}\tau} = \frac{p_i^\mu}{E_i}$$

$$\frac{\mathrm{d}p_i^\mu}{\mathrm{d}\tau} = -\sum_{k=1}^{N} \frac{1}{2E_k} \frac{\partial V_k}{\partial q_{i_\mu}} + \langle \text{collisions} \rangle$$

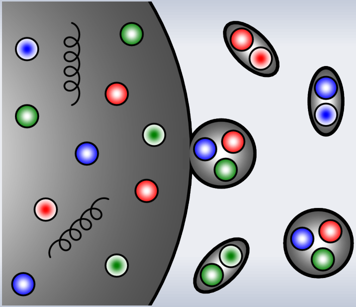
Finally, choosing the **nuclei collision frame** gives us these simple final equations of motion.

Outline

How

How does the hadronization take place ?

Local phase transition



Lattice QCD cannot be extended to describe the **dynamical phase transition** but a microscopic NJL+RQMD approach can do it.

Chiral phase transition and big cross sections are supposed to be enough to hadronize the plasma **even without confinement**.

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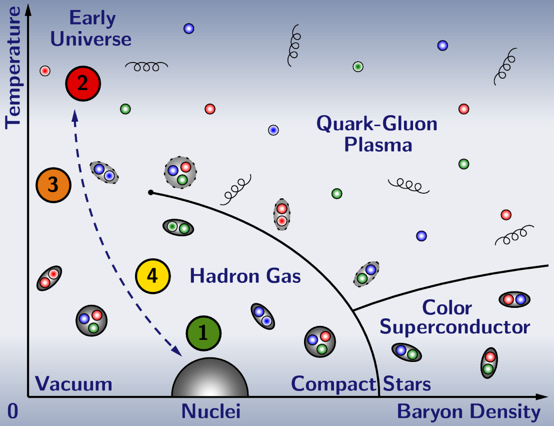
Local phase transition



We must use local (T, μ) with the NJL model for hadronization.

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Local equilibrium

We use equations of motion for **each particle** in the plasma. Assuming a **local equilibrium** we can define a (T, μ) for each particle. In the NJL model the mass plays the role of a potential :

$$\frac{\partial V_k}{\partial q_{i_\mu}} = 2m_k \, \frac{\partial m_k}{\partial q_{i_\mu}} = 2m_k \, \left(\frac{\partial m_k}{\partial T_k} \frac{\partial T_k}{\partial q_{i_\mu}} + \frac{\partial m_k}{\partial \mu_k} \frac{\partial \mu_k}{\partial q_{i_\mu}} \right)$$

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Local equilibrium

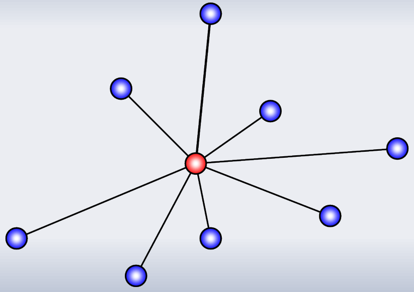
Local densities :

$$R_{ij} = \exp\left(-\frac{\Delta r_{ij}^2}{L^2}\right)$$

we *define*

$$\rho_{F\,i} = \sum_{i \neq j} R_{ij}$$

$$\rho_{B\,i} = \sum_{i \neq j} R_{ij}\ \mathrm{Sign}(j)$$



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Local potentials :

Thermodynamics gives :

$$T_i = (\hbar c)(\rho_{F\,i})^{1/3}\left(\frac{\pi^2}{g}\right)^{1/3}$$

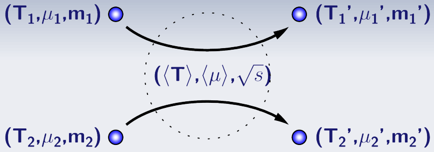
$$(\text{for } \mu \approx 0)$$

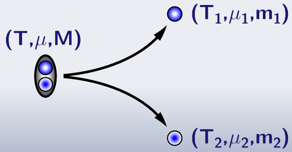
$$\mu_i = (\hbar c)(\rho_{B\,i})^{1/3}\left(\frac{6\pi^2}{g}\right)^{1/3}$$

$$(\text{for } T \approx 0)$$

Collisions and decays

Again, local (T, μ) are used for our microscopic processes





We also use an adaptative mean free path ($\propto \sigma^{-1}$) to set an **adaptative time step** $\Delta\tau$.

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Collisions and decays

Among all the possible NJL cross sections we include in our simulations :

- $q\ q \rightarrow q\ q$,
- $q\ \bar{q} \rightarrow q\ \bar{q}$,
- $q\ \bar{q} \rightarrow M\ M$,
- $M \rightarrow q\ \bar{q}$,

with scalar and pseudoscalar mesons. We finally use a **fully microscopic n-body theory** to describe the phase transition.

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Outline

Why

Why is a microscopic approach so interesting ?

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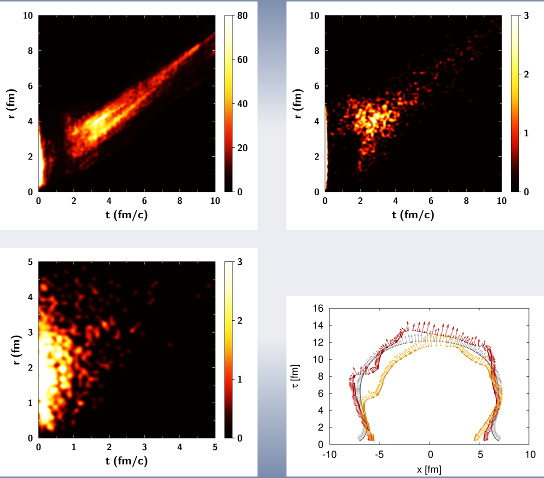
Cross-over

Let's have a look at the **microscopic scale**. We can **trace each particle** and record collisions and decays as a function of time, energy, temperature and so on.

Here is an example for $b = 6.5$ fm : the elastic and inelastic collisions, and the decay **compared to the freeze-out surface** coming from an hydrodynamical simulation with the same initial conditions (but $b = 0$ fm) :

Cross-over



(Schenke, Phys. Rev. Lett. 106 (2011))

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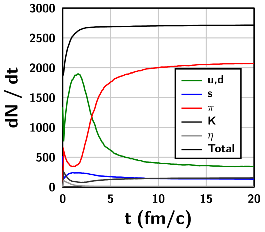
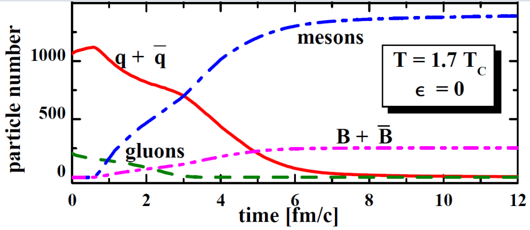
Cross-over

We can also look at the **particle multiplicity** :





Multiplicity as a function of time for us and for PHSD.
(**Cassing, Phys. Rev. C78 (2008)**)

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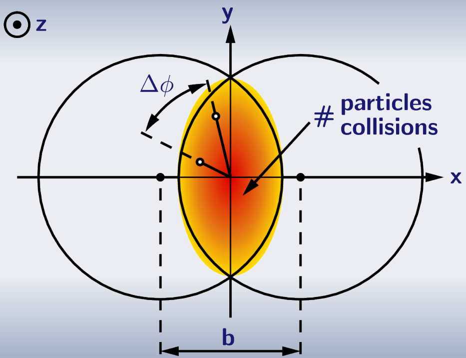
Effects of initial conditions



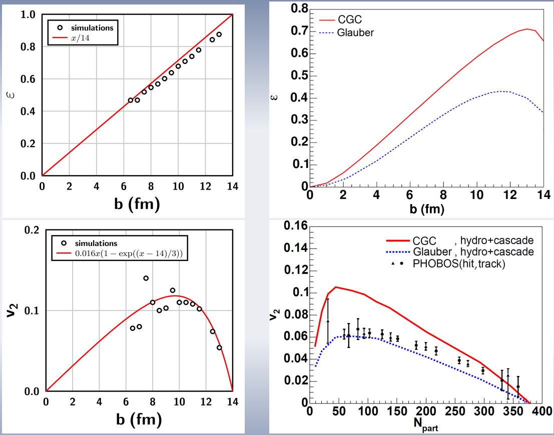
Eccentricity :

$$\varepsilon = \frac{\langle \mathbf{y}^2 - \mathbf{x}^2 \rangle}{\langle \mathbf{y}^2 + \mathbf{x}^2 \rangle}$$

Elliptic flow :

$$v_2 = \left\langle \left(\frac{\mathbf{p}_x}{\mathbf{p}_T}\right)^2 - \left(\frac{\mathbf{p}_y}{\mathbf{p}_T}\right)^2 \right\rangle$$

Effects of initial conditions



ε and v_2 as a function of b or N_{part} as compared to data (**Hirano, J. Phys. G 35 (2008)**)

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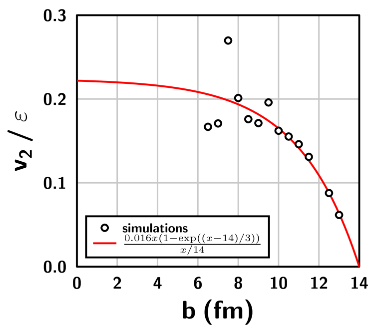
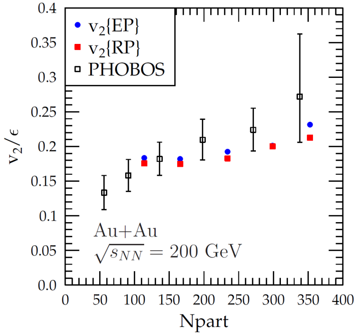
Effects of initial conditions





v_2/ε as a function of b or N_{part} as compared to data (**Holopainen, Phys. Rev. C83 (2011)**)

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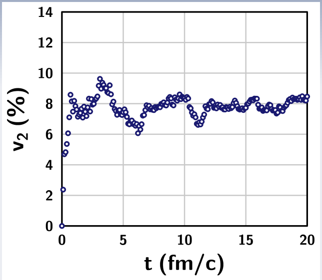
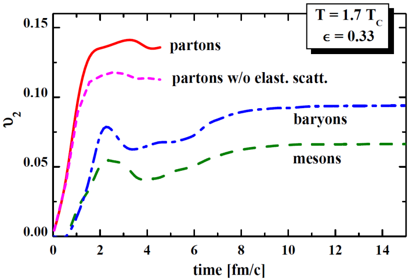
Effects of initial conditions





Formation of the v_2 as a function of time for similar event conditions in our approach and in PHSD. (**Cassing, Phys. Rev. C78 (2008)**)

Conclusion

It is possible to use a transport theory based on an **effective model** with local interactions and local equilibrium in order to reproduce the main properties of the quark gluon plasma.

The role of the **confinement** in the phase transition was not so clear but now it is possible to say that we can describe the **mechanism of hadronization** without this one.

A secondary conclusion is that it is possible to describe a **fully relativistic strongly interacting system** within a code which perfectly conserves the energy.

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Outlook

- Implement another improvement of the NJL model : the PNJL model (with Polyakov loop) which includes approximatively the confinement of color,
- Add new particles like vector mesons, baryons (to be able to compare results to experimental data), and new processes,
- Try to solve the problem of the few final free quarks . . .
- . . . and use different initial conditions (?),
- Improve the source code allowing to simulate bigger system (use parallelism on modern computer – OpenCL),
- Extension to a true event generator which can predict observables.

Thanks for your attention

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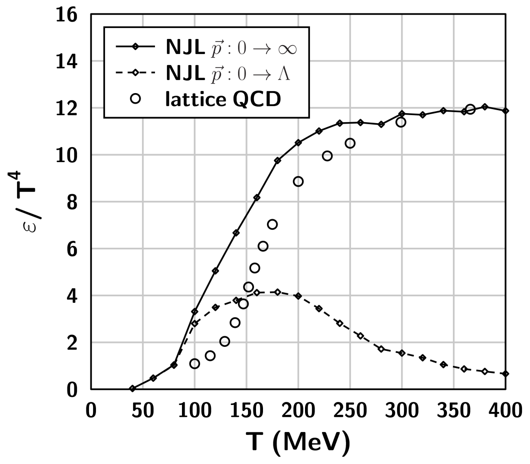
NJL equation of state



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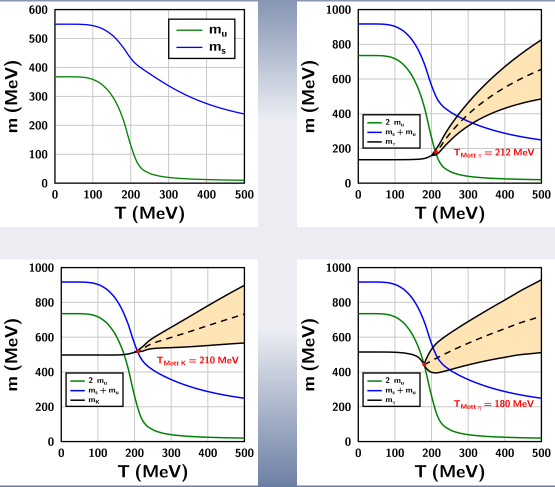
NJL masses



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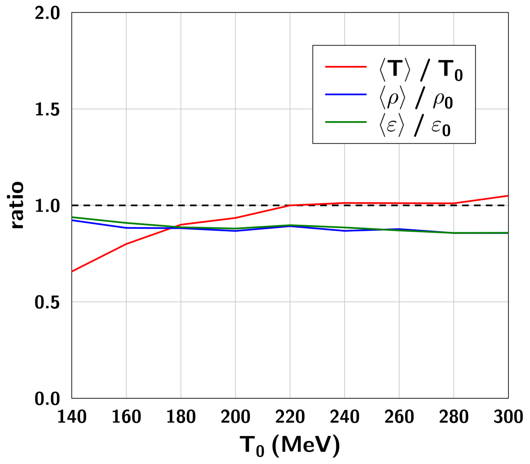
Checking local equilibrium



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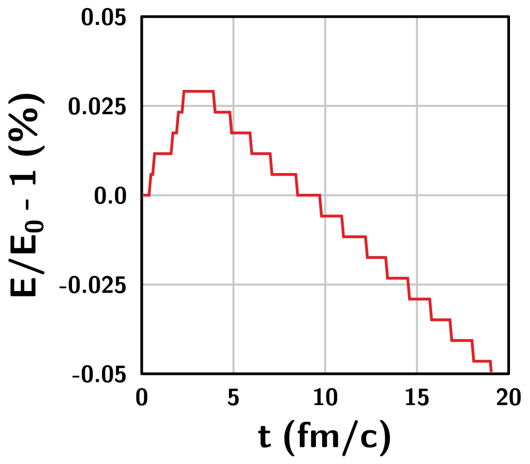
Energy conservation



Here we have density break effect (ex: mesons decay) !

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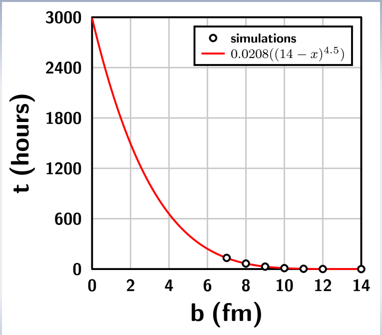
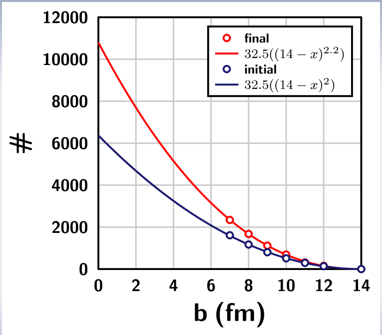
Running time





Running time (in hours) and corresponding N_{part} as a function of b.

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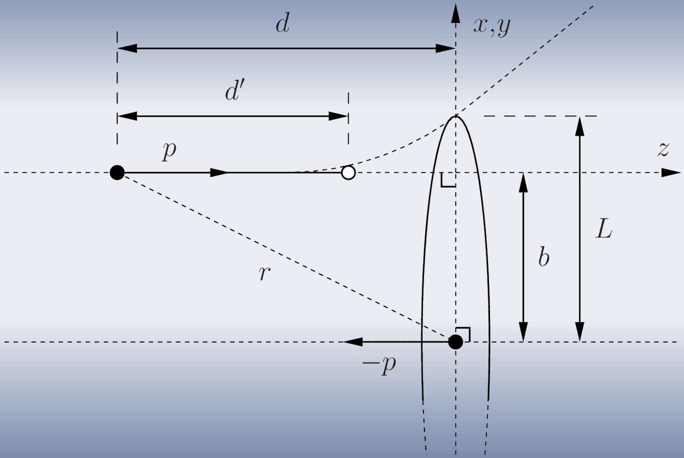
Relativistic collisions



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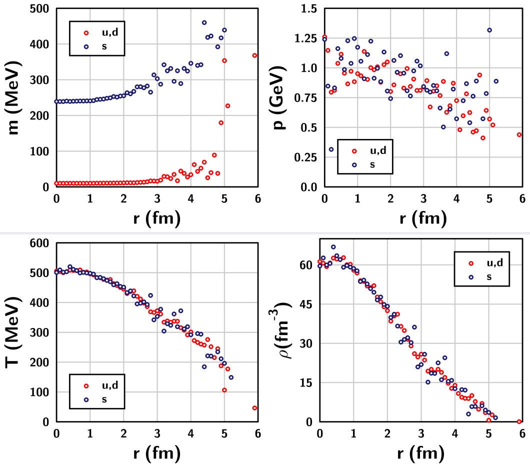
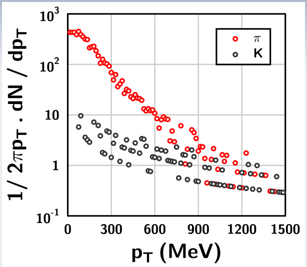
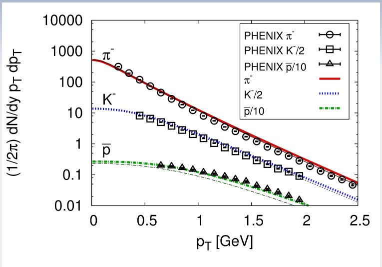
Results



Initial conditions of the movie ($b = 6.5$ fm).

Results





$\frac{\mathrm{d}^2 N}{\mathrm{d}\mathbf{p}_T^2}$ as a function of $\mathbf{p}_T$ as compared to data ($6 < b < 8$ fm).

(**Schenke, Phys. Rev. C82 (2010)**)

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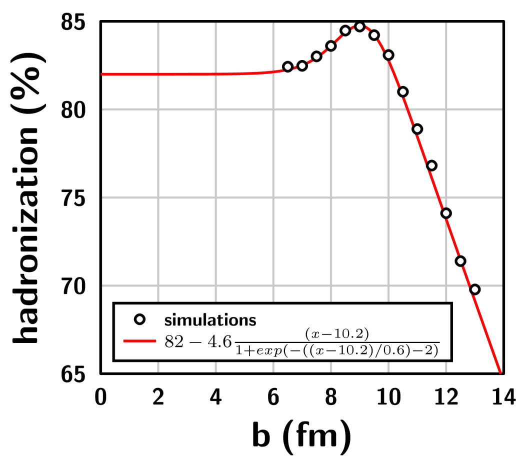
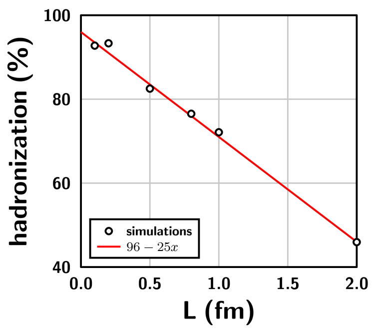
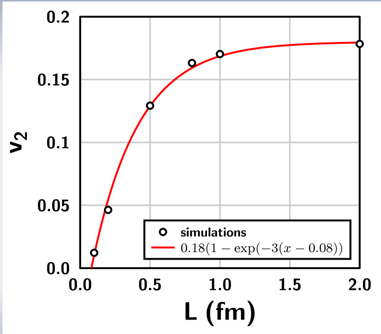
Results



Hadronization rate.

Results





Influence of L in the dynamics.