Helmholtz Research School
Quark Matter Studies

Dynamical simulation of a linear sigma model
Fluctuations at the phase transition

Christian Wesp
ITP Uni Frankfurt

with: A. Meistrenko, H. van Hees, C. Greiner

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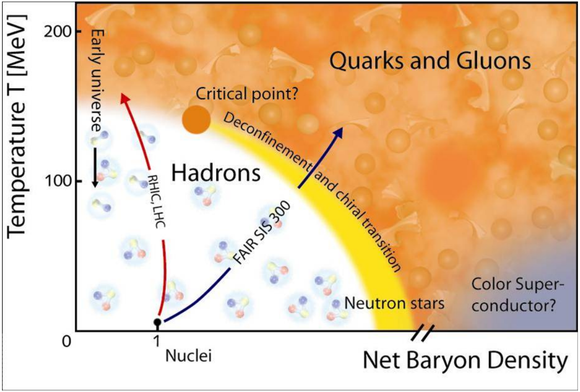
Content

Chiral Phase Transition

Use of phenomenological model to investigate:

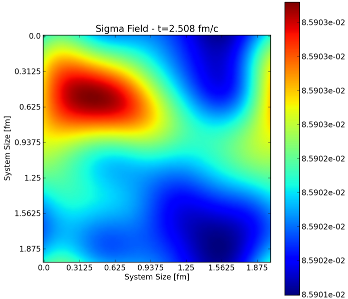
- dynamics of the phase transition
- non-equilibrium effects
- critical phenomena



The Model: Overview

Dynamical simulation of a linear sigma model with constituent quarks

- 3D+1 simulation
- quarks - quasi particles via Vlasov equation
- chiral fields - Klein-Gordon equation
- coupled PDE-solver on a 3D grid ($\sim 256^3$ points)



$$\mathcal{L} = \bar{\psi}\left[\imath\partial\!\!\!/ - g\left(\sigma + \imath\vec{\pi}\cdot\vec{\tau}\gamma_5\right)\right]\psi - \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - U\left(\sigma,\vec{\pi}\right)$$

$$U\left(\sigma,\vec{\pi}\right) = \frac{\lambda^2}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 - f_\pi m_\pi^2\sigma + U_0$$

Model Parameter

λ^2	$= 20$	self coupling parameter
g	$\approx 3\ldots 6$	Quark-sigma coupling
U_0	$= m_\pi^4/\left(4\lambda^2\right) - f_\pi^2 m_\pi^2$	Ground state
f_π	$= 93\ \mathrm{MeV}$	Pion Decay Constant
m_π	$= 138\ \mathrm{MeV}$	Pion mass
ν^2	$= f_\pi^2 - m_\pi^2/\lambda^2$	Field shift term

The Model: Equations of Motion

Meson fields σ and $\vec{\pi}$: nonlinear Klein-Gordon equations:

$$\partial_\mu \partial^\mu \ \sigma + \lambda^2 \left(\sigma^2 + \vec{\pi}^2 - \nu^2\right) \sigma + g \langle \bar{\psi} \psi \rangle - f_\pi m_\pi^2 = 0$$

$$\partial_\mu \partial^\mu \ \vec{\pi} + \lambda^2 \left(\sigma^2 + \vec{\pi}^2 - \nu^2\right) \vec{\pi} + g \langle \bar{\psi} \imath \gamma_5 \psi \rangle - f_\pi m_\pi^2 = 0$$

Quarks $\bar{\psi}$ and ψ: Vlasov equation:

$$\left[\partial_t + \frac{p}{E(t, \mathbf{r}, \mathbf{p})} \cdot \nabla_\mathbf{r} - \nabla_\mathbf{r} E(t, \mathbf{r}, \mathbf{p}) \nabla_\mathbf{p}\right] \ f(t, \mathbf{r}, \mathbf{p}) = 0$$

$$E(t, \mathbf{r}, \mathbf{p}) = \sqrt{\mathbf{p}(t)^2 + M(\mathbf{r})^2}$$

$$M(\mathbf{r})^2 = g^2 \left[\sigma(\mathbf{r})^2 + \vec{\pi}(\mathbf{r})^2\right]$$

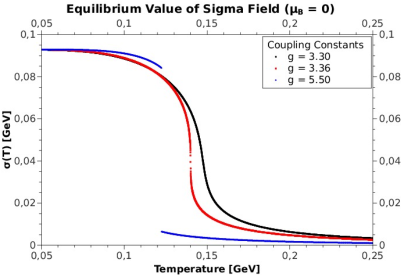
Scalar and pseudo-scalar quark densities:

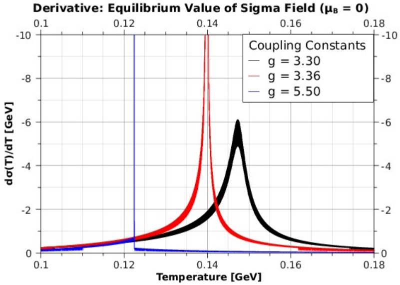
$$\langle \bar{\psi}\psi(\mathbf{r})\rangle = g\sigma(\mathbf{r}) \int d^3\mathbf{p}\ \frac{f(\mathbf{r},\mathbf{p}) + \tilde{f}(\mathbf{r},\mathbf{p})}{E(\mathbf{r},\mathbf{p})}$$

$$\langle \imath\bar{\psi}\gamma_5\psi(\mathbf{r})\rangle = g\vec{\pi}(\mathbf{r}) \int d^3\mathbf{p}\ \frac{f(\mathbf{r},\mathbf{p}) + \tilde{f}(\mathbf{r},\mathbf{p})}{E(\mathbf{r},\mathbf{p})}$$

Test particles for quarks:

$$f(t,\mathbf{r},\mathbf{p}) = \frac{1}{N_{\text{test}}} \sum_i \delta^3\left(\mathbf{r} - \mathbf{r}_i(t)\right)\ \delta^3\left(\mathbf{p} - \mathbf{p}_i(t)\right)$$





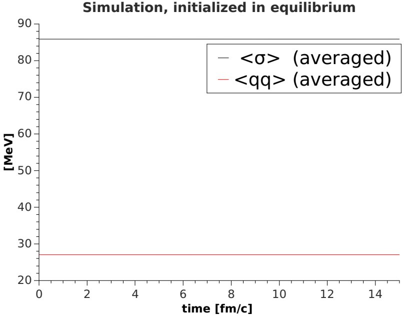
→ σ-field solving the nonlinear self-consistent equations $\partial_\mu \partial^\mu \sigma \equiv 0$:

$$\left[\lambda^2\left(\sigma_0^2 - \nu^2\right) + g^2 \int \mathrm{d}^3\mathbf{p} \frac{f(t,\mathbf{r},\mathbf{p},\sigma_0) + \tilde{f}(t,\mathbf{r},\mathbf{p},\sigma_0)}{E(t,\mathbf{r},\mathbf{p})}\right] \sigma_0 = f_\pi m_\pi^2$$

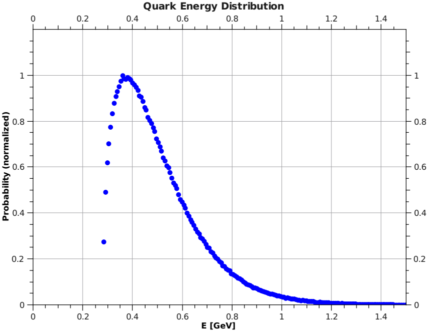
→ $f_q(t,\mathbf{r},\mathbf{p},\sigma_0)$: Fermi distribution

Test Scenario: Equilibrium

- σ and q thermal, $\pi = 0$.
- no spatial gradients, no anisotropy



Sigma field / Quark density

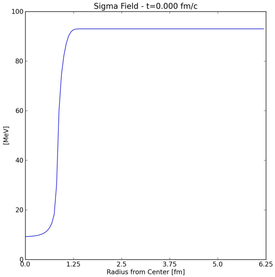


Quark energy

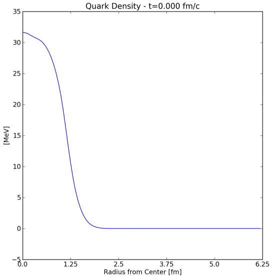
Test Scenario: Thermal Blob

- $\sigma(\mathbf{r})$ and $q(\mathbf{r})$ thermal, $\pi = 0$.
- spatial temperature / thermal 'blob'

$$T(\mathbf{r}) = \frac{T_{\text{init}}}{1 + \exp\left(\left(|\mathbf{r}| - R_0\right)/\alpha\right)}$$

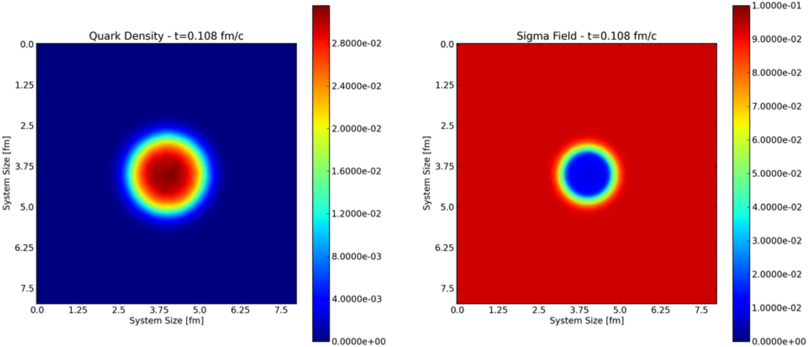


Sigma field

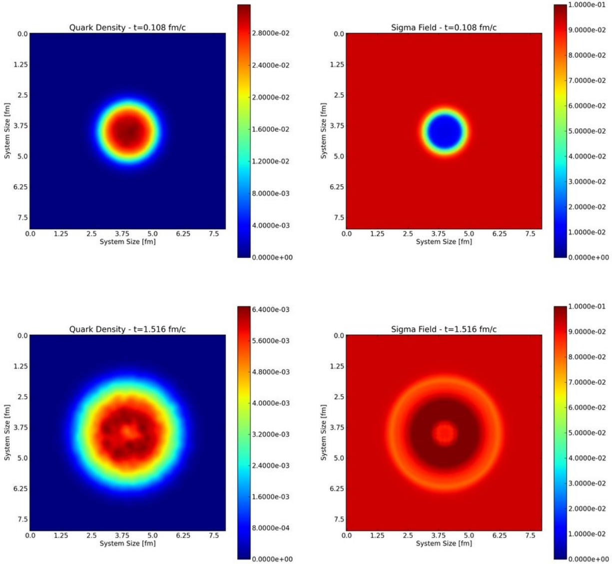


Quark density

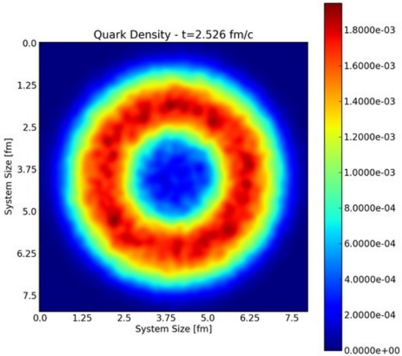
Thermal Blob Scenario

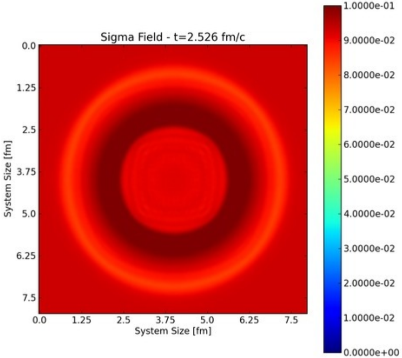


Thermal Blob Scenario

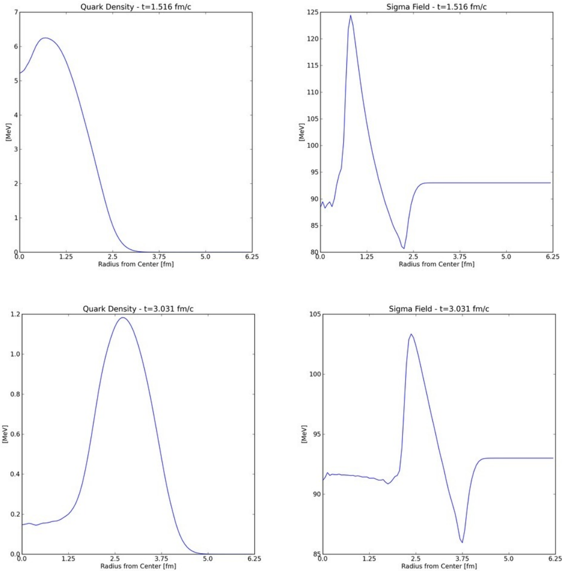


Thermal Blob Scenario



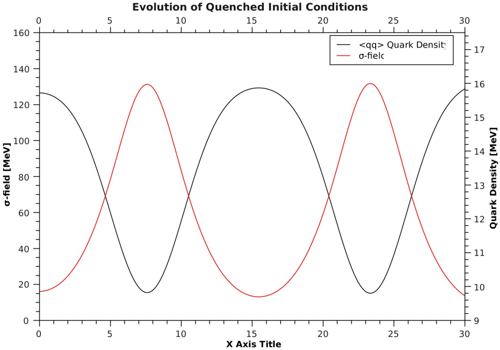


Thermal Blob Scenario



Non-Equilibrium Quench

- initialize system in equilibrium (e.g. $T = 160\ \mathrm{MeV}$)
- reinitilize quark energy and density (e.g. $T_q = 140\ \mathrm{MeV}$)
- no spatial gradients



- damping of collective behavior?
- chemical equilibration?

with $\nabla\sigma = 0$ and $\pi = 0$:

$$\partial_t\ \sigma(t) + \lambda^2\left(\sigma(t)^2 - \nu^2\right)\sigma(t) = -g\langle\bar{\psi}\psi\rangle + f_\pi m_\pi^2$$

for single-particle distribution-function:

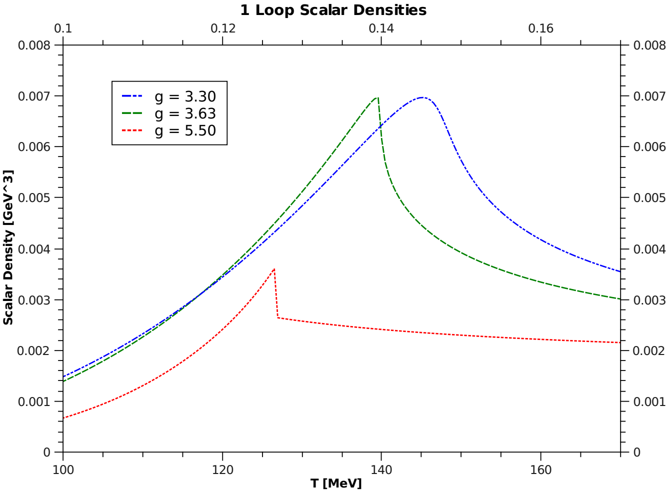
$$\begin{aligned}\langle\bar{\psi}\psi(\mathbf{r})\rangle &= g\sigma(\mathbf{r})\int d^3\mathbf{p}\ \frac{f(\mathbf{r},\mathbf{p}) + \tilde{f}(\mathbf{r},\mathbf{p})}{E(\mathbf{r},\mathbf{p})} \\ &= g\sigma(\mathbf{r})\,\langle n(\mathbf{r},T)\rangle\left\langle\frac{1}{E(\mathbf{r},T)}\right\rangle\end{aligned}$$

for massless fermi-gas:

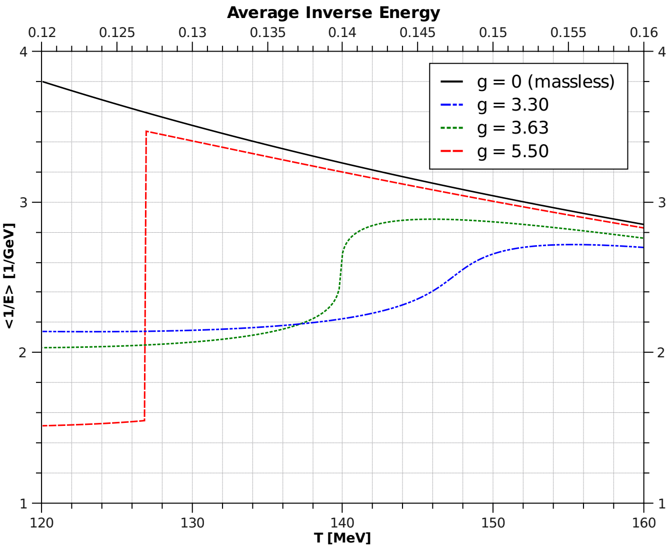
$$\langle n(T)\rangle = d_q\frac{3\ \zeta(3)}{4\pi^2}T^3 \qquad \left\langle\frac{1}{E(T)}\right\rangle = d_q\frac{\pi^2}{18\ \zeta(3)}T^{-1}$$

$$\langle n(T)\rangle\ \left\langle\frac{1}{E(T)}\right\rangle = \frac{1}{24}\frac{T^3_{\rm chem}}{T_{\rm therm}}$$

Non-Equilibrium effects of the density

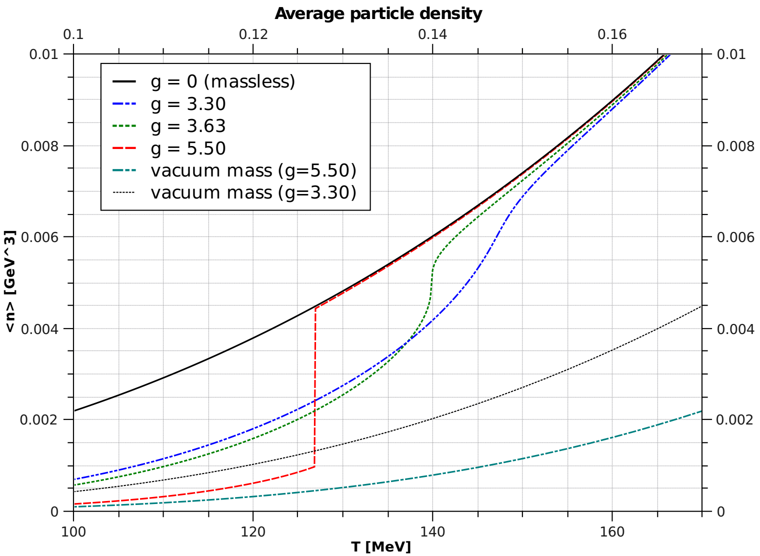


Equilibrium value



Equilibrium value

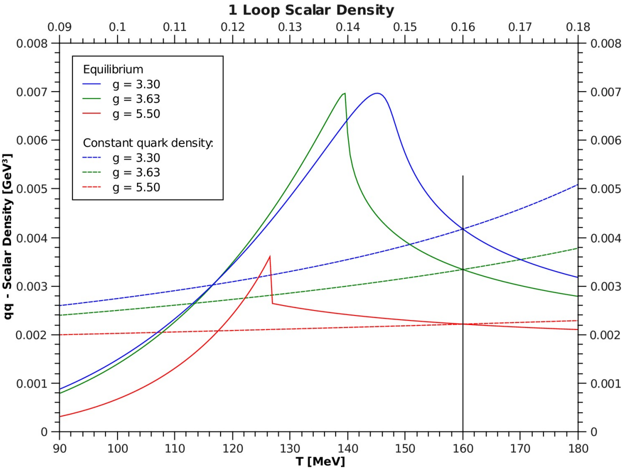
Non-Equilibrium effects of the density



Equilibrium value

Non-Equilibrium effects of the density

- toy scenario: $n = n_{T=160\ \mathrm{MeV}}$
- thermal equilibrium, but no particle production



Expansion scenario

- initial thermal blob
- cooling and density thinning by expansion
- slow expansion (σ in equilibrium)

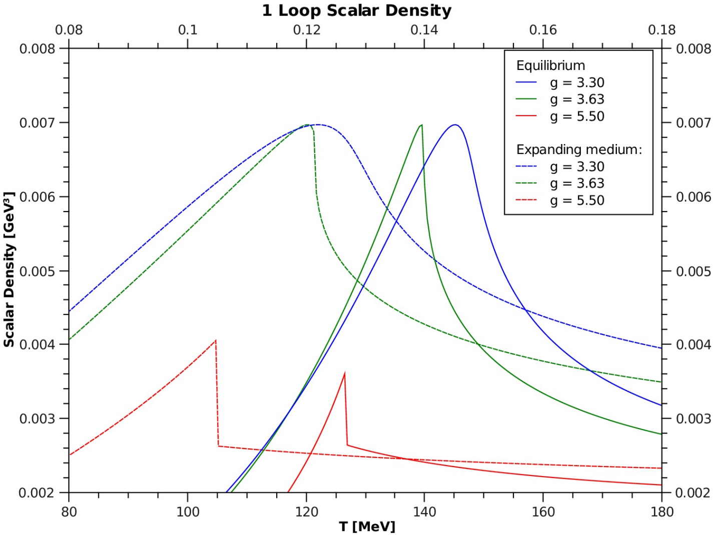
$$\begin{aligned} \text{no particle production:} \quad & n(t) \cdot V(t) &= n_0 \cdot V_0 \\ \text{adiabatic expansion:} \quad & T(t) V(t)^{\gamma-1} &= T_0 \; V_0^{\gamma-1} \end{aligned}$$

assuming an ideal gas: $\gamma = 5/3$

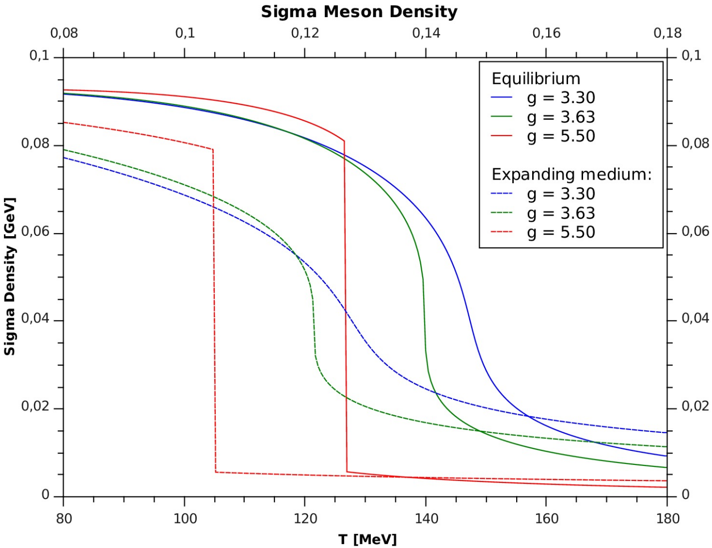
$$n(T) = n_0 \left(\frac{T}{T_0} \right)^{3/2}$$

Non-Equilibrium effects of the density

Temperature shift of phase transition



Temperature shift of phase transition

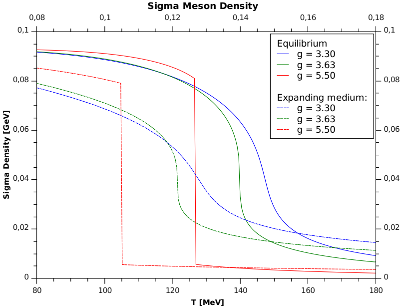


Discussion

- no phase transition in constant box scenario
- pseudo-phase transitions in expansion scenario

⇒ Non-equilibrium effects have huge impact on phase transition!

- temperature and shape of transition is shifted
- small density fluctuations can amplify sigma fluctuations
- what happens in real-time to the density?



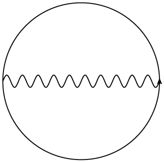
Outlook

Employ medium dependent

- binary interactions - thermal equilibration
- creation / annihilation processes - chemical equilibration
- Polyakov-loop potential - effective gluon background

Further investigation of

- non-equilibrium effects
- real-time effects
- finite time and size effects
- fluctuations



- Thanks for your attention!